Converse, inverse, contrapositive

In this presentation we will take a look at conditional statements of the form $p \rightarrow q$ and introduce the converse, the inverse and the contrapositive of such conditional statements.

Definition

Given a conditonal statement $p \rightarrow q$:

    the **converse** of this statement is $q \rightarrow p$,

    the **inverse** of this statement is $\neg p \rightarrow \neg q$,

    the **contrapositive** of this statement is $\neg q \rightarrow \neg p$.

Example 1

Consider the statement:

If it rains, then $2 + 2 = 4$.

This statement is of the form $p \implies q$, where:

p: it rains

q: $2 + 2 = 4$

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The **converse** of this statement will be $q \rightarrow p$, so it will be

If 2 + 2 = 4, then it rains.

The **inverse** of this statement will be $\neg p \rightarrow \neg q$, so it will be

If it doesn't rain, then 2 + 2 ≠ 4.

The **contrapositive** of this statement will be $\neg q \rightarrow \neg p$, so it will be

If 2 + 2 ≠ 4, then it doesn't rain.

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If $2 + 2 = 4$, then it rains.

The **inverse** of this statement will be $\neg p \rightarrow \neg q$, so it will be:

If it doesn't rain, then $2 + 2 \neq 4$.

The **contrapositive** of this statement will be $\neg q \rightarrow \neg p$, so it will be:

If $2 + 2 \neq 4$, then it doesn't rain.

Example 1

The **converse** of this statement will be $q \to p$, so it will be:

$$\text{If } 2 + 2 = 4, \text{ then it rains.}$$

The **inverse** of this statement will be $\neg p \to \neg q$, so it will be:

$$\text{If it doesn't rain, then } 2 + 2 \neq 4.$$

The **contrapositive** of this statement will be $\neg q \to \neg p$, so it will be:

$$\text{If } 2 + 2 \neq 4, \text{ then it doesn't rain.}$$

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Example 2

Consider the statement:

If a number is divisible by 4, then it is divisible by 2.

This statement is of the form $p \to q$, where:

p: a number is divisible by 4

q: a number is divisible by 2

Example 2

Consider the statement:

If a number is divisible by 4, then it is divisible by 2.

This statement is of the form $p \rightarrow q$, where:

p a number is divisible by 4

q a number is divisible by 2

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The **converse** of this statement will be $q \to p$, so it will be

If a number is divisible by 2, then it is divisible by 4.

The **inverse** of this statement will be $\neg p \to \neg q$, so it will be

If a number is not divisible by 4, then it is not divisible by 2.

The **contrapositive** of this statement will be $\neg q \to \neg p$, so it will be

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Exercise

Construct truth tables for the statement $p \rightarrow q$, its converse, its inverse and its contrapositive. Hence decide which pairs of statements are equivalent.