Tautology, contradiction, equivalence

In this presentation we will practice checking if a compound proposition is **tautology/contradiction** and if two statements are **equivalent**.

Example 1

Check if the following proposition is a tautology, contradiction or neither.

$$(p \to q) \to (\neg p \vee q)$$

We need to construct the truth table for this proposition. There are 2 simple sentences involved p and q, so the table will have four rows.

We will need columns for p, q, $p \to q$, $\neg p$, $\neg p \vee q$ and finally $(p \to q) \to (\neg p \vee q)$.

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The truth table will look as follows. Try to complete a column and then move to the next slide to check your answers.

p	q	$p \to q$	$\neg p$	$\neg p \vee q$	$(p \to q) \to (\neg p \vee q)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

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F	T	T	T	T	T
F	F	T	T	T	T

The last columns has Ts only, so the statement is always true i.e. it is a tautology.

Example 2

Check if the following compound statement is a tautology, contradiction or neither.

$$(p \wedge q) \vee (r \rightarrow \neg q)$$

We have three simple statements involved in this proposition: p,q and r. So our table will have eight rows.

We need the following columns: p, q, r and then also $p \wedge q$, $\neg q$, $r \rightarrow \neg q$ and the statement we want to check $(p \wedge q) \vee (r \rightarrow \neg q)$

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p	q	r	$p \wedge q$	$\neg q$	$r \to \neg q$	$(p \wedge q) \vee (r \to \neg q)$
T	T	T	T	F	F	T
T	T	F	T	F	T	T
T	F	T	F	T	T	T
T	F	F	F	T	T	T
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F	T	F	F	F	T	T
F	F	T	F	T	T	T
F	F	F	F	T	T	T

Example 2

The compound statement $(p \wedge q) \vee (r \rightarrow \neg q)$ is neither a tautology nor a contradiction.

Example 3

Check if the statement $(p \vee \neg p) \rightarrow (q \wedge \neg q)$ is a tautology, a contradiction or neither.

We have 2 simple statements involved: p and q, so we will have 4 rows.
We need columns for p, q, $\neg p$, $p \vee \neg p$, $\neg q$, $q \wedge \neg q$ and $(p \vee \neg p) \rightarrow (q \wedge \neg q)$

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p	q	$\neg p$	$p \vee \neg p$	$\neg q$	$q \wedge \neg q$	$(p \vee \neg p) \rightarrow (q \wedge \neg q)$
T	T	F	T	F	F	F
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F	T	T	T	F	F	F
F	F	T	T	T	F	F

Example 3

The statement $(p \vee \neg p) \rightarrow (q \wedge \neg q)$ is always false, so it is a contradiction.

Example 4

Check if the statements $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are equivalent.

We need to construct truth table for both statements. We will try to do it in one table.

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T	T	T	F	F	F	F
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The truth table will look as follows. Try to complete a column and then move to the next slide to check your answers.

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
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T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Example 4

The columns for $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are identical, so the two statements are equivalent.