Truth tables

In this presentation we will go through a few examples of truth tables for compound statements and we will introduce the notion of **tautology**.

We want to construct the truth table for the proposition:

$$(p \land q) \rightarrow (\neg p \lor \neg q)$$

The first observation is that there are two simple statements involved in this proposition, namely p and q. So our table will have four rows.

The second observation is that apart for columns for p and q and our proposition $(p \land q) \rightarrow (\neg p \lor \neg q)$, we also need columns for: $p \land q$, $\neg p$, $\neg q$

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The second observation is that apart for columns for p and q and our proposition $(p \land q) \to (\neg p \lor \neg q)$, we also need columns for: $p \land q$, $\neg p$, $\neg q$ and $\neg p \lor \neg q$.

p	q	$p \wedge q$	$\neg p$	$ \neg q $	$\neg p \lor \neg q$	$\mid (p \wedge q) ightarrow (\lnot p \lor \lnot q)$
T	Т	T	F	F	F	F
Т	F	F	F	T	T	T
F	T	F	T	F	T	T
F	F	F	Т	T	T	T

p	q	$p \wedge q$	$\neg p$	$ \neg q $	$\neg p \lor \neg q$	$(p \wedge q) ightarrow (eg p ee eg q)$
T	Т	Т	F	F	F	F
Т	F	F	F	T	Т	T
F	T	F	T	F	Т	T
F	F	F	Т	T	T	T

p	q	$p \wedge q$	$\neg p$	$ \neg q $	$\neg p \lor \neg q$	$\mid (p \wedge q) ightarrow (\lnot p \lor \lnot q)$
T	Т	Т	F	F	F	F
Т	F	F	F	T	T	T
F	T	F	Т	F	T	T
F	F	F	Т	T	T	T

p	q	$p \wedge q$	$\neg p$	$ \neg q$	$\neg p \lor \neg q$	$\mid (p \wedge q) ightarrow (\lnot p \lor \lnot q)$
Т	Т	Т	F	F	F	F
Τ	F	F	F	T	T	T
F	Т	F	Т	F	T	T
F	F	F	Т	Т	T	T

p	q	$p \wedge q$	$\neg p$	$ \neg q $	$\neg p \lor \neg q$	$\mid (p \wedge q) ightarrow (\lnot p \lor \lnot q)$
T	Т	Т	F	F	F	F
Т		F	F	Т	Т	T
F	T	F	Т	F	Т	T
F	F	F	Т	Т	Т	T

p	q	$p \wedge q$	$\neg p$	$ \neg q $	$\neg p \lor \neg q$	$(p \wedge q) o (\lnot p \lor \lnot q)$
Т	Т	Т	F	F	F	F
Τ	F	F F	F	Т	Т	
F	T	F	Т	F	Т	
F	F	F	Т	Т	Т	

p	q	$p \wedge q$	$\neg p$	$ \neg q $	$\neg p \lor \neg q$	$(p \wedge q) o (\lnot p \lor \lnot q)$
Т	Т	Т	F	F	F	F
Τ	F	F F	F	Т	Т	Т
F	T	F	Т	F	Т	
F	F	F	Т	Т	Т	

p	q	$p \wedge q$	$\neg p$	$ \neg q $	$\neg p \lor \neg q$	$(p \wedge q) o (\lnot p \lor \lnot q)$
Т	Т	Т	F	F	F	F
Τ	F	F	F	Т	Т	Т
F	T	F	Т	F	T T	Т
F	F	F	Т	Т	Т	Т

p	q	$p \wedge q$	$\neg p$	$ \neg q $	$\neg p \lor \neg q$	$\mid (p \wedge q) ightarrow (\lnot p \lor \lnot q)$
T	Т	Т	F	F	F	F
Т	F	F F	F	Т	Т	Т
F	T	F	Т	F	T	Т
F	F	F	Т	Т	T	Т

Now we want to construct the truth table for the proposition:

$$(p \vee q) \vee (\neg r \wedge \neg q)$$

This time we have three simple statements involved in this proposition:: p,q and r. So our table will have eight rows.

We need the following columns: $p,\ q,\ r$ and then also $p \lor q,\ \neg r,\ \neg q,$ $\neg r \land \neg q$ and finally column for our proposition $(p \lor q) \lor (\neg r \land \neg q)$

Now we want to construct the truth table for the proposition:

$$(p \lor q) \lor (\neg r \land \neg q)$$

This time we have three simple statements involved in this proposition: p,q and r. So our table will have eight rows.

Now we want to construct the truth table for the proposition:

$$(p \vee q) \vee (\neg r \wedge \neg q)$$

This time we have three simple statements involved in this proposition: p,q and r. So our table will have eight rows.

We need the following columns: p, q, r and then also $p \lor q$, $\neg r$, $\neg q$, $\neg r \land \neg q$ and finally column for our proposition $(p \lor q) \lor (\neg r \land \neg q)$

The truth table will look as follows. Again try to complete a column and then move to the next slide to check your answers.

р	q	r	$p \lor q$	$\neg r$	$\neg q$	$\neg r \land \neg q$	$(p \lor q) \lor (\neg r \land \neg q)$
Т	Т	Т	Т	F	F	F	T
Т	Т	F	T	Т		F	T
Т	F	Т	T	F		F	T
Т	F	F	T	Т		T	T
F	Т	Т	T	F		F	T
F	Т	F	T	T		F	T
F	F	Т	F	F		F	F
F	F	F	F	Т		T	T

р	q	r	$p \lor q$	$\neg r$	$\neg q$	$\neg r \land \neg q$	$(p \vee q) \vee (\neg r \wedge \neg q)$
Т	Т	Т	Т	F	F	F	T
Т	Т	F	Т	T	F	F	T
Т	F	Т	Т	F	T	F	T
Т	F	F	Т	T	T	T	T
F	Т	Т	Т	F	F	F	T
F	Т	F	Т	T	F	F	T
F	F	Т	F	F	T	F	F
F	F	F	F	T	T	T	Т

p	q	r	$p \lor q$	$\neg r$	$\neg q$	$\neg r \land \neg q$	$ \mid (p \lor q) \lor (\neg r \land \neg q) $
Т	Т	Т	Т	F	F	F	T
Т	Т	F	Т	Т	F	F	T
Т	F	Т	Т	F	T	F	T
Т	F	F	Т	Т	T	T	T
F	Т	Т	Т	F	F	F	T
F	Т	F	Т	Т	F	F	T
F	F	Т	F	F	T	F	F
F	F	F	F	Т	T	T	T

p	q	r	$p \lor q$	$\neg r$	$\neg q$	$\neg r \wedge \neg q$	$(p \lor q) \lor (\neg r \land \neg q)$
Т	Т	Т	Т	F	F	F	T
Т	Т	F	T	Т	F	F	T
Т	F	Т	Т	F	Т	F	T
Т	F	F	Т	Т	Т	T	T
F	Т	Т	Т	F	F	F	T
F	Т	F	Т	Т	F	F	T
F	F	Т	F	F	Т	F	F
F	F	F	F	Т	T	T	T

р	q	r	$p \lor q$	$\neg r$	$\neg q$	$\neg r \wedge \neg q$	$(p \lor q) \lor (\neg r \land \neg q)$
Т	Т	Т	Т	F	F	F	T
Т	Т	F	Т	Т	F	F	T
Т	F	Т	Т	F	Т	F	T
Т	F	F	Т	Т	Т	Т	Т
F	Т	Т	Т	F	F	F	T
F	Т	F	Т	Т	F	F	Т
F	F	Т	F	F	Т	F	F
F	F	F	F	Т	Т Т	Т	T

p	q	r	$p \lor q$	$\neg r$	$\neg q$	$\neg r \wedge \neg q$	$(p \lor q) \lor (\neg r \land \neg q)$
Т	Т	Т	Т	F	F	F	Т
Т	Т	F	Т	Т	F	F	T
Т	F	Т	Т	F	Т	F	T
Т	F	F	Т	Т	Т	Т	T
F	Т	Т	Т	F	F	F	T
F	Т	F	Т	Т	F	F	T
F	F	Т	F	F	Т	F	F
F	F	F	F	Т	Т	Т	T

p	q	r	$p \lor q$	$\neg r$	$\neg q$	$\neg r \land \neg q$	$(p \lor q) \lor (\neg r \land \neg q)$
Т	Т	Т	Т	F	F	F	Т
Т	Т	F	Т	Т	F	F	Т
Т	F	Т	Т	F	Т	F	T
Т	F	F	Т	Т	Т	Т	T
F	Т	Т	Т	F	F	F	T
F	Т	F	Т	Т	F	F	T
F	F	Т	F	F	Т	F	F
F	F	F	F	Т	Т	Т	T

p	q	r	$p \lor q$	$\neg r$	$\neg q$	$\neg r \land \neg q$	$(p \lor q) \lor (\neg r \land \neg q)$
Т	Т	Т	Т	F	F	F	Т
Т	Т	F	Т	Т	F	F	Т
Т	F	Т	Т	F	Т	F	Т
Т	F	F	Т	Т	Т	Т	T
F	Т	Т	Т	F	F	F	T
F	Т	F	Т	Т	F	F	T
F	F	Т	F	F	Т	F	F
F	F	F	F	Т	Т	T	T

р	q	r	$p \lor q$	$\neg r$	$\neg q$	$\neg r \wedge \neg q$	$ \mid (p \lor q) \lor (\neg r \land \neg q) $
Т	Т	Т	Т	F	F	F	Т
Т	Т	F	Т	Т	F	F	T
Т	F	Т	Т	F	Т	F	Т
Т	F	F	Т	Т	Т	Т	Т
F	Т	Т	Т	F	F	F	T
F	Т	F	Т	Т	F	F	T
F	F	Т	F	F	Т	F	F
F	F	F	F	Т	T	T	T

р	q	r	$p \lor q$	$\neg r$	$\neg q$	$\neg r \wedge \neg q$	$ \mid (p \lor q) \lor (\neg r \land \neg q) $
Т	Т	Т	Т	F	F	F	Т
Т	Т	F	Т	Т	F	F	T
Т	F	Т	Т	F	Т	F	Т
Т	F	F	Т	Т	Т	Т	Т
F	Т	Т	Т	F	F	F	Т
F	Т	F	Т	Т	F	F	T
F	F	Т	F	F	Т	F	F
F	F	F	F	Т	T	T	T

p	q	r	$p \lor q$	$\neg r$	$\neg q$	$\neg r \land \neg q$	$(p \lor q) \lor (\neg r \land \neg q)$
Т	Т	Т	Т	F	F	F	Т
Т	Т	F	Т	Т	F	F	Т
Т	F	Т	Т	F	Т	F	Т
Т	F	F	Т	Т	Т	Т	Т
F	Т	Т	Т	F	F	F	Т
F	Т	F	Т	Т	F	F	Т
F	F	T	F	F	T	F	F
F	F	F	F	Т	Т	Т	T

р	q	r	$p \lor q$	$\neg r$	$\neg q$	$\neg r \wedge \neg q$	$ \mid (p \lor q) \lor (\neg r \land \neg q) $
Т	Т	Т	Т	F	F	F	Т
Т	Т	F	Т	Т	F	F	T
Τ	F	Т	Т	F	Т	F	Т
Τ	F	F	Т	Т	Т	Т	Т
F	Т	Т	Т	F	F	F	Т
F	Т	F	Т	Т	F	F	Т
F	F	Т	F	F	Т	F	F
F	F	F	F	Т	T	Т	T

р	q	r	$p \lor q$	$\neg r$	$\neg q$	$\neg r \wedge \neg q$	$(p \lor q) \lor (\neg r \land \neg q)$
Т	Т	Т	Т	F	F	F	Т
Т	Т	F	Т	Т	F	F	Т
Т	F	Т	Т	F	Т	F	Т
Т	F	F	Т	Т	Т	Т	Т
F	Т	Т	Т	F	F	F	Т
F	Т	F	Т	Т	F	F	Т
F	F	Т	F	F	Т	F	F
F	F	F	F	Т	Т	Т	Т

Tautology

Definition

A statement is a tautology if it is **always** true, i.e. in the truth table the column for this statement contains only truth (T).

Check if the statement $p \to (p \lor q)$ is a tautology.

We need to construct a truth table for this statement and check if the last column contains only Ts.

Check if the statement $p o (p \lor q)$ is a tautology.

We need to construct a truth table for this statement and check if the last column contains only Ts.

The truth table will look as follows.

р	q	$p \lor q$	$\mid p ightarrow (p ee q)$
Т	Т	T	T
Т	F	T	T
F	Т		T
F	F	F	T

The statement $p \to (p \lor q)$ is always true, so it is a tautology.

The truth table will look as follows.

р	q	$p \lor q$	p o (p ee q)
Т	Т	Т	Т
Т	F	T	Т
F	Т	Т	Т
F	F	F	Т

I he statement ho
ightarrow (
ho ee g) is always true, so it is a tautology.

The truth table will look as follows.

p	q	$p \lor q$	ho o (p ee q)
Т	Т	Т	Т
Т	F	T	T
F	Т	Т	T
F	F	F	Т

The statement $p \to (p \lor q)$ is always true, so it is a tautology.

The truth table will look as follows.

р	q	$p \lor q$	$p \to (p \vee q)$
Т	Т	Т	Т
Т	F	T	Т
F	Т	Т	Т
F	F	F	Т

The statement $p \to (p \lor q)$ is always true, so it is a tautology.

The truth table will look as follows.

р	q	$p \lor q$	p o (p ee q)
Т	Т	Т	Т
Т	F	T	Т
F	Т	Т	Т
F	F	F	Т

The statement $p \to (p \lor q)$ is always true, so it is a tautology.

The truth table will look as follows.

р	q	$p \lor q$	p o (p ee q)
Т	Т	Т	Т
Т	F	T	Т
F	Т	Т	Т
F	F	F	Т

The statement $p \to (p \lor q)$ is always true, so it is a tautology.

The truth table will look as follows.

p	q	$p \lor q$	ho o (p ee q)
Т	Т	Т	Т
Т	F	T	Т
F	Т	Т	Т
F	F	F	Т

The statement $p \to (p \lor q)$ is always true, so it is a tautology.

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Check if the statement $p o (p \wedge q)$ is a tautology.

Again we need to construct a truth table for this statement and check if the last column contains only Ts.

Check if the statement $p o (p \wedge q)$ is a tautology.

Again we need to construct a truth table for this statement and check if the last column contains only Ts.

The truth table will look as follows.

p	q	$p \wedge q$	$p o (p \wedge q)$
Т	Т	T	Т
Τ	F	F	
F	Т	F	
F	F	F	

The truth table will look as follows.

p	q	$p \wedge q$	$p o (p \wedge q)$
Т	Т	Т	Т
Τ	F	F	F
F	Т	F	Т
F	F	F	Т

The truth table will look as follows.

The truth table will look as follows.

p	q	$p \wedge q$	$p o (p \wedge q)$
Т	Т	T	Т
Т	F	F	F
F	Т	F	
F	F	F	

The truth table will look as follows.

p	q	$p \wedge q$	$p o (p \wedge q)$
Т	Т	Т	Т
Т	F	F	F
F	Т	F	Т
F	F	F	

The truth table will look as follows.

p	q	$p \wedge q$	$p o (p \wedge q)$
Т	Т	Т	Т
Т	F	F	F
F	Т	F	Т
F	F	F	Т

The truth table will look as follows.

p	q	$p \wedge q$	$p o (p \wedge q)$
Т	Т	Т	Т
Τ	F	F	F
F	Т	F	Т
F	F	F	Т

Remember: a statement is a tautology if it is always true, so it has to have all Ts. If it has at least one F, then it is not a tautology.