Tautology, contradiction, equivalence

Tomasz		

In this presentation we will practice checking if a compound proposition is **tautology/contradiction** and if two statements are **equivalent**.

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Check if the following proposition is a tautology, contradiction or neither. $(p ightarrow q) ightarrow (\neg p \lor q)$

We need to construct the truth table for this proposition. There are 2 simple sentences involved *p* and *q*, so the table will have four rows.

We will need columns for $p, q, p \rightarrow q, \neg p, \neg p \lor q$ and finally $(p \rightarrow q) \rightarrow (\neg p \lor q)$.

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$$p$$
 q $p \rightarrow q$ $\neg p$ $\neg p \lor q$ $(p \rightarrow q) \rightarrow (\neg p \lor q)$ TTTTTFFFTFFFF

The last columns has Ts only, so the statement is always true i.e. it is a tautology.

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Check if the following compound statement is a tautology, contradiction or neither.

 $(p \land q) \lor (r \to \neg q)$

We have three simple statements involved in this proposition: *p*,*q* and *r*. So our table will have eight rows.

We need the following columns: p, q, r and then also $p \land q$, $\neg q$, $r \rightarrow \neg q$ and the statement we want to check $(p \land q) \lor (r \rightarrow \neg q)$

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Check if the following compound statement is a tautology, contradiction or neither.

$$(p \wedge q) \lor (r o
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The truth table will look as follows. Again try to complete a column and then move to the next slide to check your answers.

р	q	r	$p \wedge q$	$\neg q$	$r ightarrow \neg q$	$(p \wedge q) \lor (r o eg q)$
Т	Т	Т	Т	F	F	Т
Т	Т	F	Т			
Т	F	Т	F			
Т	F	F	E	T		Т
F	Т	Т	E	E		F
F	Т	F	E	E		Т
F	F	Т	E	T		Т
F	F	F	F	Т		Т

The truth table will look as follows. Again try to complete a column and then move to the next slide to check your answers.

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Т	Т	Т	Т	F	F	Т
Т	Т	F	Т			
Т	F	Т	F	T		Т
Т	F	F	F	T		Т
F	Т	Т	F	E		
F	Т	F	F	E		Т
F	F	Т	F	T		Т
F	F	F	F	Т		Т

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Т	Т	Т	Т	F	F	Т
Т	Т	F	Т	F	T	
Т	F	Т	F	Т	Т	
Т	F	F	F	Т	Т	
F	Т	Т	F	F	F	
F	Т	F	F	F	Т	
F	F	Т	F	Т	Т	
F	F	F	F	Т	Т	Т

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Т	Т	Т	Т	F	F	Т
Т	Т	F	Т	F	Т	
Т	F	Т	F	Т	Т	
Т	F	F	F	Т	Т	Т
F	Т	Т	F	F	F	F
F	Т	F	F	F	Т	Т
F	F	Т	F	Т	Т	Т
F	F	F	F	Т	Т	Т

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Т	Т	Т	Т	F	F	Т
Т	Т	F	Т	F	Т	
Т	F	Т	F	Т	Т	Т
Т	F	F	F	Т	Т	Т
F	Т	Т	F	F	F	F
F	Т	F	F	F	Т	T
F	F	Т	F	Т	Т	
F	F	F	F	Т	Т	Т

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Т	Т	Т	Т	F	F	Т
Т	Т	F	Т	F	Т	Т
Т	F	Т	F	Т	Т	
Т	F	F	F	Т	Т	Т
F	Т	Т	F	F	F	F
F	Т	F	F	F	Т	Т
F	F	Т	F	Т	Т	Т
F	F	F	F	Т	Т	Т

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Т	Т	Т	Т	F	F	Т
Т	Т	F	Т	F	Т	Т
Т	F	Т	F	T	Т	Т
Т	F	F	F	Т	Т	Т
F	Т	Т	F	F	F	F
F	Т	F	F	F	Т	Т
F	F	Т	F	Т	Т	Т
F	F	F	F	T	Т	Т

The truth table will look as follows. Again try to complete a column and then move to the next slide to check your answers.

p	q	r	$p \wedge q$	$ \neg q$	$r ightarrow \neg q$	$(p \wedge q) \lor (r o eg q)$
Т	Т	Т	Т	F	F	Т
Т	Т	F	Т	F	Т	Т
Т	F	Т	F	Т	Т	Т
Т	F	F	F	Т	Т	Т
F	Т	Т	F	F	F	
F	Т	F	F	F	Т	
F	F	Т	F	Т	Т	Т
F	F	F	F	Т	Т	Т

The truth table will look as follows. Again try to complete a column and then move to the next slide to check your answers.

p	q	r	$p \wedge q$	$ \neg q$	$r ightarrow \neg q$	$(p \wedge q) \lor (r o eg q)$
Т	Т	Т	Т	F	F	Т
Т		F		1	Т	Т
Т	F	Т	F	Т	Т	Т
Т	F	F	F	Т	Т	Т
F	Т	Т	F	F	F	F
F	Т	F	F	F	Т	
F	F	Т	F	Т	Т	Т
F	F	F	F	Т	Т	Т

The truth table will look as follows. Again try to complete a column and then move to the next slide to check your answers.

p	q	r	$p \wedge q$	$ \neg q$	$r ightarrow \neg q$	$(p \wedge q) \lor (r o eg q)$
Т	Т	Т	Т		F	Т
Т	Т	F	Т	F	Т	Т
Т	F	Т	F	Т	Т	Т
Т	F	F	F	Т	Т	Т
F	Т	Т	F	F	F	F
F	Т	F	F	F	Т	Т
F	F	Т	F	Т	Т	Т
F	F	F	F	Т	Т	Т

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Т	Т	Т	Т		F	Т
Т		F			Т	Т
Т	F	Т	F	Т	Т	Т
Т	F	F	F	Т	Т	Т
F	Т	Т	F	F	F	F
F	Т	F	F	F	Т	Т
F	F	Т	F	Т	Т	Т
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Т	Т		Т		F	Т
Т	Т	F	Т	F	Т	Т
Т	F	Т	F	Т	Т	Т
Т	F	F	F	Т	Т	Т
F	Т	Т	F	F	F	F
F	T	F	F	F	Т	Т
F	F	Т	F	Т	Т	Т
F	F	F	F F	Т	Т	Т



The compound statement $(p \land q) \lor (r \rightarrow \neg q)$ is neither a tautology nor a contradiction.

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Check if the statement $(p \lor \neg p) \rightarrow (q \land \neg q)$ is a tautology, a contradiction or neither.

We have 2 simple statements involved: p and q, so we will have 4 rows. We need columns for p, q, $\neg p$, $p \lor \neg p$, $\neg q$, $q \land \neg q$ and $(p \lor \neg p) \rightarrow (q \land \neg q)$

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$$p$$
 q $\neg p$ $p \lor \neg p$ $\neg q$ $q \land \neg q$ $(p \lor \neg p) \rightarrow (q \land \neg q)$ TTFTTFFTFFTFTTFFT

$$p$$
 q $\neg p$ $p \lor \neg p$ $\neg q$ $q \land \neg q$ $(p \lor \neg p) \rightarrow (q \land \neg q)$ TTFTFFTFFTTFFTTFFFTTFFFFTTFFFTTF

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$$p$$
 q $\neg p$ $p \lor \neg p$ $\neg q$ $q \land \neg q$ $(p \lor \neg p) \rightarrow (q \land \neg q)$ TTFTFFTFFTTFFFTTFFFTTFFFFTTFFFTTF

$$p$$
 q $\neg p$ $p \lor \neg p$ $\neg q$ $q \land \neg q$ $(p \lor \neg p) \rightarrow (q \land \neg q)$ TTFFFFTFFFFFTFFFFFFTTFFFFTTFFFTTF

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Example 3

The statement $(p \lor \neg p) \rightarrow (q \land \neg q)$ is always false, so it is a contradiction.

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Check if the statements $\neg(p \land q)$ and $\neg p \lor \neg q$ are equivalent.

We need to construct truth table for both statements. We will try to do it in one table.

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Check if the statements $\neg(p \land q)$ and $\neg p \lor \neg q$ are equivalent.

We need to construct truth table for both statements. We will try to do it in one table.

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р	q	$p \wedge q$	$ eg(p \wedge q)$	$\neg p$	eg q	$ eg p \lor eg q$
Т	Т		F	F	F	F
Т	F	F	Т	-		
F	Т	E	Т	Т		
F	F	F	Т	T		Т

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$$p$$
 q $p \land q$ $\neg (p \land q)$ $\neg p$ $\neg q$ $\neg p \lor \neg q$ TTTTFFFTFFFF

$$p$$
 q $p \land q$ $\neg (p \land q)$ $\neg p$ $\neg q$ $\neg p \lor \neg q$ TTTFTFFTFTFTFFFTFFFT

$$p$$
 q $p \land q$ $\neg (p \land q)$ $\neg p$ $\neg q$ $\neg p \lor \neg q$ TTTFFITFFTFIFTFTTIFFFTTIFFFTTI

$$p$$
 q $p \land q$ $\neg (p \land q)$ $\neg p$ $\neg q$ $\neg p \lor \neg q$ TTTFFFTFFTFTFTFTFTFTFTTFFFFTTT

$$p$$
 q $p \land q$ $\neg (p \land q)$ $\neg p$ $\neg q$ $\neg p \lor \neg q$ TTTFFFFTFFTFTTFTFTTFTFTFTTTFFFFTTT

Example 4

The columns for $\neg(p \land q)$ and $\neg p \lor \neg q$ are identical, so the two statements are equivalent.

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