1. Consider the function $f(x) = x^3 + \frac{48}{x}, x \neq 0$.

(a) Calculate $f(2)$.

(2)

(b) Sketch the graph of the function $y = f(x)$ for $-5 \leq x \leq 5$ and $-200 \leq y \leq 200$.

(4)

(c) Find $f'(x)$.

(3)

(d) Find $f'(2)$.

(2)

(e) Write down the coordinates of the local maximum point on the graph of f.

(2)

(f) Find the range of f.

(3)

(g) Find the gradient of the tangent to the graph of f at $x = 1$.

(2)

There is a second point on the graph of f at which the tangent is parallel to the tangent at $x = 1$.

(h) Find the x-coordinate of this point.

(2)
(Total 20 marks)

2. The diagram shows a sketch of the function $f(x) = 4x^3 - 9x^2 - 12x + 3$.

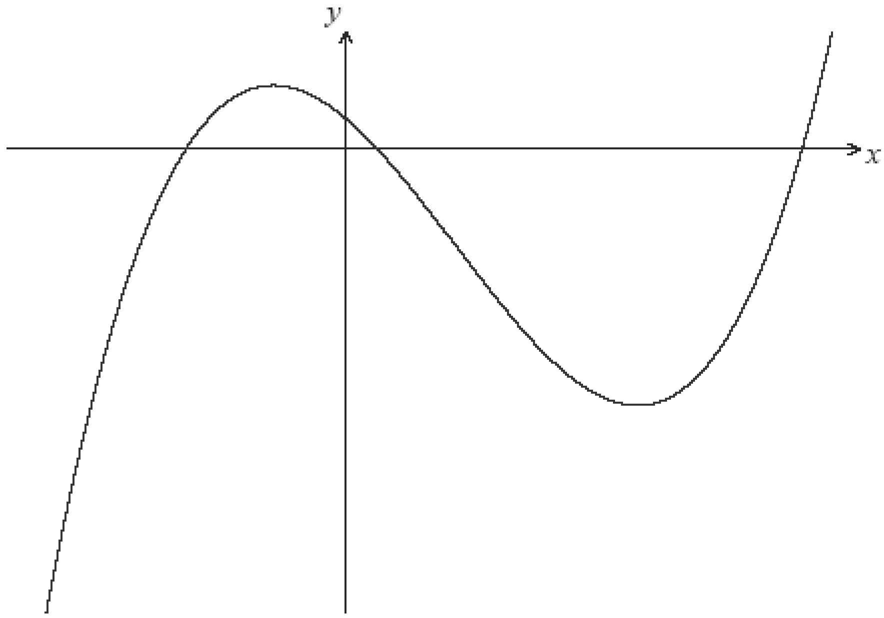

diagram not to scale

(a) Write down the values of x where the graph of $f(x)$ intersects the x-axis. **(3)**

(b) Write down $f'(x)$. **(3)**

(c) Find the value of the local maximum of $y = f(x)$. **(4)**

Let P be the point where the graph of $f(x)$ intersects the y-axis.

(d) Write down the coordinates of P. **(1)**

(e) Find the gradient of the curve at P. **(2)**

The line, L, is the tangent to the graph of $f(x)$ at P.

(f) Find the equation of L in the form $y = mx + c$.

(2)

There is a second point, Q, on the curve at which the tangent to $f(x)$ is parallel to L.

(g) Write down the gradient of the tangent at Q.

(1)

(h) Calculate the x-coordinate of Q.

(3)
(Total 19 marks)

3. Consider the function $f(x) = x^3 - 3x^2 - 24x + 30$.

(a) Write down $f(0)$.

(1)

(b) Find $f'(x)$.

(3)

(c) Find the gradient of the graph of $f(x)$ at the point where $x = 1$.

(2)

The graph of $f(x)$ has a local maximum point, M, and a local minimum point, N.

(d) (i) Use $f'(x)$ to find the x-coordinate of M and of N.

(ii) Hence or otherwise write down the coordinates of M and of N.

(5)

(e) Sketch the graph of $f(x)$ for $-5 \le x \le 7$ and $-60 \le y \le 60$. Mark clearly M and N on your graph.

(4)

Lines L_1 and L_2 are parallel, and they are tangents to the graph of $f(x)$ at points A and B respectively. L_1 has equation $y = 21x + 111$.

(f) (i) Find the x-coordinate of A and of B.

(ii) Find the y-coordinate of B.

(6)
(Total 21 marks)

4. Consider the function $f(x) = 3x + \dfrac{12}{x^2}$, $x \neq 0$.

(a) Differentiate $f(x)$ with respect to x.

(3)

(b) Calculate $f'(x)$ when $x = 1$.

(2)

(c) Use your answer to part (b) to decide whether the function, f, is increasing or decreasing at $x = 1$. Justify your answer.

(2)

(d) Solve the equation $f'(x) = 0$.

(3)

(e) The graph of f has a local minimum at point P. Let T be the tangent to the graph of f at P.

(i) Write down the coordinates of P.

(ii) Write down the gradient of T.

(iii) Write down the equation of T.

(5)

(f) Sketch the graph of the function f, for $-3 \leq x \leq 6$ and $-7 \leq y \leq 15$. Indicate clearly the point P and any intercepts of the curve with the axes.

(4)

(g) (i) On your graph draw and label the tangent *T*.

(ii) *T* intersects the graph of *f* at a second point. Write down the *x*-coordinate of this point of intersection.

(3)
(Total 22 marks)

5. A function is defined by $f(x) = \frac{5}{x^2} + 3x + c, x \neq 0, c \in \mathbb{Z}$.

(a) Write down an expression for $f'(x)$.

(4)

Consider the graph of *f*. The graph of *f* passes through the point P(1, 4).

(b) Find the value of *c*.

(2)

(c) There is a local minimum at the point Q.

(i) Find the coordinates of Q.

(ii) Find the set of values of *x* for which the function is decreasing.

(7)

Let *T* be the tangent to the graph of *f* at P.

(d) (i) Show that the gradient of *T* is –7.

(ii) Find the equation of *T*.

(4)

(e) *T* intersects the graph again at R. Use your graphic display calculator to find the coordinates of R.

(2)
(Total 19 marks)

6. Consider the curve $y = x^3 + \frac{3}{2}x^2 - 6x - 2$

(a) (i) Write down the value of y when x is 2.

(ii) Write down the coordinates of the point where the curve intercepts the y-axis.

(3)

(b) Sketch the curve for $-4 \le x \le 3$ and $-10 \le y \le 10$. Indicate clearly the information found in (a).

(4)

(c) Find $\frac{dy}{dx}$.

(3)

(d) Let L_1 be the tangent to the curve at $x = 2$.
Let L_2 be a tangent to the curve, parallel to L_1.

(i) Show that the gradient of L_1 is 12.

(ii) Find the x-coordinate of the point at which L_2 and the curve meet.

(iii) Sketch and label L_1 and L_2 on the diagram drawn in (b).

(8)

(e) It is known that $\frac{dy}{dx} > 0$ for $x < -2$ and $x > b$ where b is positive.

(i) Using your graphic display calculator, or otherwise, find the value of b.

(ii) Describe the behaviour of the curve in the interval $-2 < x < b$.

(iii) Write down the equation of the tangent to the curve at $x = -2$.

(5)
(Total 23 marks)

7. The function $g(x)$ is defined by $g(x) = \frac{1}{8}x^4 + \frac{9}{4}x^2 - 5x + 7,\ x \geq 0$.

(a) Find $g(2)$.

(2)

(b) Calculate $g'(x)$.

(3)

The graph of the function $y = g(x)$ has a tangent T_1 at the point where $x = 2$.

(c) (i) Show that the gradient of T_1 is 8.

(ii) Find the equation of T_1. Write the equation in the form $y = mx + c$.

(5)

(d) The graph has another tangent T_2 at the point $\left(1, \frac{35}{8}\right)$. T_2 has zero gradient.

Write down the equation of T_2.

(2)

(e) (i) Sketch the graph of $y = g(x)$ in the region $0 \leq x \leq 3$, $0 \leq y \leq 22$.

(ii) Add the two tangents T_1 and T_2 to your sketch, in the correct positions.

(5)
(Total 17 marks)

8. Consider the function $f(x) = \dfrac{3}{x^2} + x - 4$.

(a) Calculate the value of $f(x)$ when $x = 1$.

(2)

(b) Differentiate $f(x)$.

(4)

(c) Find $f'(l)$.

(2)

(d) Explain what $f'(l)$ represents.

(2)

(e) Find the equation of the tangent to the curve $f(x)$ at the point where $x = 1$.

(3)

(f) Determine the x-coordinate of the point where the gradient of the curve is zero.

(3)
(Total 16 marks)

9. (a) On the same graph sketch the curves $y = x^2$ and $y = 3 - \dfrac{1}{x}$ for values of x from 0 to 4 and values of y from 0 to 4. Show your scales on your axes.

(4)

(b) Find the points of intersection of these two curves.

(4)

(c) (i) Find the gradient of the curve $y = 3 - \dfrac{1}{x}$ in terms of x.

(ii) Find the value of this gradient at the point (1, 2).

(4)

(d) Find the equation of the tangent to the curve $y = 3 - \dfrac{1}{x}$ at the point (1, 2).

(3)
(Total 15 marks)