Diploma Programme
Programme du diplôme
Programa del Diploma

# Mathematical studies
# Standard level
# Paper 1

Monday 13 November 2017 (afternoon)

Candidate session number

| | | | | | | | | | |
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| | | | | | | | | | |

1 hour 30 minutes

**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

8817 – 7401

20 pages

Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Answers must be written within the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

**1.** A group of $20$ students travelled to a gymnastics tournament together. Their ages, in years, are given in the following table.

| **Age (years)** | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 22 |
|---|---|---|---|---|---|---|---|---|
| **Frequency** | 1 | 2 | 7 | 1 | 4 | 1 | 1 | 3 |

(a) For the students in this group

(i) find the mean age;

(ii) write down the median age. [3]

The lower quartile of the ages is $16$ and the upper quartile is $18.5$.

(b) Draw a box-and-whisker diagram, for these students' ages, on the following grid. [3]


**(This question continues on the following page)**

**(Question 1 continued)**

**Working:**

**Answers:**

(a) (i) . . . . . .

(ii) . . . . . .

**2.** The coordinates of point A are $(6, -7)$ and the coordinates of point B are $(-6, 2)$. Point M is the midpoint of AB.

(a) Find the coordinates of M. [2]

$L_1$ is the line through A and B.

(b) Find the gradient of $L_1$. [2]

The line $L_2$ is perpendicular to $L_1$ and passes through M.

(c) (i) Write down the gradient of $L_2$.

(ii) Write down, in the form $y = mx + c$, the equation of $L_2$. [2]

**Working:**

**Answers:**

(a) . . . . . .

(b) . . . . . .

(c) (i) . . . . . .

(ii) . . . . . .

**3.** The speed of light is $300\,000$ kilometres per second. The average distance from the Sun to the Earth is $149.6$ million $\text{km}$.

(a) Calculate the time, **in minutes**, it takes for light from the Sun to reach the Earth. [3]

A light-year is the distance light travels in one year and is equal to $9\,467\,280$ million $\text{km}$. Polaris is a bright star, visible from the Northern Hemisphere. The distance from the Earth to Polaris is $323$ light-years.

(b) Find the distance from the Earth to Polaris in millions of $\text{km}$. Give your answer in the form $a \times 10^k$ with $1 \leq a < 10$ and $k \in \mathbb{Z}$. [3]

**Working:**

**Answers:**

(a) . . . . . .

(b) . . . . . .

Please **do not** write on this page.

Answers written on this page
will not be marked.

**4.** Consider the following propositions.

$p$: The car is under warranty
$q$: The car is less than $2$ years old
$r$: The car has been driven more than $20\,000\,\text{km}$

(a) Write down in words $(q \vee \neg r) \Rightarrow p$. [3]

(b) Complete the truth table. [2]

| $p$ | $q$ | $r$ | $\neg r$ | $q \vee \neg r$ | $(q \vee \neg r) \Rightarrow p$ |
|---|---|---|---|---|---|
| T | T | T | F | | |
| T | T | F | T | | |
| T | F | T | F | | |
| T | F | F | T | | |
| F | T | T | F | | |
| F | T | F | T | | |
| F | F | T | F | | |
| F | F | F | T | | |

(c) State whether the statement $\neg p \Rightarrow \neg(q \vee \neg r)$ is the inverse, the converse or the contrapositive of the statement in part (a). [1]

**Working:**

**Answers:**

(a) . . . . . .

. . . . . .

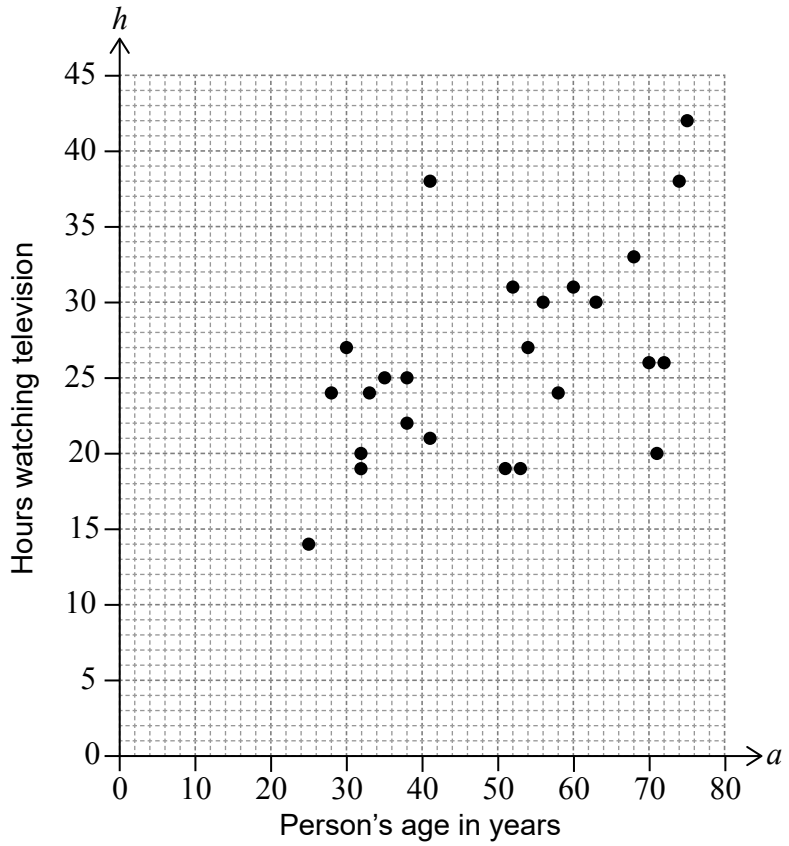
. . . . . .

. . . . . .

. . . . . .

(c) . . . . . .

**5.** A survey was carried out to investigate the relationship between a person's age in years ($a$) and the number of hours they watch television per week ($h$). The scatter diagram represents the results of the survey.




The mean age of the people surveyed was $50$.

For these results, the equation of the regression line $h$ on $a$ is $h = 0.22a + 15$.

(a) Find the mean number of hours that the people surveyed watch television per week. [2]

(b) Draw the regression line on the scatter diagram. [2]

(c) By placing a tick (✓) in the correct box, determine which of the following statements is true:

| | |
|---|---|
| The correlation between $h$ and $a$ is positive. | |
| The correlation between $h$ and $a$ is negative. | |
| There is no correlation between $h$ and $a$. | |

[1]

(d) Diogo is $18$ years old. Give a reason why the regression line should not be used to estimate the number of hours Diogo watches television per week. [1]

**(This question continues on the following page)**

**(Question 5 continued)**

**Working:**

**Answers:**

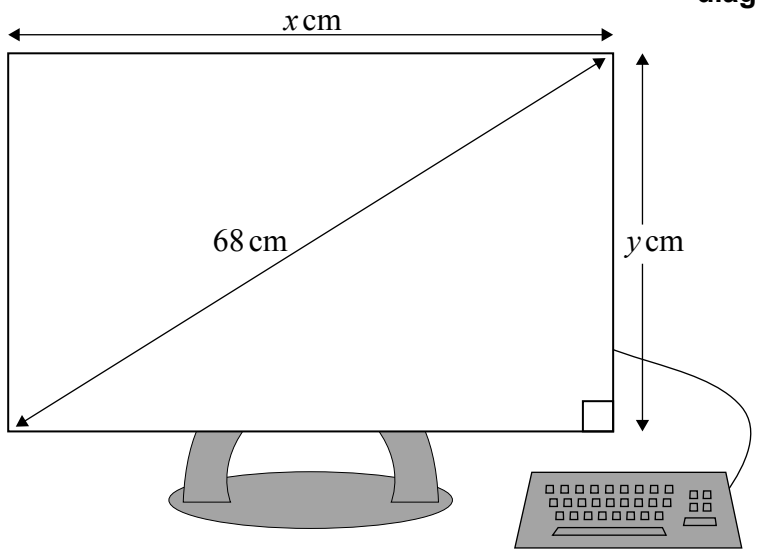
(a) . . . . . .

(d) . . . . . .

. . . . . .

**6.** The size of a computer screen is the length of its diagonal. Zuzana buys a rectangular computer screen with a size of $68\,\text{cm}$, a height of $y\,\text{cm}$ and a width of $x\,\text{cm}$, as shown in the diagram.

**diagram not to scale**




(a) Use this information to write down an equation involving $x$ and $y$. [1]

The ratio between the height and the width of the screen is $3{:}4$.

(b) Use this ratio to write down $y$ in terms of $x$. [2]

(c) Find the value of $x$ and of $y$. [3]

**Working:**

**Answers:**

(a) . . . . . .

(b) . . . . . .

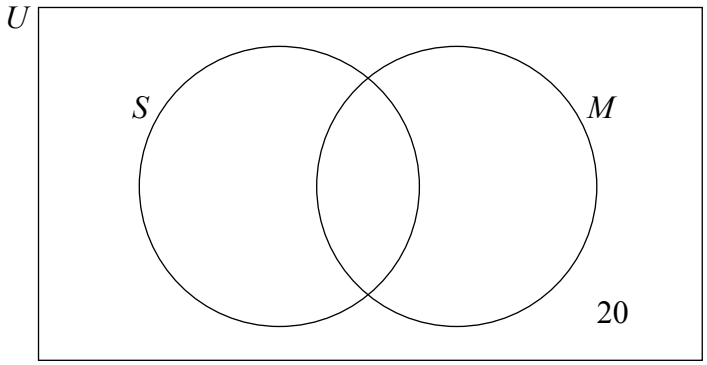
(c) . . . . . .

**7.** Rosewood College has $120$ students. The students can join the sports club ($S$) and the music club ($M$).

For a student chosen at random from these $120$, the probability that they joined both clubs is $\frac{1}{4}$ and the probability that they joined the music club is $\frac{1}{3}$.

There are $20$ students that did not join either club.

(a) Complete the Venn diagram for these students. [2]




(b) One of the students who joined the sports club is chosen at random. Find the probability that this student joined both clubs. [2]

(c) Determine whether the events $S$ and $M$ are independent. [2]

**Working:**

**Answers:**

(b) . . . . . .

(c) . . . . . .

**8. In this question, give all answers correct to 2 decimal places.**

Jose travelled from Buenos Aires to Sydney. He used Argentine pesos, $\mathrm{ARS}$, to buy $350$ Australian dollars, $\mathrm{AUD}$, at a bank. The exchange rate was $1\,\mathrm{ARS} = 0.1559\,\mathrm{AUD}$.

(a) Use this exchange rate to calculate the amount of $\mathrm{ARS}$ that is equal to $350\,\mathrm{AUD}$. [2]

The bank charged Jose a commission of $2\%$.

(b) Calculate the **total** amount of $\mathrm{ARS}$ Jose paid to get $350\,\mathrm{AUD}$. [2]

Jose used his credit card to pay his hotel bill in Sydney. The bill was $585\,\mathrm{AUD}$. The value the credit card company charged for this payment was $4228.38\,\mathrm{ARS}$. The exchange rate used by the credit card company was $1\,\mathrm{AUD} = x\,\mathrm{ARS}$. No commission was charged.

(c) Find the value of $x$. [2]

**Working:**

**Answers:**

(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

**9.** Juan buys a bicycle in a sale. He gets a discount of $30\,\%$ off the original price and pays $560$ US dollars (USD).

(a) Calculate the original price of the bicycle. [2]

To buy the bicycle, Juan takes a loan of $560\,\text{USD}$ for $6$ months at a nominal annual interest rate of $75\,\%$, **compounded monthly**. Juan believes that the total amount he will pay will be less than the original price of the bicycle.

(b) Calculate the difference between the original price of the bicycle and the total amount Juan will pay. [4]

**Working:**

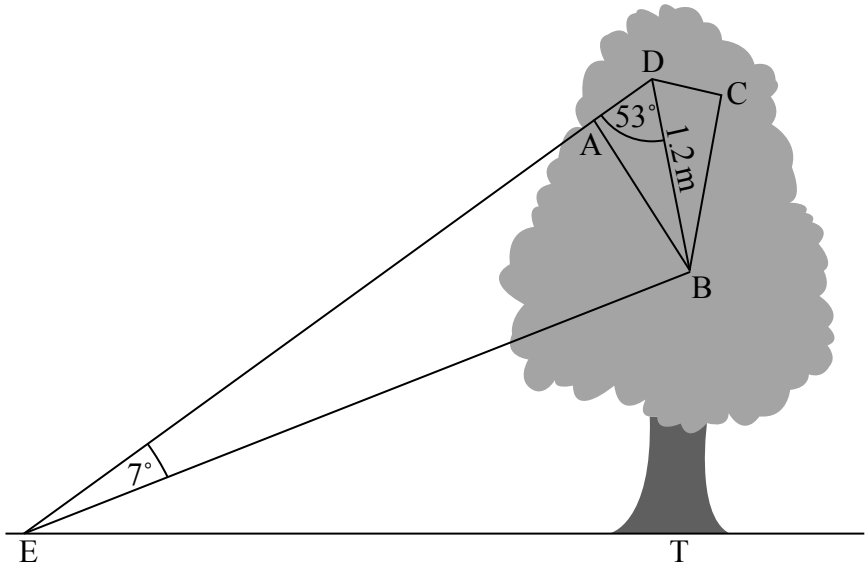
**Answers:**

(a) . . . . . .

(b) . . . . . .

**10.** Emily's kite $ABCD$ is hanging in a tree. The plane $ABCDE$ is vertical.

Emily stands at point $E$ at some distance from the tree, such that $EAD$ is a straight line and angle $BED = 7^\circ$. Emily knows $BD = 1.2$ metres and angle $BDA = 53^\circ$, as shown in the diagram.




(a) Find the length of $EB$. [3]

$T$ is a point at the base of the tree. $ET$ is a horizontal line. The angle of elevation of $A$ from $E$ is $41^\circ$.

(b) Write down the angle of elevation of $B$ from $E$. [1]

(c) Find the vertical height of $B$ above the ground. [2]

**(This question continues on the following page)**

**(Question 10 continued)**

**Working:**

**Answers:**

(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

**11.** A quadratic function $f$ is given by $f(x) = ax^2 + bx + c$. The points $(0, 5)$ and $(-4, 5)$ lie on the graph of $y = f(x)$.

(a) Find the equation of the axis of symmetry of the graph of $y = f(x)$. [2]

(b) Write down the value of $c$. [1]

The $y$-coordinate of the minimum of the graph is $3$.

(c) Find the value of $a$ and of $b$. [3]

**Working:**

**Answers:**

(a) . . . . . .

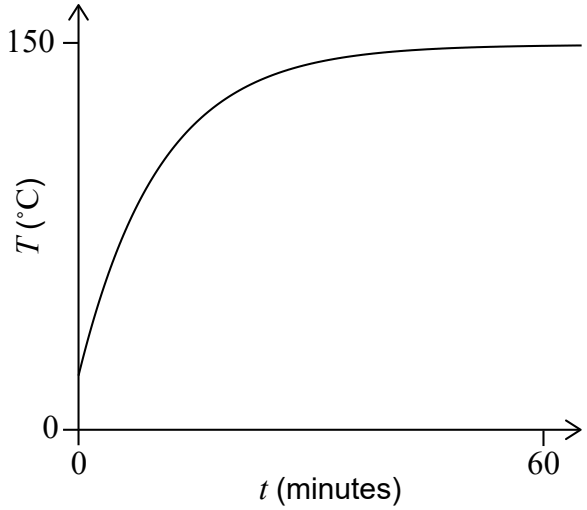
(b) . . . . . .

(c) . . . . . .

**12.** Sejah placed a baking tin, that contained cake mix, in a preheated oven in order to bake a cake. The temperature in the centre of the cake mix, $T$, in degrees Celsius ($^\circ$C) is given by

$$T(t) = 150 - a \times (1.1)^{-t}$$

where $t$ is the time, in minutes, since the baking tin was placed in the oven. The graph of $T$ is shown in the following diagram.




(a) Write down what the value of $150$ represents in the context of the question. [1]

The temperature in the centre of the cake mix was $18\,^\circ$C when placed in the oven.

(b) Find the value of $a$. [2]

The baking tin is removed from the oven $15$ minutes after the temperature in the centre of the cake mix has reached $130\,^\circ$C.

(c) Find the total time that the baking tin is in the oven. [3]

**Working:**

**Answers:**

(a) . . . . . .

. . . . . .

(b) . . . . . .

(c) . . . . . .

**13.** Applicants for a job had to complete a mathematics test. The time they took to complete the test is normally distributed with a mean of $53$ minutes and a standard deviation of $16.3$. One of the applicants is chosen at random.

(a) Find the probability that this applicant took at least $40$ minutes to complete the test. [2]

For $11\,\%$ of the applicants it took longer than $k$ minutes to complete the test.

(b) Find the value of $k$. [2]

There were $400$ applicants for the job.

(c) Estimate the number of applicants who completed the test in less than $25$ minutes. [2]

**Working:**

**Answers:**

(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

**14.** A function $f$ is given by $f(x) = 4x^3 + \frac{3}{x^2} - 3,\ x \neq 0$.

(a) Write down the derivative of $f$. [3]

(b) Find the point on the graph of $f$ at which the gradient of the tangent is equal to $6$. [3]

**Working:**

**Answers:**

(a) . . . . . .

(b) . . . . . .

**15.** Maria owns a cheese factory. The amount of cheese, in kilograms, Maria sells in one week, $Q$, is given by

$$Q = 882 - 45p,$$

where $p$ is the price of a kilogram of cheese in euros (EUR).

(a) Write down how many kilograms of cheese Maria sells in one week if the price of a kilogram of cheese is $8\,\text{EUR}$. [1]

Maria earns $(p - 6.80)\,\text{EUR}$ for each kilogram of cheese sold.

(b) Find how much Maria earns in one week, from selling cheese, if the price of a kilogram of cheese is $8\,\text{EUR}$. [2]

To calculate her weekly profit $W$, in EUR, Maria multiplies the amount of cheese she sells by the amount she earns per kilogram.

(c) Write down an expression for $W$ in terms of $p$. [1]

(d) Find the price, $p$, that will give Maria the highest weekly profit. [2]

**Working:**

**Answers:**

(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

(d) . . . . . .