1. The system of equations

$$\begin{aligned} 2x - y + 3z &= 2 \\ 3x + y + 2z &= -2 \\ -x + 2y + az &= b \end{aligned}$$

is known to have more than one solution. Find the value of a and the value of b.

(Total 5 marks)

2. (a) Find the set of values of k for which the following system of equations has no solution.

$$\begin{aligned} x + 2y - 3z &= k \\ 3x + y + 2z &= 4 \\ 5x + 7z &= 5 \end{aligned}$$

(4)

(b) Describe the geometrical relationship of the three planes represented by this system of equations.

(1)

(Total 5 marks)

3. Find the vector equation of the line of intersection of the three planes represented by the following system of equations.

$$\begin{aligned} 2x - 7y + 5z &= 1 \\ 6x + 3y - z &= -1 \\ -14x - 23y + 13z &= 5 \end{aligned}$$

(Total 6 marks)

4. (a) Show that the following system of equations will have a unique solution when $a \neq -1$.

$$\begin{aligned} x + 3y - z &= 0 \\ 3x + 5y - z &= 0 \\ x - 5y + (2 - a)z &= 9 - a^2 \end{aligned}$$

(5)

(b) State the solution in terms of a.

(6)

(c) Hence, solve

$$\begin{aligned} x + 3y - z &= 0 \\ 3x + 5y - z &= 0 \\ x - 5y + z &= 8 \end{aligned}$$

(2)
(Total 13 marks)

5. Consider the planes defined by the equations $x + y + 2z = 2$, $2x - y + 3z = 2$ and $5x - y + az = 5$ where a is a real number.

(a) If $a = 4$ find the coordinates of the point of intersection of the three planes.

(2)

(b) (i) Find the value of a for which the planes do not meet at a unique point.

(ii) For this value of a show that the three planes do not have any common point.

(6)
(Total 8 marks)

6. The three planes

$$\begin{aligned} 2x - 2y - z &= 3 \\ 4x + 5y - 2z &= -3 \\ 3x + 4y - 3z &= -7 \end{aligned}$$

intersect at the point with coordinates (a, b, c).

(a) Find the value of each of a, b and c.

(2)

(b) The equations of three other planes are

$$\begin{aligned} 2x - 4y - 3z &= 4 \\ -x + 3y + 5z &= -2 \\ 3x - 5y - z &= 6. \end{aligned}$$

Find a vector equation of the line of intersection of these three planes.

(4)
(Total 6 marks)