First practice exam
Paper 1

1. a) Given $s = \dfrac{v^2 - u^2}{2a}$, $v = 15.5$, $u = 6$, and $a = 9.8$ calculate the value of s, and

    (i) write your answer to two decimal places (2 d.p.).   *[2 marks]*

    (ii) write your answer to 3 significant figures.   *[2 marks]*

    Another student used the values of $v = 16$, $u = 6$, and $a = 10$ and calculated the value of s as 11.

  b) Calculate the percentage error in the second student's result, compared to your answer to 2 d.p.   *[2 marks]*

2. The following table shows the distribution of the numbers of pens and pencils in all grade 10 students' pencil cases.

Number of pens and pencils	Number of students
$0 \leqslant x < 5$	5
$5 \leqslant x < 10$	7
$10 \leqslant x < 15$	15
$15 \leqslant x < 20$	11
$20 \leqslant x < 25$	9
$25 \leqslant x < 30$	1

  a) Construct a histogram to represent the data.   *[3 marks]*

  b) Write down the modal group.   *[1 mark]*

  c) Calculate an estimate of the mean number of pens and pencils in a student's pencil case.   *[2 marks]*

3. a) Complete the truth table below.

| p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$(\neg p \vee \neg q)$
T	T					
T	F					
F	T					
F	F					

[4 marks]

  b) State whether $\neg(p \wedge q)$ and $(\neg p \vee \neg q)$ are logically equivalent.   *[1 mark]*

  c) Explain your answer to part b).   *[1 mark]*

4. The lines L_1 and L_2 are given on the diagram below where L_2 is normal to L_1
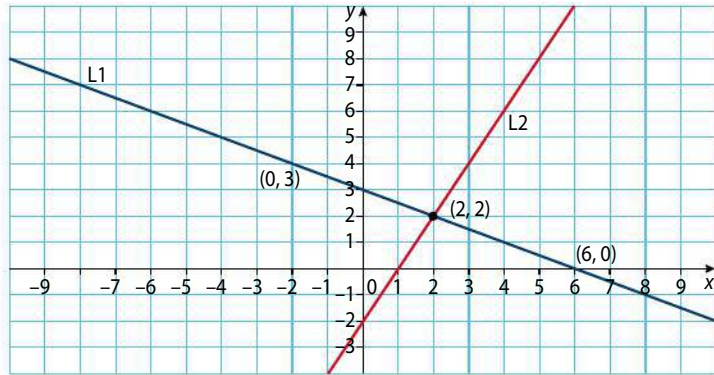
a) What is the gradient of L_1?   [*2 marks*]

b) Write down the equation for L_1.   [*2 marks*]

c) Write down the equation for L_2 in the form $ax + by + c = 0$.

     [*2 marks*]

5. Consider the arithmetic sequence 272, 265, 258, 251, …

   a) Find the value of the common difference of this sequence. [*2 marks*]

   b) Calculate the sum of the first 10 terms of this sequence.   [*2 marks*]

   c) The last number in the sequence is 6. How many terms are in the sequence?   [*2 marks*]

6. The height of a building is 345 m. From the top of the building the angle of depression to the top of a building located horizontally 276 m away is 31°.

   a) Draw a diagram to show this information.   [*2 marks*]

   b) Calculate the height of the second building.   [*4 marks*]

7. A box contains 11 green balls and 5 red balls. Bjarne chooses a ball at random from the box and does not replace the ball.

   a) (i) What is the probability that the ball is red?   [*1 mark*]

     (ii) Ali then takes a ball from the box. What is the probability that the ball is green given that Bjarne took a red?   [*1 mark*]

     (iii) What is the probability that Bjarne chose a red ball and Ali chose a green ball?   [*1 mark*]

   b) Find the probability that Bjarne and Ali chose different coloured balls.   [*3 marks*]

8. A curve of the form $y = ax^2 + bx + c$ is drawn and is shown below.
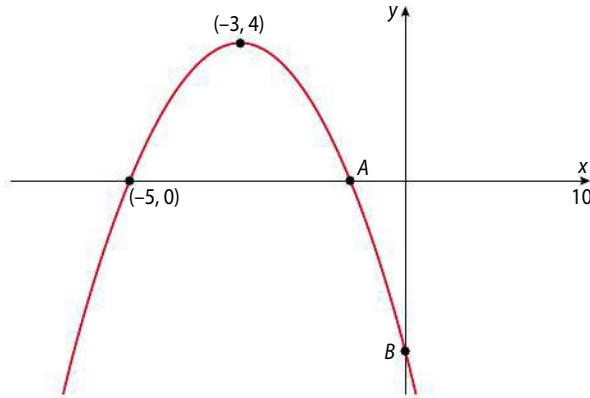
a) Find the coordinates of the point A.   [*2 marks*]

b) Find the coordinates of the point B.   [*3 marks*]

c) Write down the equation of the parabola.   [*1 mark*]

9. The volume of a can of a soft drink is approximately normally distributed with a mean of 358 ml and a standard deviation of 7 ml.

a) It is known that 80% of all cans have a volume less than v ml. Determine the value of v.   [*2 marks*]

b) Sketch a diagram of the distribution of the volume of the cans of drink, indicating the location of v on your diagram.   [*2 marks*]

c) Given that the company produces 10 000 cans of soft drink a day what is the expected number of cans that are below the required volume of 350 ml?   [*2 marks*]

10. The box-and-whisker diagram below shows the statistics for a set of data.
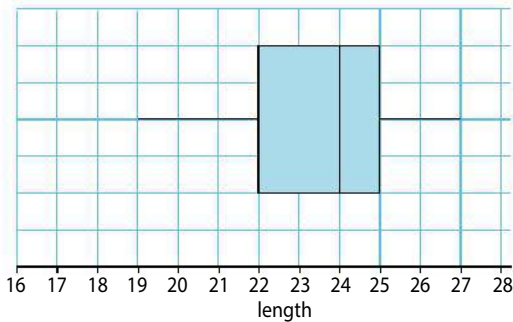

a) Write down the value of each of the following:

 (i) the median

(ii) the upper quartile

(iii) the minimum value

(iv) the interquartile range.   [*4 marks*]

b) A second box-and-whisker diagram is to be drawn on the same grid. The following information is known about the data for this box-and-whisker diagram: the range is 10, the minimum value is 18, the interquartile range is 6, the lower quartile is 19.5, and the median is 23.

(i) Draw the box-and-whisker diagram on the same grid as the first box-and-whisker diagram.

(ii) Describe the differences in the diagrams. [*2 marks*]

11. A square-based pyramid sits on top of a cube as shown.
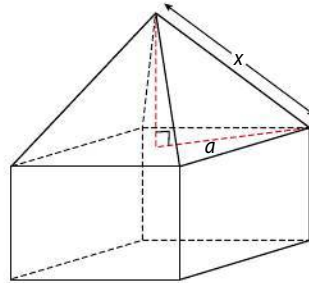
The total height of the object is 25 cm and the cube has a side length of 8 cm.

a) Write down the height of the pyramid. [*2 marks*]

b) Determine the length of *a*. [*2 marks*]

c) Determine the length of *x*. [*2 marks*]

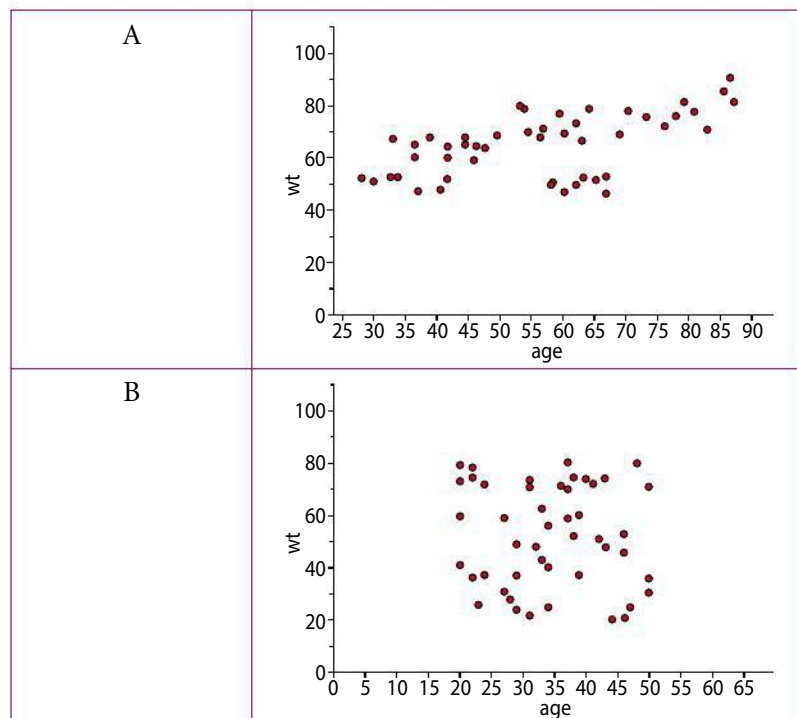12. Data comparing the heights and weights of 190 students were graphed. A researcher was trying to decide if there was any correlation between the height and weight of the students. Indicate which graph(s) show:
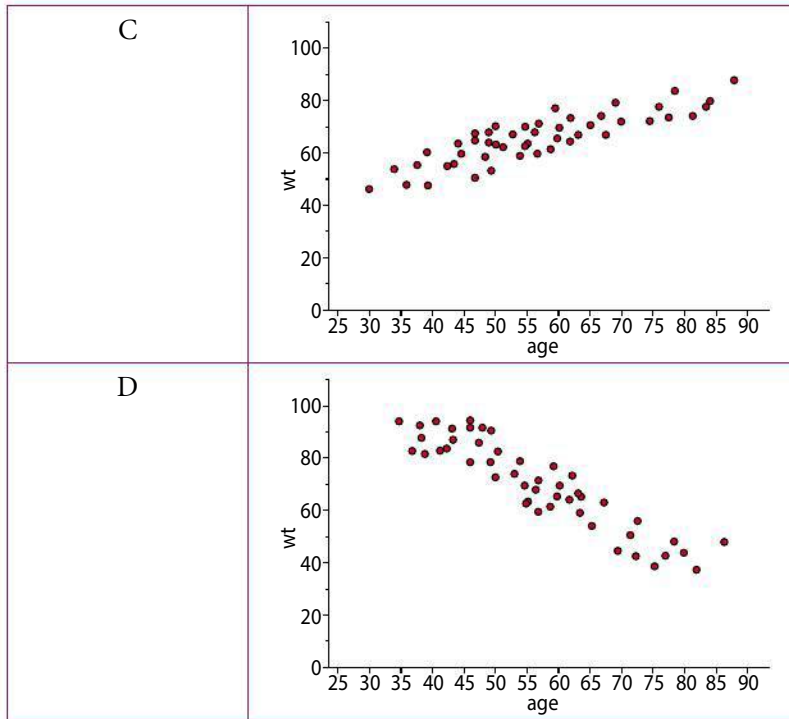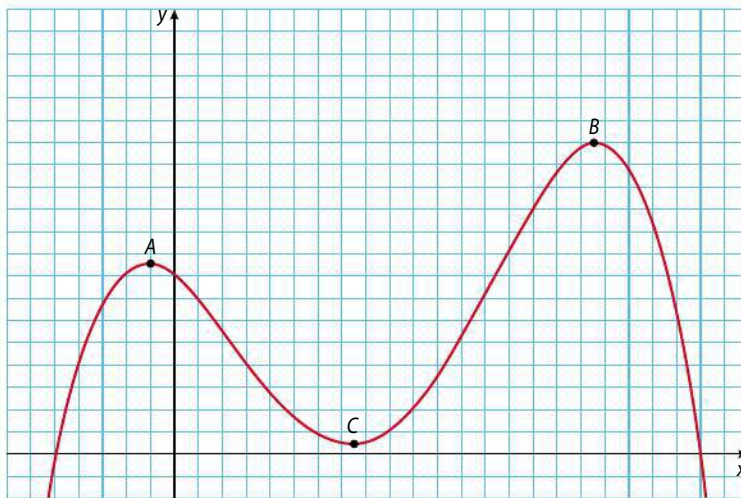
No correlation.
Strong negative correlation.
Moderate positive correlation. [*6 marks*]

| C	
D	

13. A curve is drawn and is given below.

A series of statements is given below. Choose all that are correct.

a) The function has a negative *y*-intercept.

b) The graph has three stationary points.

c) The graph has two *x*-intercepts.

d) The point *C* has zero gradient.

e) The gradient between *A* and *C* is positive.

f) *A* and *B* have the same gradient.   [*6 marks*]

14. A tower (*CD*) sits on top of a building, as shown. $A\hat{B}C$ is a right angle.
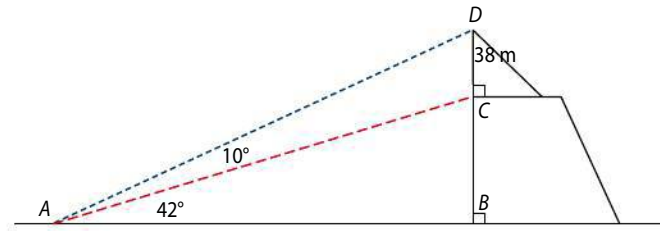
From A, the angles of elevation to C and D are 42° and 52° respectively, and it is known that the height of the tower is 38 m.

   a) Find $A\hat{D}C$.   [*3 marks*]

   b) Hence, determine the height (CB) of the building.   [*3 marks*]

15. $f(x) = \frac{1}{3}x^3 + 2x^2 + \frac{2}{x} - \frac{1}{4}$

   a) Find $f'(x)$.   [*4 marks*]

   Carlie claims that $f'(0) = 0$.

   b)   (i) Is she correct?   [*1 mark*]

        (ii) Explain your answer to b)(i).   [*1 mark*]

Paper 2

1. 113 randomly selected persons were asked what their preferred summer holiday destination was. All of the respondents travelled to the beach, to the mountains, or to another country.

   57 respondents travelled to the beach.

   21 respondents visited the mountains.

   76 respondents visited another country.

   Only 7 respondents visited the mountains and visited the beach, but did not visit another country.

   10 respondents visited the beach and another country, but did not visit the mountains.

   12 respondents visited the beach, the mountains, and another country.

   a) Represent this information on a Venn Diagram.   [*4 marks*]

   b) Calculate the number of respondents:

       (i)   who travelled to the mountains and visited another country, but did not go to the beach;

       (ii) who only visited the mountains.   [*4 marks*]

   Sana indicated in her response that she travelled to another country for her summer vacation.

   c) Find the probability that she also visited the beach.   [*2 marks*]

   Two other respondents were chosen at random from the 113 students.

   d) Find the probability that

       (i)   both respondents visited the beach;

       (ii) only one of the respondents visited the beach.   [*7 marks*]

2. A school offers 4 subjects from the IB Diploma Group 3. These are business management, economics, history, and psychology. The number of students choosing each subject by gender is given below.

	Business Management	Economics	History	Psychology
Male	27	15	7	14
Female	13	7	15	11

A χ^2 (chi-squared) test at the 5% significance level is used to determine whether the choice of subject is independent of gender.

a) What is the total number of persons in the sample and how many males and females were there in the sample? [*3 marks*]

b) Show that the expected frequency for females choosing economics is 9.2844. [*3 marks*]

c) Write down the contingency table for the expected frequencies. [*4 marks*]

d) Write down the p-value for the test. [*2 marks*]

e) State whether the null hypothesis is accepted or rejected. Give reasons for your answer. [*3 marks*]

3. Consider the function $f(x) = \frac{3}{2}x^2 - 5x - 2$.

a) Calculate $f(4)$. [*2 marks*]

b) Write down the y-intercept. [*1 mark*]

c) Determine the x-intercepts. [*3 marks*]

d) Sketch the graph of the function $y = f(x)$ for $-5 \leqslant x \leqslant 5$ and $-10 \leqslant y \leqslant 10$. [*4 marks*]

e) Find $f'(x)$. [*2 marks*]

f) Find the coordinates of the minimum point. [*2 marks*]

g) Find the gradient of the tangent at $x = 4$. [*2 marks*]

h) Determine the equation of the tangent at $x = 4$. [*2 marks*]

i) Determine the equation of the normal at $x = 4$. [*4 marks*]

4. Bob is trying to decide whether to invest \$40 000 or to purchase a new car which will cost \$80 000. Bob can invest the money, \$40 000, at 9% p.a. compounding monthly with the interest added at the end of each month.

a) Write down the investment function $I(t)$ which would be used to calculate the value of the investment at any time. [*2 marks*]

b) Hence, show that he will earn \$3752.30 in interest after the first year. [*2 marks*]

c) Write down how much money Bob will have in the bank at the end of year 2. [*1 mark*]

d) How many years will it take for his money to double? [*2 marks*]

However, Bob has realised that if he buys a new car it will gradually depreciate in value. He has been advised that the rate of depreciation for the car will be 7% per annum.

e) Write down the depreciation function $D(t)$ which would be used to calculate the value of the car at any time. [*2 marks*]

f) Hence, calculate the value of the car after one year, to the nearest dollar. [*1 mark*]

g) Sketch a graph showing both the investment function $I(t)$ and the depreciation function $D(t)$ on the same set of axes, with $0 \leqslant t \leqslant 10$. Indicate on the graph the initial values for each function. [*4 marks*]

h) Determine the value of the car when the investment function $I(t)$ and the depreciation function $D(t)$ are equal. [*4 marks*]

i) When will the value of the car equal $40 000? [*4 marks*]

5. A function is given by the equation $f(x) = 35 \times 1.3^{-0.5x} + 20$.

a) Show that at $x = 0$, $f(x) = 55$. [*2 marks*]

The following table shows values for x and $f(x)$.

x	1	2	3	4	5	6	8	10
$f(x)$	50.7	46.9	43.6	40.7	p	35.9	32.3	q

b)

(i) Write down the values of p and q. [*2 marks*]

(ii) Draw the graph of $f(x)$ for $0 \leqslant x \leqslant 10$. Use a scale of 1 cm to represent 1 on the horizontal axis and a scale of 1 cm to represent 10 on the vertical axis. [*4 marks*]

(iii) **Use your graph** to find how long it takes for $f(x)$ to decrease to 39. Show your method clearly. [*2 marks*]

(iv) Write down the horizontal asymptote. Justify your answer. [*2 marks*]

Consider the function $g(x) = 4x - 9$ for $0 \leqslant x \leqslant 10$.

c) Draw the graph of $g(x)$ on the same set of axes used for part (b). [*2 marks*]

d) **Use your graph** to solve the equation $g(x) = f(x)$. Show your method clearly. [*2 marks*]

Second practice exam
Paper 1

1. A mathematical studies textbook weighs 1.469 kg.

   A school has ordered 50 textbooks for its new Mathematical Studies class.

   a) What is the weight of the parcel? *[3 marks]*

   The delivery company, however, charges to the nearest 5 kg, so they will charge the package at 75 kg.

   b) What is the percentage error in the company's weight calculation?

   [3 marks]

2. Frederick had to change British pounds (GB£) into Swiss francs (CHF) in a bank. The exchange rate is 1 GB£ = 1.5 CHF. There is also a bank charge of 3 GB£ for each transaction.

   a) How many Swiss francs would Frederick buy with 133 GB£?

   [2 marks]

   b) Let s be the number of Swiss francs received in exchange for b GB£. Express s in terms of b. *[2 marks]*

   c) Frederick received 430 CHF. How many British pounds did he exchange? *[2 marks]*

3. In an arithmetic sequence $u_9 = -23$ and $u_{25} = 25$.

   a) Find

   (i) the common difference;

   (ii) the first term. *[4 marks]*

   b) Find S_{25}. *[2 marks]*

4. Weights of 100 apples, in grams, from an orchard were recorded

Weight of apples (g)	Number of apples	Cumulative frequency
$110 \leqslant x < 115$	13	13
$115 \leqslant x < 120$	14	27
$120 \leqslant x < 125$	18	45
$125 \leqslant x < 130$	22	p
$130 \leqslant x < 135$	17	84
$135 \leqslant x < 140$	10	94
$140 \leqslant x < 145$	5	99
$145 \leqslant x < 150$	1	100

   a) Write down the missing value p in the cumulative frequency column. *[1 mark]*

   The cumulative frequency graph showing the weights in grams of the apples is given below.
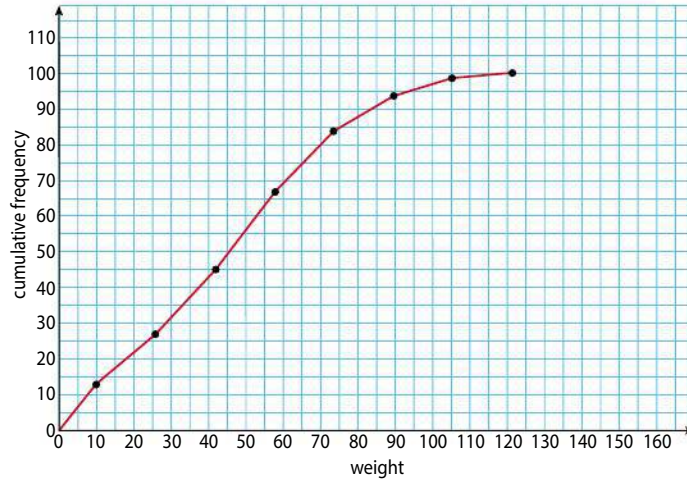
b) Write down

   (i) the median

   (ii) the lower quartile

   (iii) the upper quartile.   [*3 marks*]

c) Draw a box-and-whisker diagram to represent the information.
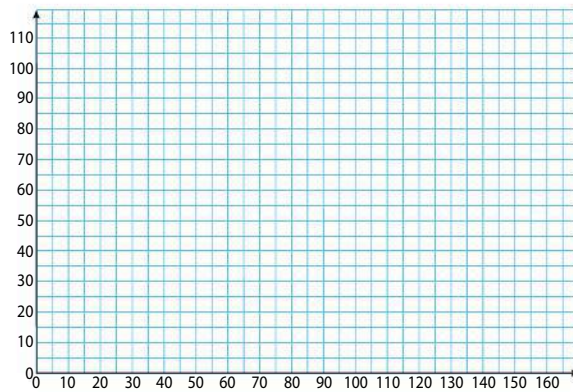
               [*2 marks*]

5. An unbiased, red, five-sided die has the numbers 1, 2, 3, 4, and 5 written on its faces. An unbiased, blue, four-sided die has the numbers 1, 2, 3, and 4 written on its faces. The two dice are rolled.
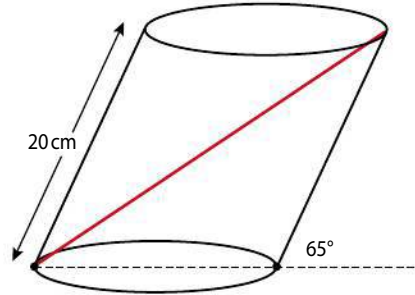
a) Complete the sample space diagram.   [*3 marks*]
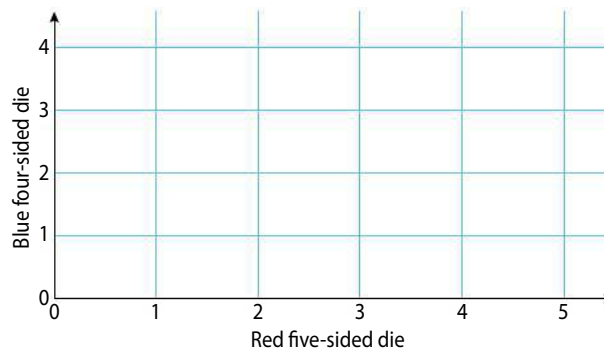
b) Find the probability that the number on the upper face of the blue die is greater than the number on the upper face of the red die.

               [*1 mark*]

c) Find the probability that the red die shows a prime number and the blue die shows a 3. [*2 marks*]

6. A drinking cup has been designed in the shape of a slanted cylinder as shown below.

20 cm

65°

The diameter of the base is 12 cm and the slant height is 20 cm. What is the longest straw that will be required to go from top to bottom as shown in the diagram? [*6 marks*]

7. a) Complete the truth table shown below. [*3 marks*]

p	q	$\neg q$	$p \Rightarrow \neg q$	$\neg p$	$(p \Rightarrow \neg q) \wedge \neg p$	$[(p \Rightarrow \neg q) \wedge \neg p] \Rightarrow q$
T	T	F				
T	F	T	T	F		
F	T		T	T		
F	F	T				

b) State whether the compound proposition $[(p \Rightarrow \neg q) \wedge \neg p] \Rightarrow q$ is a contradiction, a tautology, or neither. [*1 mark*]

Consider the following propositions

p: If the trains are running
q: Sasha will go to school.

c) Write in symbolic form the following proposition. [*2 marks*]

If Sasha does not go to school then the trains are not running.

8. The number of hours that a professional footballer trains each day in the month of June is represented in the histogram.
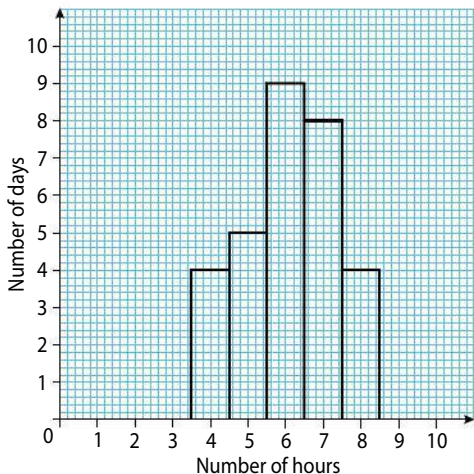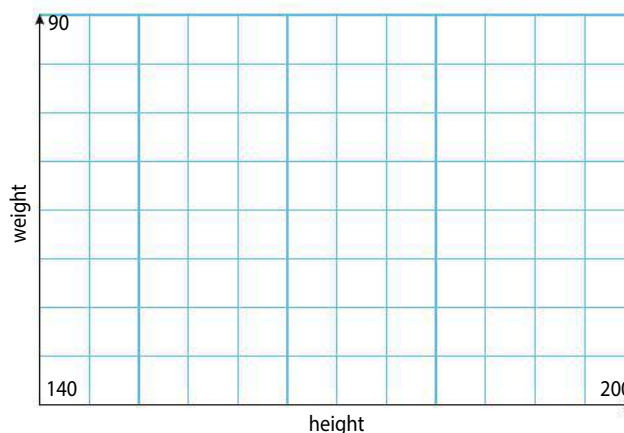
a) Write down the modal number of hours trained each day. [*2 marks*]

b) Calculate the mean number of hours he trains each day.   [*4 marks*]

9. The heights and weights of 10 football players are given below.

Height cm	193	173	178	184	189	185	179	177	174	182
Weight kg	84	61	71	77	79	72	67	66	68	73

a) Draw the data points on the following grid. [*3 marks*]

b) Determine $(\overline{x}, \overline{y})$ and mark the point on the grid. [*2 marks*]

c) Draw a line of best fit. [*1 mark*]

10. The length, width, and height of a rectangular prism are given as $2x$, $9 - x$, and $9 + x$ respectively.

a) Write an expression for the volume of the box, $V(x)$. [*2 marks*]

b) Given that $V'(x) = 162 - 6x^2$, find the value of x which corresponds to the maximum volume of the rectangular prism. [*4 marks*]

11. The straight line M passes through the point $P(3, 4)$ and is parallel to the line $3y + 2x - 6 = 0$.

a) Calculate the gradient of M. [*2 marks*]

b) Find the equation of M. [*2 marks*]

The line N is perpendicular to the line M, and passes through the point $B(-4, 0)$.

c) Determine the equation of the line N. [*2 marks*]

12. The height, in centimetres, of a type of fast-growing wheat is given by the function $h(t) = 1.3 \times 2^{0.5t}$, where t is the time in weeks.

a) Determine the height of the wheat at the end of 6 weeks. [*3 marks*]

b) When the wheat reaches a height of 1.5 metres it can be harvested. After how many weeks will the wheat be ready to harvest? [*3 marks*]

13. An investor invests $20 000 in an account which pays an annual interest rate of 2% compounded half yearly.

a) Find the value after 3 years. [*3 marks*]

b) Find the value after 3 years, if the interest was compounded quarterly.

[*3marks*]

14. The probability that the sun shines today is 0.3. If the sun shines today, then the probability that the sun shines tomorrow is 0.6. If the sun does not shine today, then the probability that it will shine tomorrow is 0.4.

a) Complete the tree diagram. [*3 marks*]

Today   **Tomorrow**

Sunshine 0.3

Sunshine

No sunshine

No sunshine

Sunshine

No sunshine
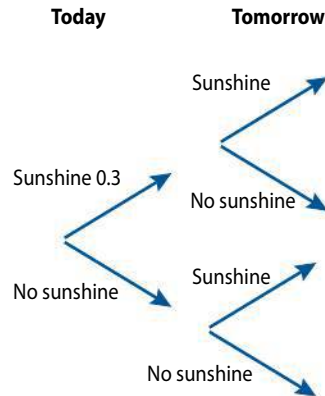
b) Calculate the probability that the sun does not shine tomorrow.

[*3 marks*]

15. A baseball is hit by the batter and it follows a parabolic trajectory. After t seconds, the vertical height above the ground is given by $H(t) = 14t - 0.25t^2$.

a) Find the height of the ball after 10 seconds.   [*2 marks*]

b) Determine the maximum height of the ball.   [*2 marks*]

c) How much time elapses before the ball lands?   [*2 marks*]

Paper 2

1. A triangular garden, *ABC*, has been constructed with $BC = 17$ m, $A\hat{B}C = 109°$, and $C\hat{A}B = 44°$.

Note: diagram not to scale.

B

109°

17 m

44°

A

C
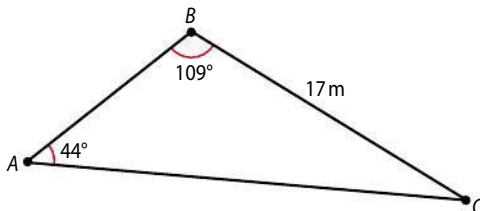
a) Calculate the length of *AC*, correct to 2 d.p.   [*3 marks*]

b) Write down the angle $A\hat{C}B$.   [*1 mark*]

The gardener wants to build a pathway from *B* to the point *D* which is halfway between *A* and *C*.

c) Calculate the length of the pathway from *B* to the midpoint of *AC* to 2 d.p. [*5 marks*]

d) Calculate the area of the triangle *BDC* to 2 d.p. [*3 marks*]

e) The fence around the triangle *BDC* is 20 cm high and the gardener intends to fill the area with soil. Find the volume of soil to be used, in cubic metres. [*3 marks*]

2. Two hundred travellers were surveyed to determine their waiting time at check-in at an international airport. These times are given in the table below.

Waiting time in minutes	Number of travellers
$0 \leqslant x < 5$	3
$5 \leqslant x < 10$	11
$10 \leqslant x < 15$	21
$15 \leqslant x < 20$	36
$20 \leqslant x < 25$	43
$25 \leqslant x < 30$	37
$30 \leqslant x < 35$	22
$35 \leqslant x < 40$	18
$40 \leqslant x < 45$	7
$45 \leqslant x < 50$	2

a) Construct a histogram to represent the data. [*3 marks*]

b) Calculate an estimate of the mean waiting time at the airport check-in. [*3 marks*]

c) Construct a cumulative frequency table [*3 marks*]

d) Draw a cumulative frequency curve on the grid below. [*3 marks*]
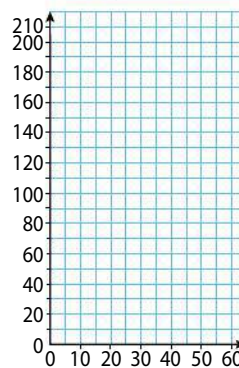
e) Using the cumulative frequency curve, or otherwise, find

   (i) the median waiting time

   (ii) the inter quartile range of the wait time. [*3 marks*]

A mathematician has determined that the waiting times are normally distributed with a mean of 24.25 and a standard deviation of 9.38.

f) Find the probability that a traveller selected at random has a wait time of less than 25 minutes. [*2 marks*]

g) The probability that a person has a wait time of less than W minutes is 0.10. Find the value of W. [*2 marks*]

h) One hundred travellers are randomly selected. Find the expected number of travellers whose wait time is between 30 and 50 minutes. [*2 marks*]

3. The following graph shows the function $f(x) = x^3 - 6x^2 + 8x + c$.
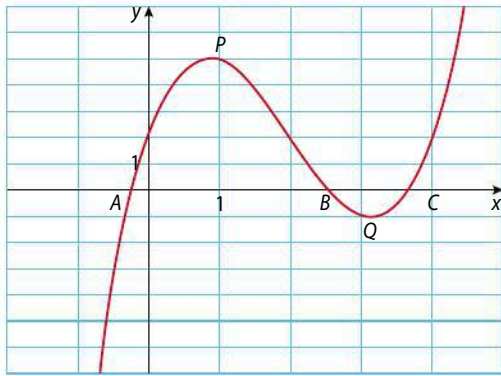
a) Given that $f(1) = 5$, show that $c = 2$. [*3 marks*]

b) The function has 3 solutions for $f(x) = 0$. Write the values of x at these points. [*3 marks*]

c) Write down the gradients at the points P and Q. [*2 marks*]

d) Determine the coordinates of the points P and Q. [*4 marks*]

e) Write down the domain where $f(x)$ is decreasing. [*1 mark*]

A function $g(x) = mx + c$ passes through the points $(0, -2)$ and $(4, 2)$.

f) Determine the equation of the line $g(x)$. [*3 marks*]

g) Using your GDC, graph the functions $f(x)$ and $g(x)$.

  (i) How many solutions are there where $f(x) = g(x)$?

  (ii) Write down the solutions. [*3 marks*]

h) Write down the interval(s) where $f(x) \geqslant g(x)$. [*3 marks*]

4. A circular based cone sits on top of a cylinder.
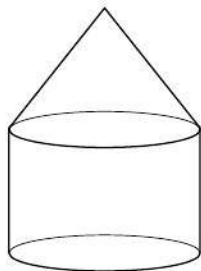
a) (i) Show that the volume of the cone and the cylinder is given by

$V = \pi r^2 (h_{cyl} + \frac{1}{3} h_{cone})$ [*3 marks*]

Isaac has designed a new container which consists of a cone placed on top of a cylinder, where the height of the cylinder = the height of the cone.

(ii) Show that the volume is now given by $V = \frac{4}{3}\pi r^2 h$. [*2 marks*]

(iii) Given that the height of the cylinder is 10 cm and the radius is 5 cm, determine the volume of the combined shape. [*3 marks*]

b) A rectangle has dimensions $(5 + 3x)$ metres and $(7 - 6x)$ metres.

(i) Show that the area, A, of the rectangle can be written as
$A(x) = 35 - 9x - 18x^2$. [*3 marks*]

(ii) Write down the equation of the axis of symmetry of the curve. [*3 marks*]

(iii) Hence, or otherwise, determine the lengths of the sides for the maximum area. [*4 marks*]

5. Data were collected to determine the ages and genders of managers of businesses registered on the local stock exchange. The results are tabulated below.

	< 40 years old	40 ⩽ age < 50	⩾ 50 years old	Total
Male	64	117	67	248
Female	87	112	53	252
Total	151	229	120	500

The table below shows the **expected number** of males and females at each age group.

	< 40 years old	40 ⩽ age < 50	⩾ 50 years old	Total
Male	a	b	60	248
Female	c	d	60	252
Total	151	229	120	500

a) (i) Show that the expected number of males under 40 years old is 75 (to 0 d.p.) [*2 marks*]

(ii) Hence find the values of b, c, and d. Write your answers to 0 d.p. [*3 marks*]

b) (i) Write a suitable null hypothesis for this data.

(ii) Write a suitable alternate hypothesis for this data. [*2 marks*]

c) (i) Write down the degrees of freedom.

(ii) Determine the p-value.

(iii) What conclusion can be drawn regarding gender and age in management positions at the 5% level of significance?

[*6 marks*]

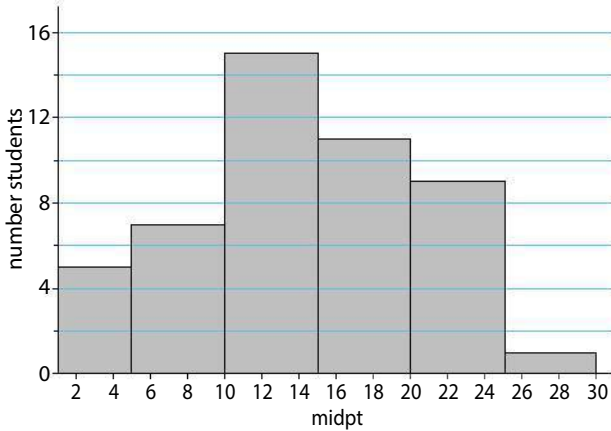# Solutions to first practice exam
Paper 1

1. a) Using our calculators we find that $s = \frac{15.5^2 - 6^2}{2 \times 9.8} = 10.4209$.

(i) To 2 decimal places, this is 10.42. [*2 marks*]

(ii) To 3 significant figures, this is 10.4. [*2 marks*]

b) $V_A = 10.42$ and $V_E = 11$: $s = \left| \dfrac{11 - 10.42}{10.42} \right| \times 100\% = 5.57\%$ to 3 s.f.

[*2 marks*]

2. a)

[*3 marks*]

b) The modal group is $10 \leqslant x < 15$. [*1 mark*]

c) Mean $= \bar{x} = \dfrac{3 \times 5 + 8 \times 7 + 13 \times 15 + 18 \times 11 + 23 \times 9 + 28 \times 1}{48}$

$= 14.6$ to 3 s.f. [*2 marks*]

3. a)

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$(\neg p \vee \neg q)$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

[*4 marks*]

b) Logically equivalent that is $\neg(p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$. [*1 mark*]

c) Logically equivalent as 4th column and 7th column are the same.

[*1 mark*]

4. a) Gradient of $L_1 m_{L1} = \dfrac{0-3}{6-0} = \dfrac{-1}{2}$ [*2 marks*]

b) Equation L_1. $y = \dfrac{-1}{2}x + 3$ [*2 marks*]

c) Gradient of $L_2 = \dfrac{-1}{m_{L1}} = 2$, $-2x + y + 2 = 0$ [*2 marks*]

5. a) Common difference $d = 265 - 272 = -7$ [*2 marks*]

b) $n = 10$, $d = -7$ then $s_n = \dfrac{10}{2}(2 \times 272 + (10-1)(-7)) = 2405$

[*2 marks*]

c) $n = 10$, $d = -7$ then $u_n = 6 = 272 + (n-1)(-7) \therefore n = 39$ [*2 marks*]

6. a)
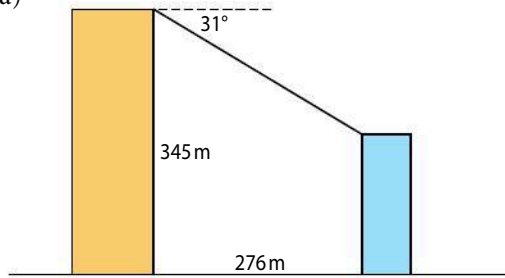
[*2 marks*]

b) Need to solve following triangle for length *l*.
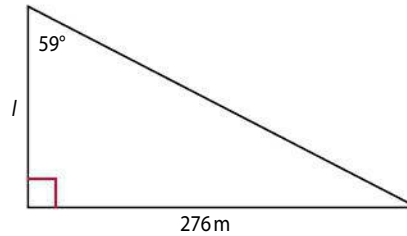
$l = \dfrac{276}{\tan(59°)} = 166$ m to 3 s.f.

Height of second building is $345 - 166 = 179$ m. [*4 marks*]

7. a) (i) Probability a ball is red is $\dfrac{5}{16}$. [*1 mark*]

   (ii) Probability a ball is green given that Bjarne chose a red is $\dfrac{11}{15}$. [*1 mark*]

   (iii) Probability is $\dfrac{55}{240} = \dfrac{11}{48}$. [*1 mark*]

   b) $\dfrac{55}{240} + \dfrac{55}{240} = \dfrac{110}{240} = \dfrac{11}{24}$. [*3 marks*]

8. a) By symmetry, the coordinates of *A* are $(-1, 0)$. [*2 marks*]

   b) The factorized form of the quadratic is written as

   $y = a(x - p)(x - q) = a(x + 1)(x + 5)$

   Expanding $y = a(x^2 + 6x + 5)$

   Substitute a point in for *x* and *y*, e.g. $(-3, 4)$, to solve for *a*.

   $4 = a(9 - 18 + 5) \therefore a = -1$

   $y = -x^2 - 6x - 5$

   Coordinates of *B* are $(0, -5)$. [*3 marks*]

   c) $y = -x^2 - 6x - 5$ [*1 mark*]

9. a) Using a GDC, $v = 367$ ml. [*2 marks*]

b)
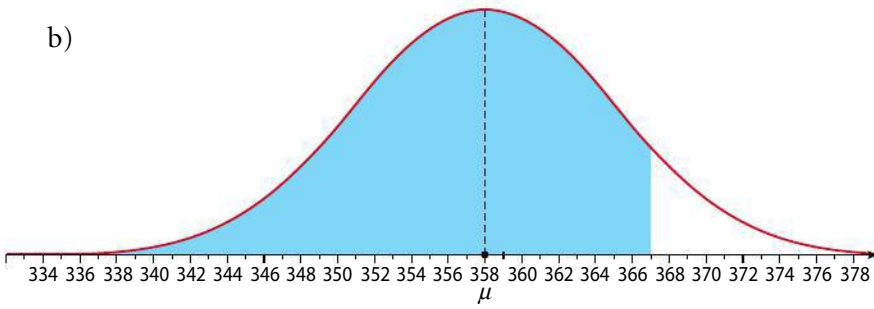
[*2 marks*]

c) Using a GDC, the probability is 0.126549.

Number of cans $= 10\,000 \times 0.126\,549 = 1260$ to 3 s.f.   [*2 marks*]

10. a) (i) the median $= 24$

(ii) the upper quartile $= 25$

(iii) the minimum value $= 19$

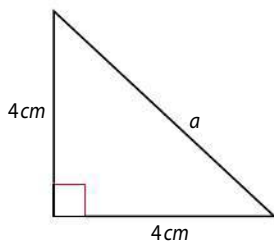(iv) the interquartile range $= 3$ [*4 marks*]

b) (i)
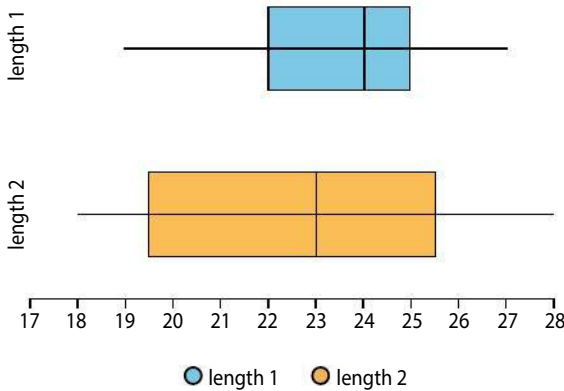
[*1 mark*]

(ii) There is a greater range, the median is lower, the interquartile range is greater, the maximum value is also larger, the data has a greater spread.   [*1 mark*]

11. a) The height of the pyramid is $25 - 8 = 17$ cm.   [*2 marks*]

b) The length a is found by using a triangle on the top of the cube as shown.

$a^2 = 4^2 + 4^2 = 32 \therefore a = \sqrt{32} = 5.657$ cm   [*2 marks*]

c) The length of the edge of the pyramid is found using the following triangle.
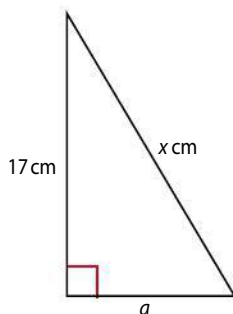
$$x^2 = 32 + 17^2 = 321 \therefore x = \sqrt{321} = 17.9 \text{ cm to 3 s.f.} \qquad \textit{[2 marks]}$$

12. B: Has no correlation.   *[2 marks]*

      D: Strong negative correlation.   *[2 marks]*

      A and C: Moderate positive correlation.   *[2 marks]*
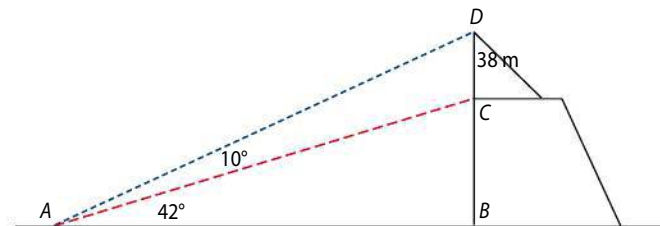
13. Correct statements are:
    b) The graph has three stationary points.
    c) The graph has two *x*-intercepts.
    d) The point *C* has zero gradient.
    f) *A* and *B* have the same gradient.   *[6 marks]*

14. A tower (*CD*) sits on top of a building, as shown. $A\hat{B}C$ is a right angle.

    a) $A\hat{C}B = 90° − 42° = 48°$ and $A\hat{C}D = 180° − 48° = 132°$

        $A\hat{D}C = 180° − 132° − 10° = 38°$

        or:

        $B\hat{A}D = 52°$ *[3 marks]*

        $A\hat{D}C = 90° − 52° = 38°$ *[3 marks]*

    b) Need to calculate length *AC*. Using the sine rule
    $\dfrac{38}{\sin(10)} = \dfrac{AC}{\sin(38)}$, $AC = 134.727$

        $CB = 134.727 \cos(42°) = 100 \text{ m to 3 s.f}$ *[3 marks]*

15. a) Using the power rule: $f'(x) = x^2 + 4x − \dfrac{2}{x^2}$ *[4 marks]*
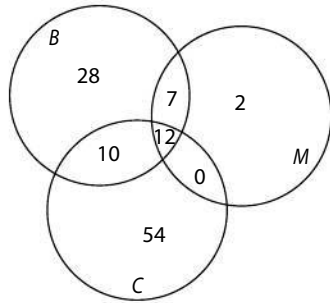
    b)   (i) No, see below.   *[1 mark]*

       (ii) $f(0) = (0)^2 + 4(0) − \dfrac{2}{(0)^2}$

          Since the last term is undefined, the whole expression is undefined.   *[1 mark]*

Paper 2

1. a) Let *B* be respondents who visited the beach, *M* those respondents who visited the mountains and *C* those who visited another country.

   B 28 7 2
   10 12
   0 M
   54
   C

   [*4 marks*]

   b) (i) Number of respondents $M \cap C \cap B' = 0$ [*4 marks*]

   (ii) Only visited the mountains = 2

   c) $P(B|C) = \dfrac{22}{76} = \dfrac{11}{38}$ [*2 marks*]

   d) (i) $\dfrac{57}{113} \times \dfrac{56}{112} = \dfrac{57}{226}$

   (ii) $\dfrac{57}{113} \times \dfrac{56}{112} + \dfrac{56}{113} \times \dfrac{57}{112} = \dfrac{57}{113}$ [*7 marks*]

2. a) Total surveyed is 109 students, 63 males, and 46 females.   [*3 marks*]

   b) Expected frequency $= \dfrac{46}{109} \times \dfrac{22}{109} \times 109 = 9.2844$ [*3 marks*]

   c)

23.1	12.7	12.7	14.4
16.9	9.28	9.28	10.6

   [*4 marks*]

   d) *p*-value = 0.0345   [*2 marks*]

   e) As $0.0345 < 0.05$ then we reject the null hypothesis.   [*3 marks*]

3. Consider the function $f(x) = \dfrac{3}{2}x^2 - 5x - 2$
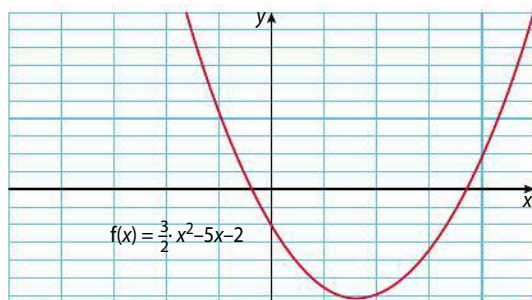
   a) $f(4) = \dfrac{3(4)^2}{2} - 5(4) - 2 = 2$ [*2 marks*]

   b) *y*-intercept = −2   [*1 mark*]

   c) Using GDC, the roots are $x = -0.360921$ and $x = 3.69425$ [*3 marks*]

   d)

 $f(x) = \dfrac{3}{2} \cdot x^2 - 5x - 2$

   [*4 marks*]

   e) $f'(x) = 3x - 5$ [*2 marks*]

● **Examiner's hint:** Use your calculator to find the expected values, and χ^2 values as the examiners are expecting you to do this.

f) At the minimum point $f'(x) = 0 = 3x - 5$.

$x = \dfrac{5}{3} = 1.67$ to 3 s.f.

By substitution or GDC, $y = -6.17$ to 3 s.f. Coordinates of minimum are $(1.67, -6.17)$.

g) Gradient of the tangent at $x = 4$: $f'(4) = 3(4) - 5 = 7$ [*2 marks*]

h) Equation of tangent: $y - y_1 = m(x - x_1)$ using $(4, 2)$

$y - (2) = 7(x - 4)$

$y = 7x - 26$ [*2 marks*]

i) Gradient of normal: at $x = 4$ is $\dfrac{-1}{7}$

Equation of normal: $y - y_1 = m(x - x_1)$ using $(4, 2)$

$y - (2) = \dfrac{-1}{7} \times (x - 4)$

$\therefore y = \dfrac{-1}{7}x + \dfrac{18}{7} = -0.143x + 2.57$ [*4 marks*]

4. a) $I(t) = 40\,000 \left(1 + \dfrac{9}{100 \times 12}\right)^{12t}$ [*2 marks*]

b) $I(1) = 40\,000 \left(1 + \dfrac{9}{100 \times 12}\right)^{12(1)} = 43\,752.30$

Interest earned is $\$43\,752.30 - 40\,000 = 3752.30$ [*2 marks*]

c) $I(2) = 40\,000 \left(1 + \dfrac{9}{100 \times 12}\right)^{12(2)} = \$47\,856.50$ [*1 mark*]

d) Solve $80\,000 = 40\,000 \left(1 + \dfrac{9}{100 \times 12}\right)^{12t}$ for t. Using GDC:


```
Intersection
X=7.7304805 Y=80000
```
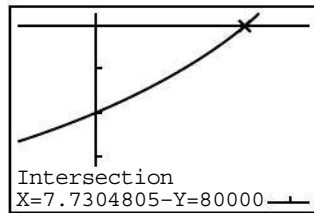
Bob will have $\$80\,000$ after 7 years 9 months, approximately.

     [*2 marks*]

e) $D(t) = 80\,000 \left(1 - \dfrac{7}{100}\right)^{t}$ [*2 marks*]

f) $D(1) = 80\,000 \left(1 - \dfrac{7}{100}\right)^{1} = \$74\,400$ [*1 mark*]
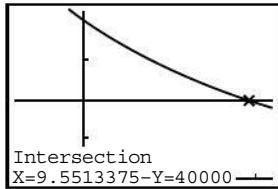
g)

(4.27, 58 700)
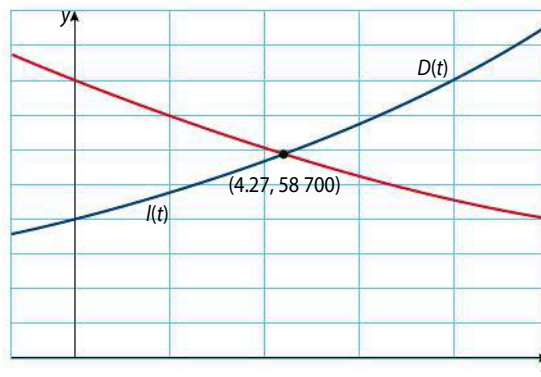
     [*4 marks*]

h) As shown in graph, the values will be equal after 4.27 years.

[*4 marks*]

i) Solve $40\,000 = 80\,000 \left(1 - \dfrac{7}{100}\right)^t$ for t.

Graphically


```
Intersection
X=9.5513375  Y=40000
```

The car will have a value of $40\,000 after 9.55 years.   [*4 marks*]

5. A function is given by the equation $f(x) = 35 \times 1.3^{-0.5x} + 20$.

   a) $f(0) = 35 \times 1.3^{-0.5x} + 20 = 35(1.3)^{-0.5(0)} + 20 = 35 + 20 = 55$

[*2 marks*]

   b) (i) $p = 38.2$, $q = 29.43$ [*2 marks*]

      (ii)

[*4 marks*]

      (iii) $x = 4.7$ [*2 marks*]

      (iv) The horizontal asymptote is $y = 20$. As x becomes larger, the
           graph will approach $y = 20$.   [*2 marks*]

   c) Consider the function $g(x) = 4x - 9$ for $0 \leqslant x \leqslant 10$.

[*2 marks*]

   d) $g(x) = f(x)$ when the graphs intersect. Thus, the solution is $(9.7, 30)$
      by reading from the graph.   [*2 marks*]

Solutions to second practice exam
Paper 1

1. a) The weight of the parcel $= 50 \times 1.469 = 73.45$ kg   [*3 marks*]

   b) Percentage Error $= \left| \dfrac{75 - 73.45}{73.45} \right| \times 100 = 2.11\%$ [*3 marks*]

2. a) Since there is a charge of 3 GB£, our £133 becomes £130 before any conversion occurs. We then convert to CHF:

   $130 \times 1.5 = 195$ CHF   [*2 marks*]

   b) First we subtracted 3 GB£, then we multiplied by 1.5. In a formula, this is equivalent to $s = 1.5(b - 3)$.   [*2 marks*]

   c) We substitute our value of 430 in our formula, and solve for b.
   Hence, $450 = 1.5(b - 3)$

   $\dfrac{450}{1.5} = b - 3$

   $300 + 3 = b$

   $b = 303$ GB£   [*2 marks*]

● **Examiner's hint:** Be careful when asked to create an expression that you do not simplify it unnecessarily. If they do not ask for it simplified, do not waste your valuable exam time simplifying it.

3. a) $u_9 = -23 = u_1 + (9 - 1)d$; $u_{25} = 25 = u_1 + (25 - 1)d$
   $u_1 + 8d = -23$
   $u_1 + 24d = 25$

   Solving equations simultaneously with GDC.

   ```
              SOLUTION
    X1■-47
    X2 = 3


   MAIN MODE SYSM STD F◀▶D
   ```
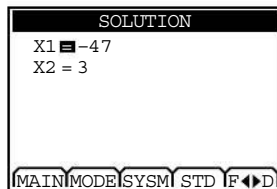
   (i) $u_1 = -47$ and $d = 3$ [*4 marks*]
   (ii) $S_{25} = \dfrac{25}{2}[2 \times -47 + (25 - 1)3] = -275$ [*2 marks*]
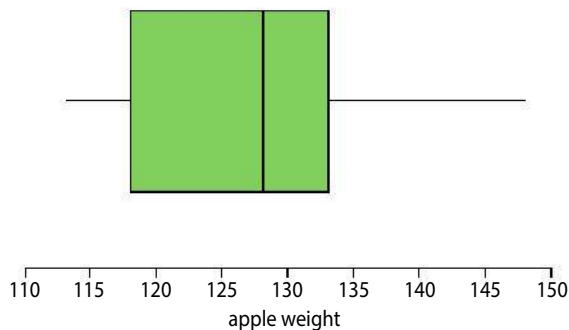
4. a) $p = 67$ [*1 mark*]

   b) (i) median $= 128$

   (ii) the lower quartile $= 118$

   (iii) the upper quartile $= 133$ [*3 marks*]
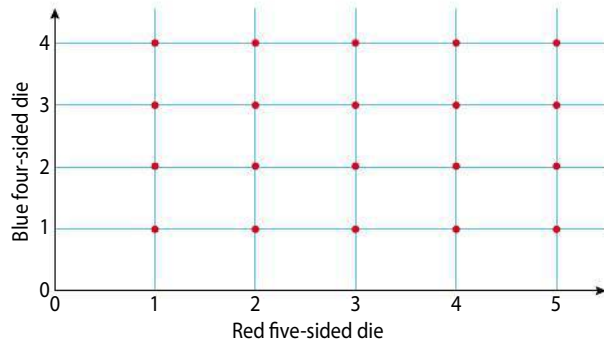
   c)

   apple weight

        [*2 marks*]

5. a) Sample space diagram is:
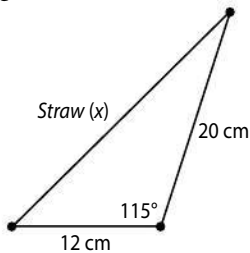

There are 20 possible outcomes. [*3 marks*]

b) There are 6 relevant outcomes. The probability is $\frac{6}{20} = \frac{3}{10}$. [*1 marks*]

c) Prime numbers are 2, 3, 5. The probability is $\frac{3}{20}$. [*2 marks*]

6. Angle between the side and the base is $180° - 65° = 115°$.

Using the cosine rule

$x^2 = 20^2 + 12^2 - 2 \times 20 \times 12 \times \cos(115°) = 746.857$

$x = \sqrt{746.857} = 27.3$ cm. [*6 marks*]

7. a)

p	q	$\neg q$	$p \Rightarrow \neg q$	$\neg p$	$(p \Rightarrow \neg q) \wedge \neg p$	$[(p \Rightarrow \neg q) \wedge \neg p] \Rightarrow q$
T	T	F	F	F	F	T
T	F	T	T	F	F	T
F	T	F	T	T	T	T
F	F	T	T	T	T	F

[*3 marks*]

b) As the final column does not contain all T's or all F's then compound proposition is neither a tautology nor a contradiction.

[*1 mark*]

c) $\neg q \Rightarrow \neg p$ [*2 marks*]

8. a) The modal number of hours is the most frequent number, which is 6 hours. [*2 marks*]
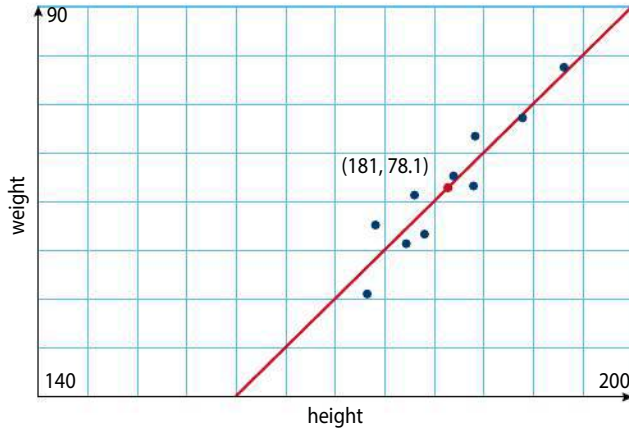
b) The total number of hours is $4 \times 4 + 5 \times 5 + 6 \times 9 + 7 \times 8 + 8 \times 4$
$= 183.$

The total number of days is $4 + 5 + 9 + 8 + 4 = 30$.

The mean is therefore $\frac{183}{30} = 6.10$ hours. [*4 marks*]
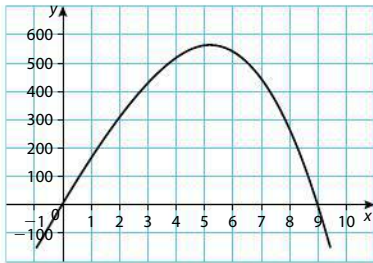
9. a)

[*3 marks*]

b) $\bar{x} = 181, \bar{y} = 78.1$ [*2 marks*]

c) Shown on graph. [*1 mark*]

● **Examiner's hint:** Often a sketch of a graph can be the fastest way to identify a local extreme as a maximum or a minimum. Be sure to include a reasonably neat sketch with your solution, should you use this method.

10. a) The volume of a rectangular prism is $V = length \times width \times height$ which becomes

$$V = 2x(9 - x)(9 + x)$$ [*2 marks*]

b) We set $V'(x) = 0$ and solve for x first. Hence,

$$0 = 162 - 6x^2$$
$$6x^2 = 162$$
$$x^2 = 27$$
$$x = \pm\sqrt{27} \approx \pm 5.20$$

Since we require x to be positive $x = \sqrt{27} \approx 5.20$ is our solution. [*4 marks*]

11. a) Gradient of $3y + 2x - 8 = 0$, rearrange $y = -\frac{2}{3}x - 8$

gradient of $M = \frac{-2}{3}$ [*2 marks*]

b) $y - 4 = -\frac{2}{3}(x - 3)$

Equation of line M: $y = -\frac{2}{3}x + 6$ [*2 marks*]

c) Gradient of line $N = \frac{3}{2}$

$y - 0 = \frac{3}{2}(x + 4)$

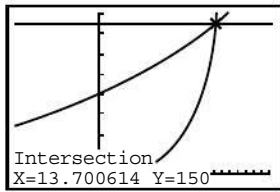Equation of line N: $y = \frac{3}{2}(x + 4)$ [*2 marks*]

12. a) $h(t) = 1.3 \times 2^{0.5 \times 6} = 10.4$ cm [*3 marks*]

b) Need to solve for 1.5 metres (150 cm.)

$$1.3 \times 2^{0.5t} = 150$$

Using GDC, draw the graph for $Y1 = 1.3 \times 2^{0.5t}$ and $Y2 = 150$ and find the point of intersection.

Intersection
X=13.700614 Y=150

The wheat will be ready to harvest at 13.7 weeks. [*3 marks*]
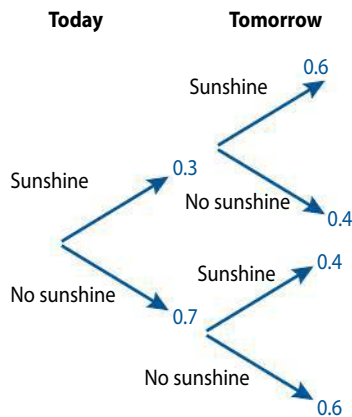
13. a) Using the compound interest formula,

$$FV = 20\,000 \left(1 + \frac{2}{100 \times 2} \right)^{2 \times 3} = \$21\,230.40 \text{ to the nearest 10 cents.}$$
[*3 marks*]

b) If the interest is compounded quarterly then the equation is given by $FV = 20\,000 \left(1 + \frac{2}{100 \times 4} \right)^{4 \times 3} = \$21\,233.60$

The difference in interest is $\$21\,233.60 - \$21\,230.40 = \$3.20$ [*3 marks*]

14. a) Tree diagram

Today   **Tomorrow**

Sunshine
0.3
Sunshine 0.6
No sunshine 0.4

No sunshine
0.7
Sunshine 0.4
No sunshine 0.6

[*3 marks*]

b) Probability that the sun does not shine tomorrow is
$0.3 \times 0.4 + 0.7 \times 0.6 = 0.54$ [*3 marks*]
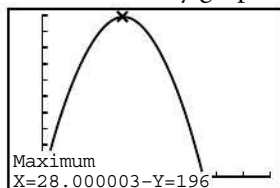
15. a) $H(10) = 14(10) - 0.25(10)^2 = 115$ m [*2 marks*]

b) Maximum height occurs at axis of symmetry
$$t = \frac{-b}{2a} = \frac{-14}{2 \times (-0.25)} = 28$$

$H(28) = 14(28) - 0.25(28)^2 = 196$ m

Alternatively graphically.

Maximum
X=28.000003 Y=196

[*2 marks*]

c) Need to solve for the *x*-axis intercepts. i.e. $14t - 0.25t^2 = 0$

$14t - 0.25t^2 = t(14 - 0.25t) = 0$, $t = 0, 56$

This can also be done by symmetry. As there is an axis of symmetry at $t = 28$, then the ball lands at $t = 2 \times 28 = 56$ seconds.   [*2 marks*]

Paper 2

1. a) Using the sine rule, $\dfrac{AC}{\sin(109)} = \dfrac{17}{\sin(44)}$.
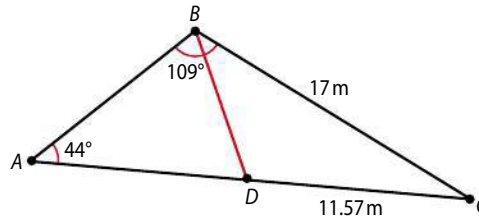
$AC = \dfrac{17}{\sin(44)} \sin(109)$

$AC = 23.1$ m   [*3 marks*]

b) $A\hat{C}B = 180° - (109° + 44°) = 27°$.   [*1 marks*]

c)

Distance between C and $D = \dfrac{1}{2}AC = \dfrac{1}{2} \times 23.1392 = 11.57$ m

Using the cosine rule

$BD^2 = 17^2 + 11.57^2 - 2 \times 17 \times 11.57 \times \cos(27°) = 72.36$

$BD = 8.51$ m   [*5 marks*]

d) Area of triangle $BDC = \dfrac{1}{2} \times 17 \times 11.57 \times \sin(27°) = 44.65$ m².   [*3 marks*]
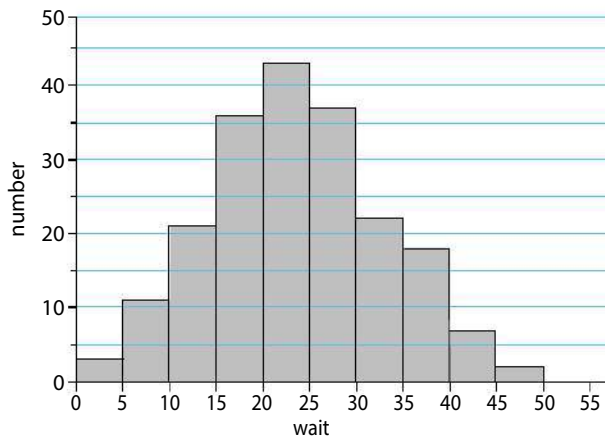
e) Convert 20 cm to metres, i.e. 0.2 m. Volume $= 0.2 \times 44.65 = 8.93$ m³   [*3 marks*]

• **Examiner's hint:** Remember to always express the answer in the form requested, in this case 2 d.p. or to 3 significant figures. Either way, do not forget to include the appropriate units for the question.

2. a)

[*3 marks*]

• **Examiner's hint:** As this question involves a lot of data, it would be helpful to enter the data into your GDC before doing any calculations.

b) $$\frac{3 \times 3 + 11 \times 8 + 21 \times 13 + 36 \times 18 + 43 \times 23 + 37 \times 28 + 22 \times 33 + 18 \times 38 + 7 \times 43 + 2 \times 48}{200}$$

= 24.25 to 3 s.f. = mean

Alternatively using GDC.

```
    1–Var Stats
x̄=24.25
 x=4850
 x²=135200
Sx=9.401031698
 x=9.377499667
↓n=200
```

[*3 marks*]

c)

Waiting time in minutes	Number of travellers	Cumulative frequency
$0 \leqslant x < 5$	3	3
$5 \leqslant x < 10$	11	14
$10 \leqslant x < 15$	21	35
$15 \leqslant x < 20$	36	71
$20 \leqslant x < 25$	43	114
$25 \leqslant x < 30$	37	151
$30 \leqslant x < 35$	22	173
$35 \leqslant x < 40$	18	191
$40 \leqslant x < 45$	7	198
$45 \leqslant x < 50$	2	200

```
L1      L2      L3      3
 3       3     -------
 8      11
13      21
18      36
23      43
28      37
33      22
L3=cumSum(L2)
```

• **Examiner's hint:** cumSum on GDC can be used.

[*3 marks*]

d)

• **Examiner's hint:** When 'otherwise' is used in a question it is left to the candidate to choose their preferred method.
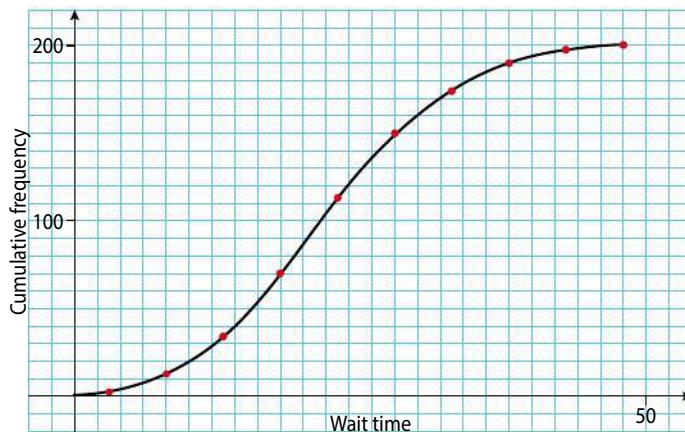
[*3 marks*]

e) (i) Median waiting time is approximately 23 minutes.   [*1 mark*]
    (ii) Interquartile range is = $Q_3 - Q_1 = 28 - 18 = 10$.   [*2 marks*]
f) Normal distribution with mean $\mu = 24.25$ and $\sigma = 9.38$.
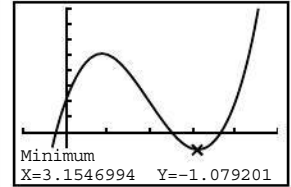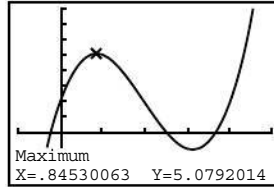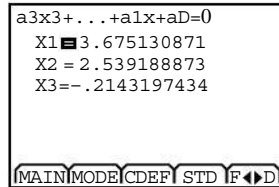   Using GDC $P(X < 25) = 0.532$.   [*2 marks*]
g) Inverse normal calculation $W = 12.2$ minutes.   [*2 marks*]

```
normalcdf(-1E99▶
    .5318644822
invNorm(.1,24.2!▶
    12.22904631
```

h) Using GDC, the lower bound is 30, the upper bound is 50 with $\mu = 24.25$ and $\sigma = 9.38$. Probability $= 0.2669$. Expected number of passengers is $0.2669 \times 100 = 26.7 \approx 27$ passengers. *[2 marks]*

3. a) $f(x) = x^3 - 6x^2 + 8x + c$

$f(1) = 1^3 - 6(1)^2 + 8(1) + c = 5$

$3 + c = 5$

$c = 5 - 3 = 2$ *[3 marks]*

b) Using GDC and plysmlt2. Solve $x^3 - 6x^2 + 8x + 2 = 0$ for the x-intercepts.


```
a3x3+...+a1x+aD=0
 X1▣3.675130871
 X2 = 2.539188873
 X3=-.2143197434

MAIN MODE CDEF STD F◀▶D
```

```
Maximum
X=.84530063  Y=5.0792014
```

```
Minimum
X=3.1546994  Y=-1.079201
```

$A(-0.214, 0)$, $B(2.54, 0)$ and $C(3.68, 0)$ *[3 marks]*

c) P and Q are turning points and the gradient in each case is zero. *[2 marks]*

d) Using GDC to find maximum and minimum:

$P(0.845, 5.08)$ and $Q(3.15, -1.08)$ *[4 marks]*

e) $f(x)$ is decreasing when $0.845 < x < 3.15$ *[1 mark]*
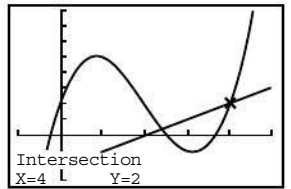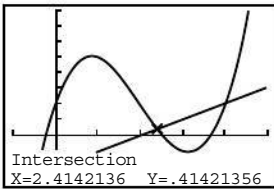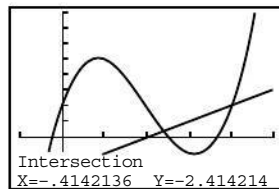
f) Gradient $m = \dfrac{2 - (-2)}{4 - 0} = 1$

$y - y_1 = m(x - x_1)$

Using $(0, -2)$ $y - (-2) = 1(x - 0)$

$g(x) = x - 2$ *[3 marks]*

g) (i) There are 3 solutions, points of intersection.

(ii) Using GDC:

```
Intersection
X=-.4142136  Y=-2.414214
```

```
Intersection
X=2.4142136  Y=.41421356
```

```
Intersection
X=4  L      Y=2
```

Solutions are at $x = -0.414$, 2.41, 4 *[3 marks]*

h) $f(x) \geqslant g(x)$ when $-0.414 \leqslant x \leqslant 2.41$ and $x \geqslant 4$ *[3 marks]*

4. a) (i) Volume of the cone $V_{\text{cone}} = \dfrac{1}{3}\pi r^2 h_{\text{cone}}$ and

Volume of cylinder $V_{\text{cyl}} = \pi r^2 h_{\text{cyl}}$

Combined volume

$V = \pi r^2 h_{\text{cyl}} + \dfrac{1}{3}\pi r^2 h_{\text{cone}} = \pi r^2\left(h_{\text{cyl}} + \dfrac{1}{3}h_{\text{cone}}\right)$ *[3 marks]*

(ii) New volume $V = \pi r^2\left(h + \dfrac{1}{3}h\right)$

$V = \dfrac{4}{3}\pi r^2 h$ *[2 marks]*

(iii) $V = \frac{4}{3}\pi \quad 5^2 \quad 10 = 1047.2 \quad 1050$ cubic cm to 3 s.f.   [3 marks]

b) A rectangle has dimensions $(5 + 3x)$ metres and $(7 - 6x)$ metres.

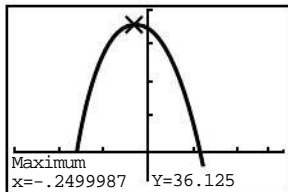(i) $A(x) = (5 + 3x) \quad (7 - 6x) = 35 + 21x - 30x - 18x^2$

   $A(x) = 35 - 9x - 18x^2$ [3 marks]

(ii) Axis of symmetry $= x = \frac{-b}{2a} = \frac{9}{2 \quad -18} = \frac{-1}{4}$.   [3 marks]

(iii) Hence, or otherwise, determine the length of the sides for the maximum area.

Using GDC:


```
Maximum
x=-.2499987   Y=36.125
```

Maxmimum area occurs at $x = -\frac{1}{4}$

$(5 + 3x) = 5 + 3 \quad \frac{-1}{4} = 4\frac{1}{4} = 4.25$ m and $(7 - 6x) = 7 - 6 \quad \frac{-1}{4}$

$= 8\frac{1}{2} = 8.5$ m length $= 8.5$ m and width $= 4.25$ m

[4 marks]

5. a) (i) Expected number of males under 40 $= \frac{248}{500} \quad \frac{151}{500} \quad 500$

   $= 74.9 \quad 75$ [2 marks]

(ii) b, c, and d are shown in the table.   [3 marks]

	< 40 years old	40 ≤ age < 50	≥ 50 years old	Total
Male	75	114	60	248
Female	76	115	60	252
Total	151	229	120	500

b) (i) Null hypothesis; The age group of managers is independent of gender.   [1 mark]

(ii) Alternate hypothesis: The age group of managers is not independent of gender.   [1 mark]

c) (i) degrees of freedom $= 2$

(ii) p-value from GDC $= 0.0591$

(iii) As $0.0591 > 0.05$ then we do not reject the null hypothesis.

[6 marks]