YEAR 1 REVISION

1. Prove that if $ab > 0$, then $\frac{a}{b} + \frac{b}{a} \geqslant 2$.
2. Find all natural numbers n, for which $\frac{2n+5}{n+1}$ is also a natural number.
3. Prove that $2^1 + 2^2 + 2^3 + ... + 2^{88}$ is divisible by 3 and 5.
4. Prove that the fraction $\frac{6k+1}{15k+2}$ is irreducible for all integer values k.
5. Find all values of x for which the sum $|x-1| + |x+2|$ has the least value.
6. Solve the inequality $\sqrt{(2-x)^2} > 3$.
7. Find the value of k, for which the equation:
$$|x-6| + |x+1| = k$$
has two solutions. Find these solutions for $k = 10$.
8. Calculate $\log_7 16$, given that $\log_{14} 2 = a$.
9. Calculate $\log_{\sqrt{ab}} (\frac{a}{b})$, given that $\log_a b = 2$.
10. Show that $\log_3 12 \times (2 - \log_{12} 48)$ is an integer.
11. Prove that $3^{\log 2} - 2^{\log 3} = 0$.
12. Sketch the following functions:
    a) $f(x) = \max(4, |x+1|)$;
    b) $g(x) = \min(1, x^2)$;