

Imię i nazwisko:

Klasa:

Grupa 1

Wynik:

Question 1 (1 pt)

$\cos(495^\circ)$ is equal to:

- A. $-\frac{\sqrt{3}}{2}$ B. $-\frac{\sqrt{2}}{2}$ C. $\frac{\sqrt{2}}{2}$ D. $\frac{\sqrt{3}}{2}$

Question 2 (1 pt)

If $\sin \alpha = \frac{2}{5}$ and α is an obtuse angle, then:

- A. $\cos \alpha = \frac{\sqrt{21}}{5}$ B. $\cos \alpha = -\frac{\sqrt{21}}{5}$ C. $\operatorname{tg} \alpha = \frac{\sqrt{21}}{5}$ D. $\operatorname{tg} \alpha = -\frac{\sqrt{21}}{5}$

Question 3 (1 pt)

The value of $\operatorname{tg} 35^\circ \times \operatorname{tg} 40^\circ \times \operatorname{tg} 45^\circ \times \operatorname{tg} 50^\circ \times \operatorname{tg} 55^\circ \times \operatorname{tg} 60^\circ$ is

- A. 0 B. $\frac{\sqrt{3}}{3}$ C. 1 D. $\sqrt{3}$

Question 4 (1 pt)

If the value of $\operatorname{tg} \alpha + \operatorname{ctg} \alpha = 5$, then $\operatorname{tg}^2 \alpha + \operatorname{ctg}^2 \alpha$ is equal to:

- A. 3 B. 23 C. 25 D. 27

Question 5 (1 pt)

In a triangle ABC , $|AB| = 10$, $|AC| = 7$ and $\cos \angle BAC = \frac{1}{5}$. The length of BC is equal to:

- A. 11 B. $\sqrt{149}$ C. 13 D. $\sqrt{177}$

Question 6 (3 pts)

Prove that if α is an acute angle, then:

$$\sqrt{\frac{1 + \sin \alpha}{1 - \sin \alpha}} + \sqrt{\frac{1 - \sin \alpha}{1 + \sin \alpha}} = \frac{2}{\cos \alpha}$$

$$\begin{aligned} LHS &= \sqrt{\frac{1 + \sin \alpha}{1 - \sin \alpha}} + \sqrt{\frac{1 - \sin \alpha}{1 + \sin \alpha}} = \\ &= \sqrt{\frac{(1 + \sin \alpha)^2}{1 - \sin^2 \alpha}} + \sqrt{\frac{(1 - \sin \alpha)^2}{1 - \sin^2 \alpha}} = \\ &= \sqrt{\frac{(1 + \sin \alpha)^2}{\cos^2 \alpha}} + \sqrt{\frac{(1 - \sin \alpha)^2}{\cos^2 \alpha}} = \\ &= \left| \frac{1 + \sin \alpha}{\cos \alpha} \right| + \left| \frac{1 - \sin \alpha}{\cos \alpha} \right| = \\ &= \frac{1 + \sin \alpha}{\cos \alpha} + \frac{1 - \sin \alpha}{\cos \alpha} = \\ &= \frac{2}{\cos \alpha} = RHS \end{aligned}$$

Question 7 (3 pts)

Given that α is acute and $\sin \alpha - \cos \alpha = \frac{3}{5}$, find the value of $\sin^3 \alpha - \cos^3 \alpha$.

$$\begin{aligned} \sin \alpha - \cos \alpha &= \frac{3}{5} \\ (\sin \alpha - \cos \alpha)^2 &= \frac{9}{25} \\ 1 - 2 \sin \alpha \cos \alpha &= \frac{9}{25} \\ \sin \alpha \cos \alpha &= \frac{8}{25} \end{aligned}$$

$$\sin^3 \alpha - \cos^3 \alpha = (\sin \alpha - \cos \alpha)^2 + 3 \sin \alpha \cos \alpha (\sin \alpha - \cos \alpha) = \frac{27}{125} + 3 \times \frac{8}{25} \times \frac{3}{5} = \frac{99}{125}$$

Question 8 (3 pts)

Prove that in an acute triangle with heights h_a and h_b and the angle γ , the area is given by the formula:

$$P = \frac{h_a \times h_b}{2 \sin \gamma}$$

We have $P_{\Delta} = \frac{a \times h_a}{2}$ and $\sin \gamma = \frac{h_b}{a}$. The second equation gives:
 $a = \frac{h_b}{\sin \gamma}$. The result follows.

Question 9 (3 pts)

Given a triangle ABC with side-lengths: $|AB| = 5, |AC| = 7$ and $|BC| = 8$, find the size of the angle $\angle ABC$.

$$\cos \beta = \frac{5^2 + 8^2 - 7^2}{2 \times 5 \times 8}$$

$$\cos \beta = \frac{1}{2}$$

$$\beta = 60^\circ$$

Question 10 (3 pts)

In an obtuse triangle ABC , $|AB| = 8$, $|AC| = 4\sqrt{6}$ and $\angle ACB = 45^\circ$. Find the size of the other two angles of the triangle.

$$\frac{8}{\sin 45^\circ} = \frac{4\sqrt{6}}{\sin \beta}$$

$$\sin \beta = \frac{\sqrt{3}}{2}$$

So $\beta = 60^\circ$ or $\beta = 120^\circ$. So $\alpha = 75^\circ$ or $\alpha = 15^\circ$. The first option gives an acute triangle, so: $\beta = 120^\circ$ and $\alpha = 15^\circ$