EXAMPLE 2.22

Solve the simultaneous equations
$$\begin{aligned} x+2y &= 10 \\ 3x+2y-4z &= 18 \\ y+z &= 3 \end{aligned}$$

Solution

We label the equations as follows;
$$\begin{aligned} x+2y &= 10 & -(1) \\ 3x+2y-4z &= 18 & -(2) \\ y+z &= 3 & -(3) \end{aligned}$$

We eliminate x using equations (1) and (2):

$(2) - 3\times(1)$: $\quad -4y-4z = -12$

$\Leftrightarrow y+z = 3 \quad -(4)$

We are now left with equations (3) and (4). However, these two equations are identical.

To obtain the solution set to this problem we introduce a **parameter**, we let z be any arbitrary value, say $z = k$ where k is some real number.
Then, substituting into equation (4), we have:

$$y+k = 3 \Rightarrow y = 3-k.$$

Next, we substitute into (1) so that $\quad x+2(3-k) = 10 \Rightarrow x = 4+2k.$

Therefore, the solution is given by, $x = 4+2k, y = 3-k, z = k$.

Notice the nature of the solution, each of the variables is expressed as a linear function of k. This means that we have a situation where the three original planes meet along a straight line.

EXERCISES 2.3.3

1. Solve the simultaneous equations

(a) $$\begin{aligned} 6x+4y-z &= 3 \\ x+2y+4z &= -2 \\ 5x+4y &= 0 \end{aligned}$$

(b) $$\begin{aligned} x+y+z &= 2 \\ 4x+y &= 4 \\ -x+3y+2z &= 8 \end{aligned}$$

(c) $$\begin{aligned} 4x+9y+13z &= 3 \\ -x+3y+24z &= 17 \\ 2x+6y+14z &= 6 \end{aligned}$$

(d) $$\begin{aligned} x-2y-3z &= 3 \\ x+y-2z &= 7 \\ 2x-3y-2z &= 0 \end{aligned}$$

(e) $$\begin{aligned} x-y-z &= 2 \\ 3x+3y-7z &= 7 \\ x+2y-3z &= 3 \end{aligned}$$

(f) $$\begin{aligned} x-2y &= -1 \\ -x-y+3z &= 1 \\ y-z &= 0 \end{aligned}$$

(g) $$\begin{aligned} x+y+z &= 1 \\ x-y+z &= 3 \\ 4x+2y+z &= 6 \end{aligned}$$

(h) $$\begin{aligned} -2x+y-2z &= 5 \\ x+4z &= 1 \\ x+y+10z &= 10 \end{aligned}$$