Pre-IB Syllabus 2017/2018

First semester:

1. Sets and set operations. Intervals. Venn diagrams. HL: includes application of inclusion-exclusion principle to counting elements of finite sets.
2. Linear equations and inequalities. Includes systems of up to 2(SL)/3(HL) equations.
3. Equations and inequalities with absolute value. HL: may include two or more absolute values.
4. Exponential and logarithmic equations.
5. Binomial Theorem.
6. Basic statistics. Includes interquartile range and cumulative frequency tables.

Second semester:

7. Quadratic equations and inequalities. HL: includes "disguised" quadratics with quadratic, exponential and logarithmic expressions.
8. Functions and transformations of functions. Includes composition of functions and inverse functions. HL: inculdes the following transformations: $|f(x)|$, $f(|x|)$ and $\dfrac{1}{f(x)}$.
9. Linear functions with applications.
10. Quadratic functions with applications. Includes optimization problems.
11. Rational functions of the form $f(x) = \dfrac{ax+b}{cx+d}$.
12. Exponential and logarithmic functions with applications. HL: includes setting up discrete and continuous exponential growth models.
13. Probability. Includes conditional probability. HL: includes application of basic combinatorics to probability.

14. Triangle trigonometry. Includes cosine and sine rule (with the ambiguous case).

15.* Sequences. Includes sigma notation and infinite geometric series.

The IB qualification exam will include topics 1 to 11 only.

Many of the questions on the exam will test the knowledge of more than one of the above topics. Additionally, skills and topics such as number arithmetic, rearranging formulae, rounding numbers (decimal places or significant figures) may be examined implicitly. Questions similar to those marked **SL** may appear on both SL and HL exam. Questions similar to those marked **HL** will not appear on the SL exam. The **HL** exam may also include questions on mathematical induction. The numbers next to the questions indicate the topic the questions pertain to.

* If time permits.

Sample exam questions

1. **(SL: 1)** A survey of 100 families was carried out, asking about the pets they own. The results are given below.

   56 owned dogs (D)

   38 owned cats (C)

   22 owned birds (B)

   16 owned dogs and cats, but not birds

   8 owned birds and cats, but not dogs

   3 owned dogs and birds, but not cats

   4 owned all three types of pets

   (a) Represent this information on a Venn diagram.

   (b) Find the number of families who own no pets.

   (c) Find the percentage of families that own exactly one pet.

   (d) Find the percentage of families that own a dog or a cat.

2. **(SL: 1)** There are 49 mice in a pet shop.

   30 mice are white.

   27 mice are male.

   18 mice have short tails.

   8 mice are white and have short tails.

   11 mice are male and have short tails.

   7 mice are male but neither white nor short-tailed.

   5 mice have all three characteristics.

   2 have none.

   (a) Represent this information on a Venn diagram.

   (b) Find the number of mice who are male and white.

   (c) Find the number of mice who have short tails and are female.

3. **(SL: 1)** There are 20 students in an IB class. 12 of those chose Biology as their group 3 subject, 9 chose Chemistry. There are twice as many students who chose both Biology and Chemistry than those who didn't choose any of the two subjects. Find the number of students who chose Biology only.

4. **(SL: 1)** In a survey about sport activities 40% of the respondents answered that they go to the gym, while 25% answered that they go to swimming pool. Out of those who go to the gym exactly half also goes to the swimming pool. Find the percentage of respondents who do go neither to the gym nor to the swimming pool.

5. **(HL: 1)** Find the number of three digit numbers divisible by 2 or 3 or 5.

6. **(SL: 2, 7)** Find the values of m for which the following system of equations is inconsistent:
$$\begin{cases} 2x - my = 3 \\ mx - y = 1 \end{cases}$$

7. **(HL: 2, 7)**
   (a) Find the values of m for which the following system of equations is inconsistent:
   $$\begin{cases} mx + z = 1 \\ 3x - y + mz = 1 \\ 5x + 3y + z = 1 \end{cases}$$
   (b) Find the solution to the above system when $m = 3$.

8. **(SL: 1, 3)** Conisder the following inequalities:
$$(1) \qquad 2|x+3| - 1 < 5$$
$$(2) \qquad |x-2| + 1 \geqslant 8$$
   Let A and B be the sets of solutions to the first and second inequality respectively. Find
   i. $A \cup B$
   ii. $A \cap B$
   iii. $(A \cap B)'$

9. **(HL: 3)** Solve the inequality:
$$|x+2| - |x-3| < 2$$

10. **(HL: 4)** Solve the equation:

$$\log_2 x + \log_3 x = 1 + \log_4 x$$

give your answer correct to 3 significant figures.

11. **(HL: 4, 7)** Solve the equation:

$$5 \times 2^x - 4^x = 4$$

12. **(SL: 4)** Let $\log_2 x = a$ and $\log_2 y = b$. Express in terms of a and b
    (a) $\log_2 \dfrac{2x}{\sqrt{y}}$,
    (b) $\log_x \sqrt[5]{8y}$

13. **(HL: 4, 7)** Solve the equation:

$$\log_4 x + \log_x 4 = 4$$

giving your answers correct to 3 significant figures.

14. **(HL: 4, 7)** Solve the equation

$$3\log_2 x + 2 = \frac{1}{\log_2 x}$$

15. **(SL: 2, 4)** Solve the following system of equations:

$$\begin{cases} \log_3 x^2 + \log_3 y = 1 \\ \log_3 x^3 - \log_3 y = 9 \end{cases}$$

16. **(SL: 4)** Solve $\log_3 x - \log_3(x-5) = 1$, for $x > 5$.
17. **(SL: 4, 7)** Solve $\log_2 x + \log_2(x-2) = 3$, for $x > 2$.
18. **(SL: 4, 7)** Find the exact solution to the equation:

$$4^{x^2-x} = 8^{x+1}$$

19. **(SL: 5)** Find the coefficient of x^3 in the expansion of $(2x - \frac{1}{x})^7$.
20. **(SL: 5)** Find the constant term in the expansion of $(x - \frac{3}{x^2})^9$.
21. **(SL: 5)** Find the coefficient of x^8 in the expansion of $(x^2 + \frac{2}{x})^{10}$

22. **(HL: 5)** Find the coefficient of x^5 in the expansion of $(x^2+3)(2x-\frac{1}{x})^7$.

23. **(SL: 6)** The average age of a player in a football team (consisting of 11 players) is 24. If we include the coach the average is 26. How old is the coach?

24. **(SL: 6)** 9 students from the preIB basics class received the following scores on their last math test:

$$22, 17, 25, 39, 11, 36, 48, 22, 26$$

Find the (a) mean, (b) mode, (c) median, (d) range and (e) interquartile range of this data set.

25. **(SL: 4, 6)** Find the median and interquartile range of the following numbers:

$$\log_4 8,\quad 2^{2\log_2 3},\quad 2^{\log_4 9},\quad \left(\frac{1}{2}\right)^{-2},\quad \log_{11}(11^3),\quad \log 20+\log 50,\quad (\sqrt{2})^4,\quad (\sqrt{17})^0$$

26. **(SL: 6)** Consider the following table of the IB scores at a certain school:

score	frequencey	cumulative frequency
24	1	1
26	1	2
27	1	3
30	2	**p**
31	**q**	8
32	2	10
34	4	14
35	5	19
36	5	24
38	2	26
39	2	28
42	1	29
45	1	30

(a) Find the values of **p** and **q**.

(b) Find the median score and the interquartile range of scores.

27. **(SL: 6)** Consider four numbers a, b, c, d with $a \leqslant b \leqslant c \leqslant d$, and $a, b, c, d \in \mathbb{Z}^+$ and:

    the mean of these numbers is 4,

    the mode is 3,

    the median is 3,

    the range is 6.

    Find the value of a, b, c and d.

28. **(SL: 7)** Find 3 consecutive natural numbers whose product is twice the largest of them.

29. **(SL: 7)** Find a two-digit number which is four times as large as the sum of its digits and twice as large as the product of its digits.

30. **(SL: 7)** Find the value of k for which the equation:

    $$kx = 3x^2 + 2$$

    has two distinct solutions. Find these solutions when $k = 6$ giving your answers (i) exactly and (ii) correct to 3 significant figures.

31. **(SL: 7)** A train usually covers a journey of $240km$ with the average speed of $v\ \frac{km}{h}$. One day, due to adverse weather conditions, it reduces its speed by $40\ \frac{km}{h}$ and the journey take an hour longer. Find the value of v.

32. **(HL: 7)** Solve

    $$x - 9\sqrt{x} + 20 = 0$$

33. **(HL: 7)** By using substitution $p = x + \frac{1}{x}$, show that the equation

    $$2x^4 + x^3 - 6x^2 + x + 2 = 0$$

    reduces to $2p^2 + p - 10 = 0$.
    Hence solve $2x^4 + x^3 - 6x^2 + x + 2 = 0$.

34. **(SL: 7)** The quadratic equation

    $$x^2 + 6x + 1 = k(x^2 + 1)$$

    has a repeated root. Find the possible values of k.

35. **(SL: 7, 10)** Consider a quadratic function $f(x) = 2x - kx^2 + 1$.
   (a) Find the set of values of k for which the graph of f does not intersect the x-axis.

   Let $k = 2$.
   (b) Write down the y-intercept of the graph of f and find the coordinates its vertex. Hence sketch the graph of f.

36. **(SL: 7, 8, 10)** Consider the function $f(x) = 2(x - 3)(x + 5)$.
   (a) Write down the zeros of $f(x)$.
   (b) Find the coordinates of the vertex of the graph of f.
   The graph of g has been formed by translating the graph of f by a vector $\vec{v} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$.
   (c) Write down the zeros and the coordinates of the vertex of g.
   (d) Find the y-intercept of g.
   (e) Sketch the graph of g.

37. **(HL: 7)** If α and β are roots of the equation $3x^2 + 5x + 4 = 0$, find the values of:

$$\text{a) } \alpha^2 + \beta^2 \qquad \text{b) } \alpha^3 + \beta^3 \qquad \text{c) } \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$$

38. **(HL: 7)** If α and β are roots of the equation $3x^2 + 6x - 2 = 0$, find a quadratic equation whose roots are:

$$\text{a) } 2\alpha \text{ and } 2\beta \qquad \text{b) } \alpha^2 \text{ and } \beta^2 \qquad \text{c) } \alpha + 5\beta \text{ and } 5\alpha + \beta$$

39. **(SL: 7, 8, 9)** Consider a function $f(x) = \frac{x^2 - x - 6}{x + 2}$.
   (a) Write down the domain of $f(x)$.
   (b) Factorize the numerator and hence simplify the expression for $f(x)$.
   (c) Sketch the graph of f, indicate clearly the axes intercepts and the discontinuity.

40. **(SL: 7, 10)** A rectangle has perimeter equal to 20 cm. Let x cm be the length of one of the sides. Express the area of the rectangle in terms of x, and hence find the maximum possible area.

41. **(SL: 7, 10)** A string of length $60cm$ is cut into two pieces and each piece is formed into a rectangle. The first rectangle has width of $6cm$, and the second rectangle is 3 times as long as it is wide. Let A denote the total area of both rectangles. Find the minimum value of A.

42. **(HL: 7, 10)** A wire of length $1m$ has been cut into two pieces. The first piece has been bent into a square, the second piece has been bent into a circle. What should the lengths of the pieces be, so that the total area of the square and the circle is minimal.

43. **(HL: 7, 10)** Two cars are travelling along two straight roads which are perpendicular to each other and meet the point O. The first car starts 50km west of O and travels east with a constant speed of 20km/h. The second car starts 30km south of O at the same time and travels north with a constant speed of 15km/h.
    (a) Show that at time t, the distance d between the two cars satisfies the equation
    $$d^2 = 625^2 - 2900t + 3400 \qquad (1)$$
    (b) Hence find the shortest distance between the two cars.

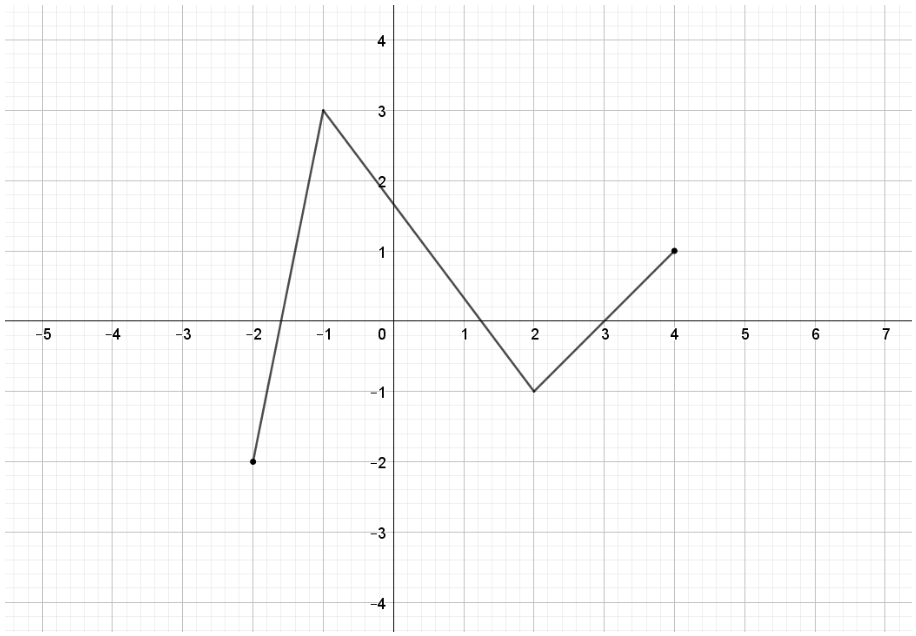
44. **(HL: 7, 10)** Find the points on the graph of the curve $y = x^2 + 1$ which are closest to the point $(0, 4)$.

45. **(HL: 7, 8)** Function f and g are defined such that $g(x) = x^2 + 7$ and $(g \circ f)(x) = 9x^2 + 6x + 8$. Find possible expressions for $f(x)$.

46. **(SL: 7, 8, 10)** Consider the function $f(x) = x^2 - x - 12$.
    (a) Sketch the graph of $f(x)$ indicating clearly the intercepts and the vertex.
    (b) The graph of g has been formed by reflecting the graph of f in the y-axis.
        i. Write down the formula for $g(x)$.
        ii. Solve the equation $f(x) + g(x) = 0$
    (c) **(HL: 8)** Sketch the graph of $\dfrac{1}{f(x)}$ indicating clearly the intercepts and the maximum.

47. **(SL: 8)**

The diagram below shows the graph of a function $f(x)$ for $-2 \leqslant x \leqslant 4$



(a) Find the range of this function.

(b) Write down the number of solutions to the equation $f(x) = \frac{1}{2}$.

On the same diagram draw the functions:

(c) $g(x) = -f(x-2) + 1$

(d) $h(x) = \frac{1}{2}f(x+1) - 2$

48. **(SL: 8, 9)** Let f be linear function. The graph of f is perpendicular to the line l with the equation $y = -2x + 3$ and passes through the point $(4, 3)$.

(a) Find the equation for f.

(b) Find the point where the graph of f intersects the line l.

(c) Find the area of the quadrilateral enclosed by the graph of f, the line l and the positive x and y axes.

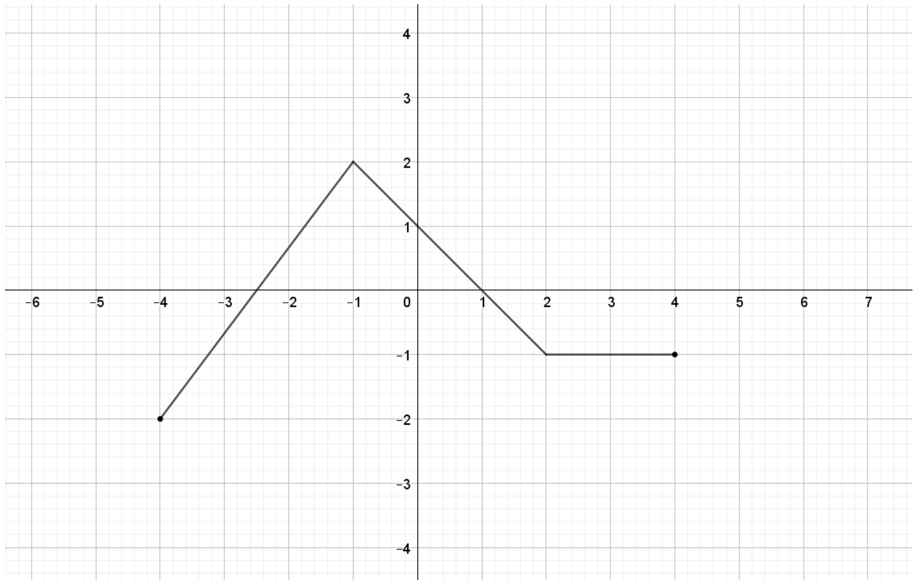
49. **(SL: 8, 9)** The function $f(x)$ is defined by the equation $f(x) = 2x + 1$ on the interval $[0, 2)$ and is periodic with a period of 2.

(a) Sketch the graph of f for $x \in [-4, 6)$.

(b) State the number of solutions to the equation $f(x) = 3$ for $x \in [-4, 6)$.

50. **(SL: 8)**

The diagram below shows the graph of a function $f(x)$ for $-4 \leqslant x \leqslant 4$



(a) Find the range of this function.

(b) Write down the number of solutions to the equation $f(x) = 3$

On the same diagram draw the following functions and state their domains and ranges:

(c) $g(x) = 2f(-x + 1)$

(d) $h(x) = f(2x) - 1$

51. **(SL: 8, 9)** Conisder two lines: l with equation $y = 3x - 1$ and m with equation $y = px + 2$, where p is a constant.

(a) Write down the value of p if:

i. l and m are parallel,

ii. l and m are perpendicular.

Let $p = 1$ and let A and B be the points of intersection of l and m respectively with the x-axis and C the point of intersection of l and m.

(b) Find the area of $\triangle ABC$.

52. **(SL: 8, 9)** Let $f(x) = (k-1)x + 3k - 2$, where k is a constant. Find the values of k for which:

    i. f is increasing,

    ii. the y-intercept of f is greater than 0,

    iii. the x-intercept of f is greater than 0.

53. **(SL: 8)** Consider the functions $f(x) = \dfrac{1}{x+1}$ and $g(x) = \sqrt{1-x^2}$

    (a) Find the domains of f and g.

    (b) Calculate $(f \circ g)(-1)$ and $(g \circ f)(2)$.

    (c) Find an expression for $(g \circ f)(x)$.

    (d) **(HL: 7, 8)** Find the domain of $(g \circ f)(x)$.

54. **(SL: 9)** Line l_1 is given by the equation:

$$2y + 5x - 1 = 0$$

    (a) Find the gradient of l_1,

    (b) hence state whether the function, whose graph is l_1, is increasing or decreasing.

    Line l_2 is given by the equation $y = \frac{5}{2}x$.

    (c) State whether l_1 and l_2 are parallel, perpendicular or neither. Justify your answer.

    Points $(1, p)$ and $(q, 13)$ lie on l_1.

    (d) Find p and q.

55. **(SL: 9)** The line l_1 is given by the equation $y = \frac{1}{3}x - 4$. Line l_2 passes through $(2, 5)$ and is perpendicular to $2x - y + 6 = 0$. Sketch both l_1 and l_2. Find the coordinates of the point of intersection of these lines.

56. **(SL: 9)** Consider the points $A(-2, 5)$ and $B(6, -1)$.

    (a) Find the equation of a line passing through A and B.

    (b) Find the midpoint M of the line segment AB.

    (c) Find the equation of the perpendicular bisector of AB.

57. **(SL: 9)** Consider the points $A(1,-1)$, $B(3,2)$ and $C(7,8)$.
    (a) Determine whether the points A, B and C are collinear.
    (b) Let M be the midpoint of BC. Find the length of AM.
58. **(SL: 9)** Determine the value of k so that the points $A(7,3)$, $B(k,-2)$ and $C(-1,0)$ are the vertices of a right triangle with the right angle at B.
59. **(SL: 9)** Use gradients to show that the points $(8,0)$, $(-1,-2)$, $(-2,3)$ and $(7,5)$ are the vertices of a parallelogram.
60. **(SL: 8, 9)** Let $f(x) = 2x - 5$, $g(x) = \frac{x}{3} - 1$ and $h(x) = 3x + 2$.
    (a) Find $f(g(x))$ and $f(h(x))$.
    (b) Find $f^{-1}(x)$, $g^{-1}(x)$ and $h^{-1}(x)$.
    (c) Find the coordinates of the point of intersection of the graphs of $f(h(x))$ and $g^{-1}(x)$.
    (d) Find the set of values of x for which $f(h(x)) \geqslant g^{-1}(x)$.
61. **(SL: 9)** Consider the line given by the equation $kx - 3y = 4k$. For what values of k will this line:
    (a) have a gradient equal to 1,
    (b) have y-intercept equal 2,
    (c) pass through the point $(2,4)$,
    (d) be parallel to the line $2x - 4y = 1$,
    (e) be perpendicular to the line $x - 6y = 2$,
    (f) be parallel to the x-axis.
62. **(SL: 8, 9, 10)** Consider the functions $f(x) = x^2 - 3$, $x \in \mathbb{R}$ and $g(x) = x + 3$, $x \in \mathbb{R}$.
    (a) Find the formula for $(f \circ g)(x)$.
    (b) State the range of (i) $f(x)$, (ii) $g(x)$, (iii) $(f \circ g)(x)$.
63. **(SL: 8, 9)** Consider the function $f(x) = 2x - 3$. Find the inverse f^{-1} of f. Sketch both $f(x)$ and its inverse on the same diagram.

64. **(SL: 9)** Conisder the function

$$f(x) = \begin{cases} x+1 & x < 1 \\ 3-x & x \geqslant 1 \end{cases}$$

(a) Sketch the graph of $f(x)$.

(b) State the values of x for which $f(x)$ is

i. increasing,

ii. positive,

iii. greater than 3.

65. **(SL: 8, 10)** Consider the function $f(x) = 2(x-1)^2 + 3$.

(a) Write down the coordinates of the vertex of the graph of f.

(b) The graph of f has been first translated by a vector $\vec{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and the reflected in the x-axis to form the graph of g. Find the expression for $g(x)$ and state the coordinates of the vertex of the graph of g.

66. **(HL: 3, 8, 9)** Consider the graphs of $y = |x|$ and $y = b - |x|$, where b is a positive integer.

(a) Sketch both graphs of the same set of axes.

(b) Given that the graphs enclose an area of 18 square units, find the value of b.

67. **(SL: 8, 9)** A straight line l passes through the point $A(6, 10)$ and is perpendicular to the line m which has equation $3y + x = 6$.

(a) Find the equation of l.

(b) Find the coordinates of the point of intersection of l and m. Deduce the shortest distance from the point A to the line m.

68. **(HL: 8)** The function f is defined by $f(x) = 2x^3 + 5$, where $-2 \leqslant x \leqslant 2$.

(a) Find the range of f.

(b) Find an expression for $f^{-1}(x)$.

(c) Write down the domain and range of $f^{-1}(x)$.

69. **(SL: 7, 10)** Find the set of values of k for which the function $f(x) = x^2 - 6x + 2k$ does not intersect the x-axis.

70. **(HL: 8, 10)** Consider a function $f(x) = x^2 - m^2 \quad x \in \mathbb{R}$, where m is a positive constant. Sketch the graph of the following curves, indicating clearly all axis intercepts, maximum/minimum points and asymptotes.

    i. $y = f(x)$,

    ii. $y = f(x + m)$,

    iii. $y = \dfrac{1}{f(x)}$,

    iv. $y = \left|\dfrac{1}{f(x)}\right|$.

71. **(SL: 7, 10)**

    (a) The graph of $y = x^2 - 6x + k$ has its vertex on the x-axis. Find the value of k.

    (b) A second parabola has its vertex at $(-2, 5)$ and passes through the vertex of the first graph. Find the equation of the second graph in the form $y = ax^2 + bx + c$.

    (c) Find the coordinates of the other point of intersection between the two graphs.

72. **(HL: 8, 10)** Let $f(x) = x^2 - 8x + 12$. Sketch the graph of $y = |f(x)| - 1$ indicating clearly the intercepts and maxima/minima. Hence find the set of values of p, for which the equation

$$|f(x)| - 1 = p$$

    has exactly two solutions.

73. **(SL: 10)** Consider the quadratic function $f(x) = -x^2 + 6x - 2$, $x \in \mathbb{R}$.

    (a) Find the range of $f(x)$.

    The graph of f has been translated by the vector $\vec{v} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ and then reflected in the x-axis to form the graph of g.

    (b) Find the equation for $g(x)$ in the standard form.

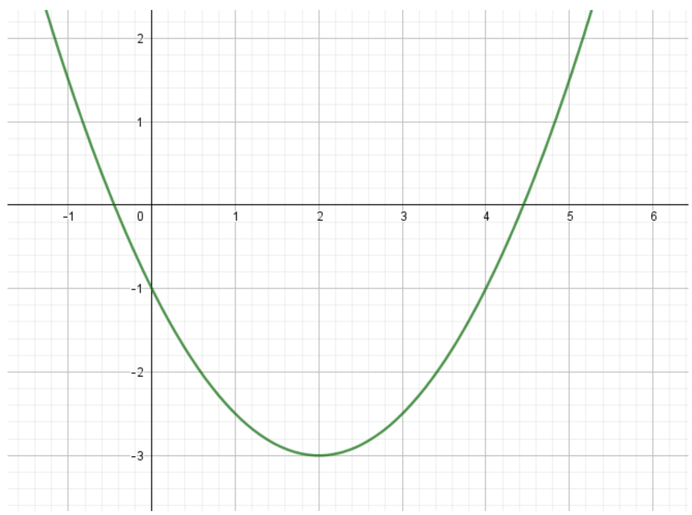
    (c) Find the range of $g(x)$.

74. **(HL: 7, 10)** The positive difference between the zeros of the quadratic function $f(x) = x^2 + kx + 3$ is $\sqrt{69}$. Find the possible values of k.

75. **(SL: 8, 10)** Let $f(x) = x^2 - 8x + 12$.
   (a) Write $f(x)$ in the vertex form and hence state the coordinates of the vertex of the graph of f.

   The graph of f has been translated by the vector $\vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and then reflected in the y-axis to form the graph of $g(x)$.
   (b) Write down the equation of $g(x)$ and state the coordinates of its vertex.

76. **(SL: 10)** Part of the graph of a quadratic function $f(x)$ is shown below.

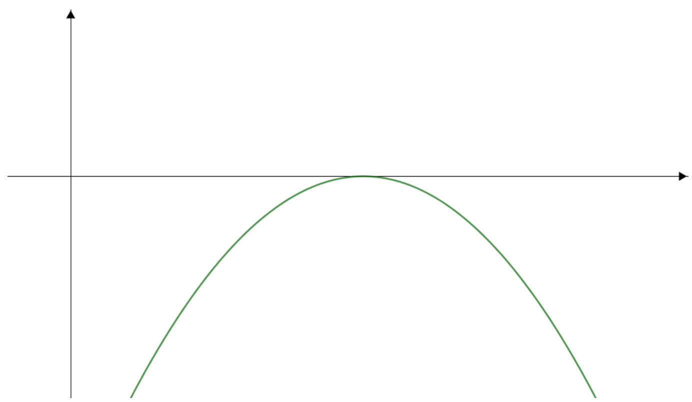


The graph passes through $(0, -1)$ and has its vertex at $(2, -3)$.
   (a) Find the equation of $f(x)$ in *(i)* vertex form *(ii)* standard form.
   (b) Find the x-intercepts of f.

77. **(SL: 9)** The cost of producing x units of a certain item is given by the formula $C(x) = 50x + 1200$. The income from selling x units is given by the equation $I(x) = 2x^2$. The profit $P(x) = I(x) - C(x)$.
   (a) Find the least number of units that need to be produced and sold in order for the profit to be positive.
   (b) Find the number of units that need to be produced and sold in order for the profit to be equal to 1300.

78. **(SL: 8, 9)** Consider the lines l_1 and l_2 given by the equations $y = 2x + 7$ and $y = 2x + 2$ respectively.
   (a) Write down the y-intercept of l_1.
   (b) Find an equation of a line m which is perpendicular to l_1 and l_2 and intersects the y-axis at the same point as l_1.
   (c) Find the point of intersection of m and l_2.
   (d) Hence find the distance between l_1 and l_2.

79. **(SL: 10)** The diagram shows the graph of a quadratic function $y = ax^2 + bx + c$.



Complete the table below to show whether each expression is positive, negative or zero.

Expression	Positive	Negative	Zero
a			
c			
Δ			
b			

80. **(SL: 7, 10)** A quadratic function f can be written in the form $f(x) = a(x - p)(x - 3)$. The graph of f has axis of symmetry $x = 2.5$ and y-intercept at $(0, -6)$.
   (a) Find the value of p.
   (b) Find the value of q.
   (c) The line $y = kx - 5$ is tangent to the graph of f. Find the value of k.

81. **(HL: 3, 8, 9)** Sketch the graph of $f(x) = \Big|\big||x-1|-4\big|-1\Big|$. Hence find the set of values of k for which the equation $f(x) = k$ has exactly 4 solutions.

82. **(HL: 8, 9, 10)** On the same diagram sketch the graphs of $y = |x+2|$ and $y = |x^2+2x-3|$. Hence state the number of solutions to the equation $|x^2+2x-3| = |x+2|$.

83. **(HL: 8, 10)** Consider the function $g(x) = x^2 + 3x + 2$.
    (a) Write $g(x)$ in:
        i. factored form,
        ii. vertex form.
    Let $f(x) = \dfrac{1}{g(x)}$.
    (b) Find the domain of $f(x)$.
    (c) Sketch the graph of $f(x)$ clearly labelling the intercepts, maxima/minima and asymptotes.

84. **(SL: 7, 10)** Let $f(x) = x^2 + 2x - 5$ and $g(x) = 3x + 1$. Find the set of values of x for which $f(x) \leqslant g(x)$.

85. **(SL: 7, 10)** Let $f(x) = 2x^2 + 5x + 6$ and $g(x) = 4x - 5$. Show that the graphs of these functions do not intersect.

86. **(HL: 8, 9)**
    (a) Given that $f(x) = ax + b$ and $(f \circ f \circ f)(x) = 64x + 21$, find the values of constants a and b.
    (b) Find an expression in terms of n for $(\underbrace{f \circ f \circ f \circ \ldots \circ f}_{n \text{ times}})(x)$.

87. **(SL: 8, 10)** Consider the function $f(x) = x^2 + 4x - 3$ for $x \geqslant -2$.
    (a) Write $f(x)$ in the vertex form.
    (b) State the range of f.
    (c) Find an expression for $f^{-1}(x)$ and state its domain and range.

88. **(HL: 9, 10)** Find the point on the line $2x + y + 1 = 0$, so that the product of its coordinates is maximal.

89. **(HL: 7, 8)** The functions g and h are defined as follows:

$$g(x) = \frac{5}{x-3}, \;\; x \in \mathbb{R} - \{3\} \qquad \text{and} \qquad h(x) = x^2 + 4, \;\; x \in \mathbb{R}^+$$

Find:

(a) an expression for the inverse function $g^{-1}(x)$,

(b) an expression for the composite function $(g \circ h)(x)$,

(c) the solution to the equation $3g^{-1}(x) = 10(g \circ h)(x) + 9$

90. **(SL: 7, 8)** The function f is defined by $f(x) = 1 - x^2$ for $-1 < x \leqslant 1$. Given that f is periodic with a period of 2, find all solutions to the equation

$$f(x) = \frac{3}{4}$$

in the interval $0 < x \leqslant 8$.

91. **(SL: 7, 11)** Consider functions $f(x) = \dfrac{x-1}{x+2}$ $x \in \mathbb{R} - \{-2\}$ and $g(x) = \dfrac{x+2}{x-3}$ $x \in \mathbb{R} - \{3\}$.

(a) Sketch the graphs of f and g clearly labelling all the intercepts and asymptotes.

(b) Find expressions for $(f \circ g)(x)$ and $(g \circ f)(x)$.

(c) Find expressions for $f^{-1}(x)$ and $g^{-1}(x)$.

(d) Solve the equation $(f \circ g)(x) = g^{-1}(x)$.

92. **(SL: 11)** Consider the function $g(x) = \frac{2x+6}{x-1}$.

(a) Write down the horizontal and vertical asymptotes of the graph of g.

(b) Find the intercepts of the graph of g with the axes.

(c) Sketch the graph of g.

93. **(HL: 8, 11)** Consider the function $f(x) = \dfrac{x-2}{x-1}$ $x \in \mathbb{R} - \{1\}$.

(a) Sketch the graph of $|f(|x|)|$ clearly labelling all the intercepts and asymptotes.

(b) Find the set of values of p for which the equation $|f(|x|)| = p$ has exactly two solutions.

94. **(SL: 8, 11)** Consider the function $f(x) = \frac{3x+12}{x-2}$.

(a) Write down the horizontal and vertical asymptotes of the graph of f.

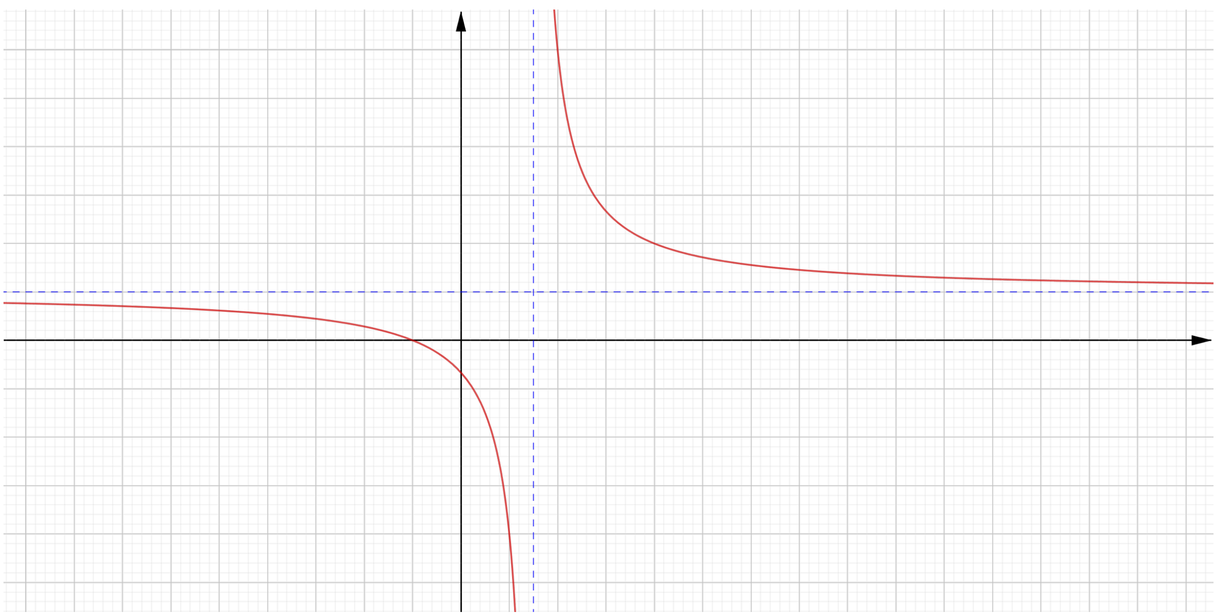
The graph of f has been translated by a vector $\vec{v} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

(b) Write down the horizontal and vertical asymptotes of the graph of g.

(c) Find the intercepts of the graph of g with the axes.

(d) Sketch the graph of g.

95. **(SL: 8, 11)** The diagram below shows the graph of a function $f(x) = \dfrac{ax+b}{x-c}$. The graph has a vertical asymptote at $x = 3$ and horizontal asymptote at $y = 2$. The graph intersects the x-axis at $(-2, 0)$.



(a) Write down the values of a, b and c.

(b) Find the y-intercept.

(c) **(HL)** Sketch the graph $f(|x|)$.

(d) **(HL)** Find the set of values of p such that the equation $f(|x|) = p$ has no solutions.

96. **(HL: 8, 11)** The graph of $f(x) = \frac{p}{x-q}$, where p and q are constants, intersects the y-axis at $(0, -3)$ and has a vertical asymptote with the equation $x = 2$ and a horizontal asymptote $y = 0$.
   (a) Write down the value of q.
   (b) Find the value of p.
   (c) Sketch the graph of $y = |f(x)|$.
   (d) Solve $|f(x)| = 3$.
   (e) Find the set of values of x for which $|f(x)| < 3$.

97. **(HL: MI)** Prove that for all positive integers n such that $n \geqslant 4$, it is true that $n^2 \leqslant 2^n$.

98. **(HL: MI)** Prove that $4^n + 15n - 1$ is divisible by 9 for all positive integers n.

99. **(HL: MI)** Prove that:
$$\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)...\left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}$$
for all positive integers n with $n \geqslant 2$.

100. **(HL: MI)** Prove that:
$$1^2 - 2^2 + 3^2 - 4^2 + ... + (-1)^{n-1}n^2 = (-1)^{n-1}\frac{n(n+1)}{2}$$
for all positive integers n.