1. **Give all answers in this question to the nearest whole currency unit.**

   Ying and Ruby each have 5000 USD to invest.

   Ying invests his 5000 USD in a bank account that pays a nominal annual interest rate of 4.2 % **compounded yearly**. Ruby invests her 5000 USD in an account that offers a fixed interest of 230 USD each year.

   (a) Find the amount of money that Ruby will have in the bank after 3 years.

   (2)

   (b) Show that Ying will have 7545 USD in the bank at the end of 10 years.

   (3)

   (c) Find the number of complete years it will take for Ying's investment to first exceed 6500 USD.

   (3)

   (d) Find the number of complete years it will take for Ying's investment to exceed Ruby's investment.

   (3)

   Ruby moves from the USA to Italy. She transfers 6610 USD into an Italian bank which has an exchange rate of 1 USD = 0.735 euros. The bank charges 1.8 % commission.

   (e) Calculate the amount of money Ruby will invest in the Italian bank after commission.

   (4)

   Ruby returns to the USA for a short holiday. She converts 800 euros at a bank in Chicago and receives 1006.20 USD. The bank advertises an exchange rate of 1 euro = 1.29 USD.

   (f) Calculate the percentage commission Ruby is charged by the bank.

   (5)
   (Total 20 marks)

2. Eva invests USD2000 at a nominal annual interest rate of 8 % **compounded half-yearly**.

   (a) Calculate the value of her investment after 5 years, correct to the nearest dollar.

   (3)

Toni invests USD1500 at an annual interest rate of 7.8 % **compounded yearly**.

(b)   Find the number of **complete** years it will take for his investment to double in value.

(3)
(Total 6 marks)

3.   Bob invests 600 EUR in a bank that offers a rate of 2.75% compounded annually. The interest is added on at the end of each year.

(a)   Calculate how much money Bob has in the bank after 4 years.

(b)   Calculate the number of years it will take for the investment to double.

Ann invests 600 EUR in another bank that offers interest compounded annually. Her investment doubles in 20 years.

(c)   Find the rate that the bank is offering.

(Total 6 marks)

4.   William invests $1200 for 5 years at a rate of 3.75% compounded annually.

(a)   Calculate the amount of money he has in total at the end of the 5 years.

(b)   The interest rate then drops to 3.25%. If he continues to leave his money in the bank find how much it will be worth after a further 3 years.

(Total 6 marks)

5.   The exchange rate between Indian rupees (INR) and Singapore dollars (S$) is
100 INR = S$3.684

Kwai Fan changes S$500 to Indian rupees.

(a)   Calculate the number of Indian rupees she will receive using this exchange rate.
Give your answer correct to the nearest rupee.

(2)

On her return to Singapore, Kwai Fan has 2500 Indian rupees left from her trip. She wishes to exchange these rupees back to Singapore dollars. There is a 3% commission charge for this transaction and the exchange rate is 100 INR = S$3.672.

(b) Calculate the commission in Indian rupees that she is charged for this exchange.

(2)

(c) Calculate the amount of money she receives in Singapore dollars, **correct to two decimal places**.

(2)

(Total 6 marks)

6. Clara visits Britain from the United States and exchanges 1000 US dollars (USD) into pounds (GBP). The exchange rate is 1 USD = 0.543 GBP. The bank charges 2% commission for each transaction.

(a) Calculate how many GBP she receives.

Next Clara wants to travel to France. She changes 150 GBP to euros (€) at a rate of 1 GBP = 1.35 €. The bank charges commission and then gives Clara 200 €.

(b) Find the amount of commission in GBP.

(Total 6 marks)

7. (a) Given $x = 2.6 \times 10^4$ and $y = 5.0 \times 10^{-8}$, calculate the value of $w = x \times y$. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

(b)   Which **two** of the following statements about the nature of x, y and w above are **incorrect**?

(i) $x \in \mathbb{N}$

(ii) $y \in \mathbb{Z}$

(iii) $y \in \mathbb{Q}$

(iv) $w < y$

(v) $x + y \in \mathbb{R}$

(vi) $\dfrac{1}{w} < x$

(Total 8 marks)

8.   Consider the numbers 5, 0.5, $\sqrt{5}$ and –5. Complete the table below, showing which of the number sets, $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{Q}$ these numbers belong to.

Answers:

	$\mathbb{N}$	$\mathbb{R}$	$\mathbb{Q}$
5			✔
0.5	✘		
$\sqrt{5}$	✘		
–5		✔	

(Total 8 marks)

9.   The sets A, B and C are subsets of U. They are defined as follows:

$U = \{$positive integers less than 16$\}$

$A = \{$prime numbers$\}$

$B = \{$factors of 36$\}$

$C = \{$multiples of 4$\}$

(a)   List the elements (if any) of the following:

   (i)   *A*;

   (ii)   *B*;

   (iii)   *C*;

   (iv) $A \cap B \cap C$.

(4)

(b)   (i)   Draw a Venn diagram showing the relationship between the sets *U*, *A*, *B* and *C*.

        (ii)   Write the elements of sets *U*, *A*, *B* and *C* in the appropriate places on the Venn diagram.

(4)

(c)   From the Venn diagram, list the elements of each of the following

   (i) $A \cap (B \cup C)$;

   (ii) $(A \cap B)'$;

   (iii) $(A \cap B)' \cap C$.

(3)

(d)   Find the probability that a number chosen at random from the universal set *U* will be

   (i)   a prime number;

   (ii)   a prime number, but **not** a factor of 36;

   (iii)   a factor of 36 or a multiple of 4, but **not** a prime number;

   (iv)   a prime number, given that it is a factor of 36.

(6)
(Total 17 marks)

10. If $x = 3.1 \times 10^4$ and $y = 2.4 \times 10^{-7}$, calculate the values of the following, expressing your answers in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

(a) x^2

(b) $\dfrac{x}{y}$

(Total 4 marks)

11. Let $\mathscr{E} = \{x : 1 \leq x < 17, x \in \mathbb{N}\}$.

P, Q and R are the subsets of $\mathscr{E}$ such that

$$P = \{\text{multiples of four}\};$$

$$Q = \{\text{factors of 36}\};$$

$$R = \{\text{square numbers}\}.$$

(a)   List the elements of

(i) $\mathscr{E}$

(ii) $P \cap Q \cap R$.

(2)

(b)   Describe in words the set $P \cup Q$.

(1)

(c)   (i)   Draw a Venn diagram to show the relationship between sets P, Q and R.

(2)

(ii)   Write the elements of $\mathscr{E}$ in the appropriate places on the Venn diagram.

(3)

(d) Let p, q and r be the statements

$$p: x \text{ is a multiple of four;}$$

$$q: x \text{ is a factor of 36;}$$

$$r: x \text{ is a square number.}$$

(i) Write a sentence, in words, for the statement

$$(p \vee r) \wedge \neg q$$

(2)

(ii) Shade the region on your Venn diagram in part (c)(i) that represents $(p \vee r) \wedge \neg q$

(1)

(iii) (a) Use a truth table to determine the values of $(p \vee r) \wedge \neg q$. Write the first three columns of your truth table in the following format.

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

(3)

(b) Write down one possible value of x for which $(p \vee r) \wedge \neg q$ is true.

(1)
(Total 15 marks)

12. Shiyun bought a car in 1999. The value of the car V, in USD, is depreciating according to the exponential model

$$V = 25\,000 \times 1.5^{-0.2t}, \, t \geq 0,$$

where t is the time, in years, that Shiyun has owned the car.

(a)   Write down the value of the car when Shiyun bought it.

(1)

(b)   Calculate the value of the car three years after Shiyun bought it. Give your answer correct
to **two decimal places**.

(2)

(c)   Calculate the time for the car to depreciate to half of its value since Shiyun bought it.

(3)
(Total 6 marks)

13.   Consider the function $f(x) = p(0.5)^x + q$ where p and q are constants. The graph of $f(x)$ passes
through the points (0, 6) and (1, 4) and is shown below.
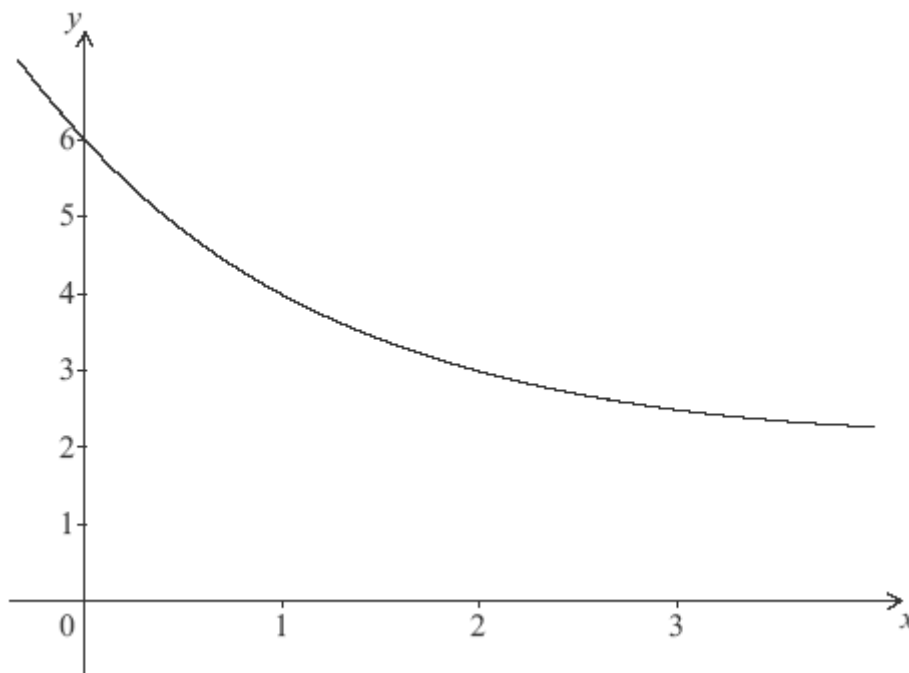
(a)   Write down two equations relating p and q.

(2)

(b)   Find the value of p and of q.

(2)

(c)   Write down the equation of the horizontal asymptote to the graph of $f(x)$.

(2)
(Total 6 marks)

14.   In an experiment researchers found that a specific culture of bacteria increases in number

according to the formula

$$N = 150 \times 2^t,$$

where N is the number of bacteria present and t is the number of hours since the experiment began.

Use this formula to calculate

(a)   the number of bacteria present at the start of the experiment;

(b)   the number of bacteria present after 3 hours;

(c)   the number of hours it would take for the number of bacteria to reach 19 200.

(Total 4 marks)

15.   The population of fleas on a dog after t days, is modelled by

$$N = 4 \times (2)^{\frac{t}{4}} , t \geq 0,$$

Some values of N are shown in the table below.

t	0	4	8	12	16	20
N	p	8	16	32	q	128

(a)   Write down the value of

(i) p;

(ii) q.

(3)

(b)   Using the values in the table above, draw the graph of N for $0 \leq t \leq 20$.
Use 1 cm to represent 2 days on the horizontal axis and 1 cm to represent 10 fleas on the vertical axis.

(6)

(c)   **Use your graph** to estimate the number of days for the population of fleas to reach 55.

(2)
(Total 11 marks)

16.   A rumour spreads through a group of teenagers according to the exponential model

$$N = 2 \times (1.81)^{0.7t}$$

where N is the number of teenagers who have heard the rumour t hours after it is first started.

(a)   Find the number of teenagers who started the rumour.

(2)

(b)   Write down the number of teenagers who have heard the rumour five hours after it is first started.

(1)

(c)   Determine the length of time it would take for 150 teenagers to have heard the rumour.
Give your answer correct to the nearest minute.

(3)
(Total 6 marks)

17. The diagrams below are sketches of some of the following functions.
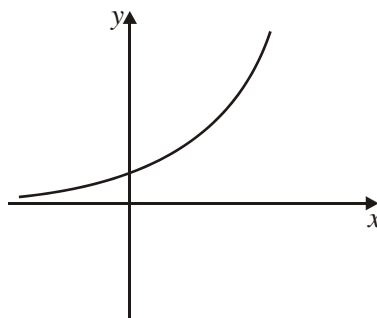
(i) $y = a^x$ (ii) $y = x^2 - a$ (iii) $y = a - x^2$
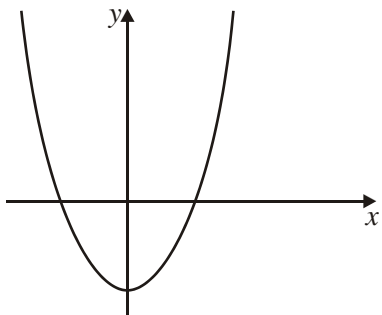
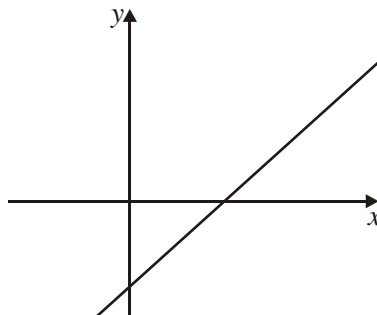(iv) $y = a - x$ (v) $y = x - a$

(a)

(b)
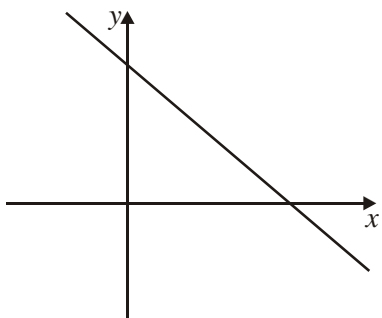
DIAGRAMS NOT TO SCALE

(c)

(d)

Complete the table to match each sketch to the correct function.

Sketch	Function
(a)	
(b)	
(c)	
(d)	

(Total 8 marks)

18. The diagram below shows a part of the graph of $y = a^x$. The graph crosses the y-axis at the point P. The point Q (4, 16) is on the graph.
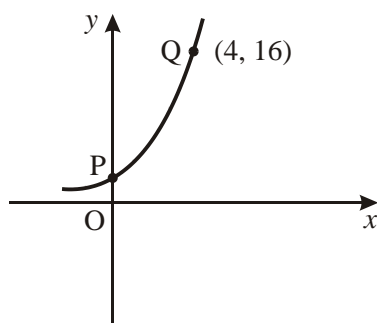

Diagram not to scale

Find

(a) the coordinates of the point P;

(b) the value of a.

(Total 8 marks)

19. The number of cells, C, in a culture is given by the equation $C = p \times 2^{0.5t} + q$, where t is the time in hours measured from 12:00 on Monday and p and q are constants.

The number of cells in the culture at 12:00 on Monday is 47.
The number of cells in the culture at 16:00 on Monday is 53.

Use the above information to

(a) write down two equations in p and q;
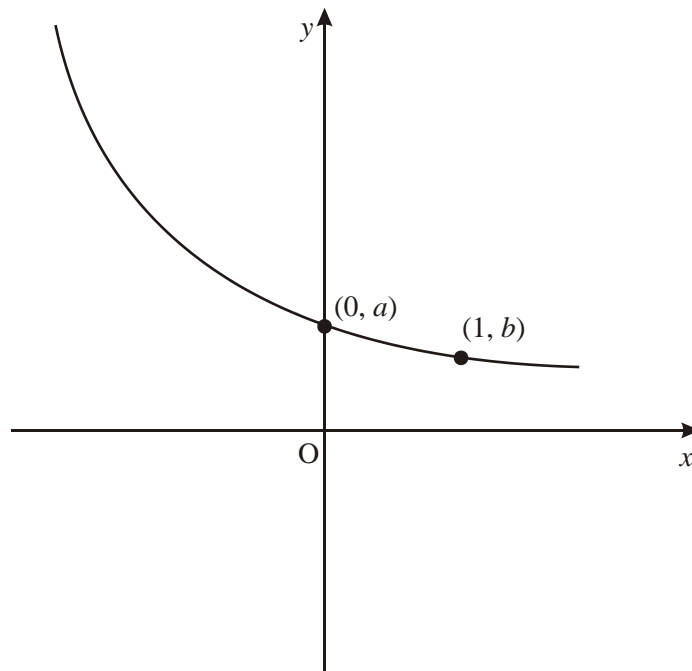
(2)

(b) calculate the value of p and of q;

(2)

(c) find the number of cells in the culture at 22:00 on Monday.

(2)
(Total 6 marks)

20. The following diagram shows the graph of $y = 3^{-x} + 2$. The curve passes through the points (0, a) and (1, b).

Diagram not to scale

(a) Find the value of

    (i) a;

    (ii) b.

(b) Write down the equation of the asymptote to this curve.

(Total 8 marks)