Truth tables

In this presentation we will go through a few examples of truth tables for compound statements and we will introduce the notion of **tautology**.

Example 1

We want to construct the truth table for the proposition:

$$(p \wedge q) \rightarrow (\neg p \vee \neg q)$$

The first observation is that there are two simple statements involved in this proposition, namely p and q. So our table will have four rows.

The second observation is that apart for columns for p and q and our proposition $(p \wedge q) \rightarrow (\neg p \vee \neg q)$, we also need columns for: $p \wedge q$, $\neg p$, $\neg q$ and $\neg p \vee \neg q$.

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We want to construct the truth table for the proposition:

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The second observation is that apart for columns for p and q and our proposition $(p \wedge q) \to (\neg p \vee \neg q)$, we also need columns for: $p \wedge q$, $\neg p$, $\neg q$ and $\neg p \vee \neg q$.

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The truth table will look as follows. Try to complete a column and then move to the next slide to check your answers.

p	q	$p \wedge q$	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$(p \wedge q) \rightarrow (\neg p \vee \neg q)$
T	T					
T	F					
F	T					
F	F					

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T	F	F	F	T	T	T
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F	F	F	T	T	T	T

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T	T	T	F			
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T	F	F	F	T	T	T
F	T	F	T	F	T	T
F	F	F	T	T	T	T

Example 2

Now we want to construct the truth table for the proposition:

$$(p \vee q) \vee (\neg r \wedge \neg q)$$

This time we have three simple statements involved in this proposition: p,q and r. So our table will have eight rows.

We need the following columns: p, q, r and then also $p \vee q$, $\neg r$, $\neg q$, $\neg r \wedge \neg q$ and finally column for our proposition $(p \vee q) \vee (\neg r \wedge \neg q)$

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The truth table will look as follows. Again try to complete a column and then move to the next slide to check your answers.

p	q	r	$p \vee q$	$\neg r$	$\neg q$	$\neg r \wedge \neg q$	$(p \vee q) \vee (\neg r \wedge \neg q)$
T	T	T	T	F	F	F	T
T	T	F	T	T	F	F	T
T	F	T	T	F	T	F	T
T	F	F	T	T	T	T	T
F	T	T	T	F	F	F	T
F	T	F	T	T	F	F	T
F	F	T	F	F	T	F	F
F	F	F	F	T	T	T	T

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T	T	F	T	T	F	F	T
T	F	T	T	F	T	F	T
T	F	F	T	T	T	T	T
F	T	T	T	F	F	F	T
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F	F	T	F	F	T	F	F
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T	T	T	T	F	F	F	T
T	T	F	T	T	F	F	T
T	F	T	T	F	T	F	T
T	F	F	T	T	T	T	T
F	T	T	T	F	F	F	T
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T	T	T	T	F	F		
T	T	F	T	T	F		
T	F	T	T	F	T		
T	F	F	T	T	T		
F	T	T	T	F	F		
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T	T	F	T	T	F	F	
T	F	T	T	F	T	F	
T	F	F	T	T	T	T	
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T	T	F	T	T	F	F	
T	F	T	T	F	T	F	
T	F	F	T	T	T	T	
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T	T	F	T	T	F	F	T
T	F	T	T	F	T	F	T
T	F	F	T	T	T	T	T
F	T	T	T	F	F	F	T
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T	F	T	T	F	T	F	T
T	F	F	T	T	T	T	
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T	T	F	T	T	F	F	T
T	F	T	T	F	T	F	T
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F	T	T	T	F	F	F	T
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F	T	T	T	F	F	F	T
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F	T	T	T	F	F	F	T
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F	T	T	T	F	F	F	T
F	T	F	T	T	F	F	T
F	F	T	F	F	T	F	F
F	F	F	F	T	T	T	T

Tautology

Definition

A statement is a tautology if it is **always** true, i.e. in the truth table the column for this statement contains only truth (T).

Example 3

Check if the statement $p \to (p \lor q)$ is a tautology.

We need to construct a truth table for this statement and check if the last column contains only Ts.

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We need to construct a truth table for this statement and check if the last column contains only Ts.

Example 3

The truth table will look as follows.

p	q	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

The statement $p \rightarrow (p \vee q)$ is always true, so it is a tautology.

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The truth table will look as follows.

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T	T	T	T
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T	T	T	T
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T	T	T	T
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T	T	T	T
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Check if the statement $p \to (p \wedge q)$ is a tautology.

Again we need to construct a truth table for this statement and check if the last column contains only Ts.

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Check if the statement $p \to (p \land q)$ is a tautology.

Again we need to construct a truth table for this statement and check if the last column contains only Ts.

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The truth table will look as follows.

p	q	$p \wedge q$	$p \rightarrow (p \wedge q)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

The statement $p \rightarrow (p \vee q)$ is not always true (the second row shows F), so it is **not** a tautology.

Example 4

The truth table will look as follows.

p	q	$p \wedge q$	$p \to (p \wedge q)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

The statement $p \to (p \vee q)$ is not always true (the second row shows F), so it is **not** a tautology.

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p	q	$p \wedge q$	$p \rightarrow (p \wedge q)$
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The statement $p \rightarrow (p \vee q)$ is not always true (the second row shows F), so it is **not** a tautology.

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T	F	F	F
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The statement $p \rightarrow (p \vee q)$ is not always true (the second row shows F), so it is **not** a tautology.

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p	q	$p \wedge q$	$p \to (p \wedge q)$
T	T	T	T
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The statement $p \to (p \vee q)$ is not always true (the second row shows F), so it is **not** a tautology.

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The statement $p \to (p \vee q)$ is not always true (the second row shows F), so it is **not** a tautology.

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The truth table will look as follows.

p	q	$p \wedge q$	$p \to (p \wedge q)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

The statement $p \to (p \vee q)$ is not always true (the second row shows F), so it is **not** a tautology.

Remember: a statement is a tautology if it is always true, so it has to have all Ts. If it has at least one F, then it is not a tautology.