Imię i nazwisko:
Klasa:
Grupa 2
Wynik:

Question 1 ***(1 pt)***
$\cos(510°)$ is equal to:

A. $-\frac{\sqrt{3}}{2}$ B. $-\frac{\sqrt{2}}{2}$ C. $\frac{\sqrt{2}}{2}$ D. $\frac{\sqrt{3}}{2}$

Question 2 ***(1 pt)***
If $\sin\alpha = \frac{2}{5}$ and α is an acute angle, then:

A. $\cos\alpha = \frac{\sqrt{21}}{5}$ B. $\cos\alpha = -\frac{\sqrt{21}}{5}$ C. $\operatorname{tg}\alpha = \frac{\sqrt{21}}{5}$ D. $\operatorname{tg}\alpha = -\frac{\sqrt{21}}{5}$

Question 3 ***(1 pt)***
The value of $\operatorname{tg} 30° \times \operatorname{tg} 35° \times \operatorname{tg} 40° \times \operatorname{tg} 45° \times \operatorname{tg} 50° \times \operatorname{tg} 55°$ is

A. 0 B. $\frac{\sqrt{3}}{3}$ C. 1 D. $\sqrt{3}$

Question 4 ***(1 pt)***
If the value of $\operatorname{tg}\alpha - \operatorname{ctg}\alpha = 5$, then $\operatorname{tg}^2\alpha + \operatorname{ctg}^2\alpha$ is equal to:

A. 3 B. 23 C. 25 D. 27

Question 5 ***(1 pt)***
In a triangle ABC, $|AB| = 10$, $|AC| = 7$ and $\cos\angle BAC = -\frac{1}{5}$. The length of BC is equal to:

A. 11 B. $\sqrt{149}$ C. 13 D. $\sqrt{177}$

Question 6 ***(3 pts)***

Prove that if α is an acute angle, then:

$$\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}+\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}=\frac{2}{\sin\alpha}$$

$$\begin{aligned}
LHS &= \sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}+\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}= \\
&= \sqrt{\frac{(1+\cos\alpha)^2}{1-\cos^2\alpha}}+\sqrt{\frac{(1-\cos\alpha)^2}{1-\cos^2\alpha}}= \\
&= \sqrt{\frac{(1+\cos\alpha)^2}{\sin^2\alpha}}+\sqrt{\frac{(1-\cos\alpha)^2}{\sin^2\alpha}}= \\
&= \left|\frac{1+\cos\alpha}{\sin\alpha}\right|+\left|\frac{1-\cos\alpha}{\sin\alpha}\right|= \\
&= \frac{1+\cos\alpha}{\sin\alpha}+\frac{1-\cos\alpha}{\sin\alpha}= \\
&= \frac{2}{\sin\alpha}=RHS
\end{aligned}$$

Question 7 ***(3 pts)***

Given that α is acute and $\sin\alpha\times\cos\alpha=\frac{1}{4}$, find the value of $\sin^3\alpha+\cos^3\alpha$.

$$\begin{aligned}
(\sin\alpha+\cos\alpha)^2 &= \sin^2\alpha+2\sin\alpha\cos\alpha+\cos^2\alpha \\
(\sin\alpha+\cos\alpha)^2 &= \frac{3}{2} \\
\sin\alpha+\cos\alpha &= \frac{\sqrt{6}}{2}
\end{aligned}$$

$$\sin^3\alpha+\cos^3\alpha=(\sin\alpha+\cos\alpha)^3-3\sin\alpha\cos\alpha(\sin\alpha+\cos\alpha)=\frac{6\sqrt{6}}{8}-\frac{3\sqrt{6}}{8}=\frac{3\sqrt{6}}{8}$$

Question 8 ***(3 pts)***

Prove that in an acute triangle with heights h_a and h_b and the angle γ, the area is given by the formula:

$$P = \frac{h_a \times h_b}{2 \sin \gamma}$$

We have $P_{\triangle} = \dfrac{a \times h_a}{2}$ and $\sin \gamma = \dfrac{h_b}{a}$. The second equation gives: $a = \dfrac{h_b}{\sin \gamma}$. The result follows.

Question 9 ***(3 pts)***

Given a triangle ABC with side-lengths: $|AB| = 7$,$|AC| = 8$ and $|BC| = 5$, find the size of the angle $\angle ACB$.

$$\cos \gamma = \frac{5^2 + 8^2 - 7^2}{2 \times 5 \times 8}$$

$$\cos \gamma = \frac{1}{2}$$

$$\gamma = 60^\circ$$

Question 10 ***(3 pts)***

In an obtuse triangle ABC, $|AB| = 4$, $|AC| = 2\sqrt{6}$ and $\angle ACB = 45°$. Find the size of the other two angles of the triangle.

$$\frac{4}{\sin 45°} = \frac{2\sqrt{6}}{\sin \beta}$$

$$\sin \beta = \frac{\sqrt{3}}{2}$$

So $\beta = 60°$ or $\beta = 120°$. So $\alpha = 75°$ or $\alpha = 15°$. The first option gives an acute triangle, so: $\beta = 120°$ and $\alpha = 15°$