Diploma Programme
Programme du diplôme
Programa del Diploma

**Mathematical studies
Standard level
Paper 1**

Thursday 4 May 2017 (afternoon)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

20 pages

20EP01

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Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Answers must be written within the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. The lengths of trout in a fisherman's catch were recorded over one month, and are represented in the following histogram.
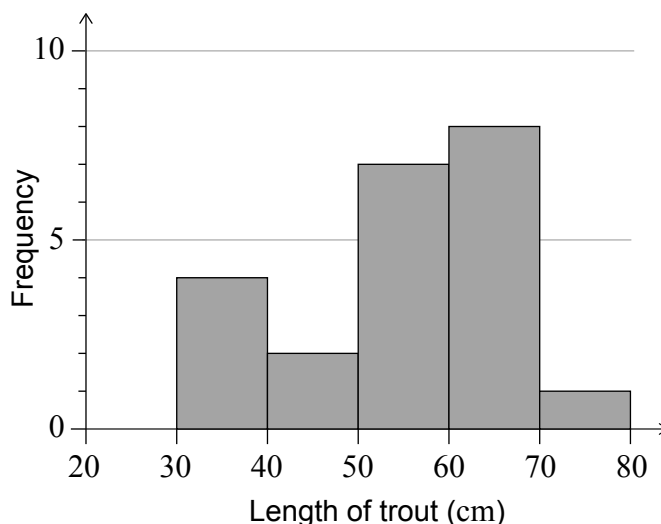
(a) Complete the following table.

Length of trout	Frequency
$20\,\text{cm} < \text{trout length} \leq 30\,\text{cm}$	0
$30\,\text{cm} < \text{trout length} \leq 40\,\text{cm}$	
$40\,\text{cm} < \text{trout length} \leq 50\,\text{cm}$	
$50\,\text{cm} < \text{trout length} \leq 60\,\text{cm}$	
$60\,\text{cm} < \text{trout length} \leq 70\,\text{cm}$	
$70\,\text{cm} < \text{trout length} \leq 80\,\text{cm}$	1

[2]

(b) State whether **length of trout** is a continuous or discrete variable. [1]

(c) Write down the modal class. [1]

Any trout with length $40\,\text{cm}$ or less is returned to the lake.

(d) Calculate the percentage of the fisherman's catch that is returned to the lake. [2]

(This question continues on the following page)

(Question 1 continued)

Working:

Answers:

(b) . . . .

(c) . . . .

(d) . . . .

2.   In the Canadian city of Ottawa:

97% of the population speak English,
38% of the population speak French,
36% of the population speak both English and French.

(a)   Calculate the percentage of the population of Ottawa that speak English but not French.   [2]

The total population of Ottawa is $985\,000$.

(b)   Calculate the number of people in Ottawa that speak both English and French.   [2]

(c)   Write down your answer to part (b) in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.   [2]

Working:

Answers:

(a)   . . . .

(b)   . . . .

(c)   . . . .

3. Consider the following propositions.

p: I completed the task
q: I was paid

(a) Write down in words $\neg q$.   [1]

(b) Write down in symbolic form the compound statement:

If I was paid then I completed the task.   [1]

(c) (i) Complete the following truth table.

p	q	$\neg q$	$p \vee \neg q$	$q \Rightarrow p$
T	T	F		
T	F	T		
F	T	F		
F	F	T		

(ii) State whether the statements $p \vee \neg q$ and $q \Rightarrow p$ are logically equivalent.
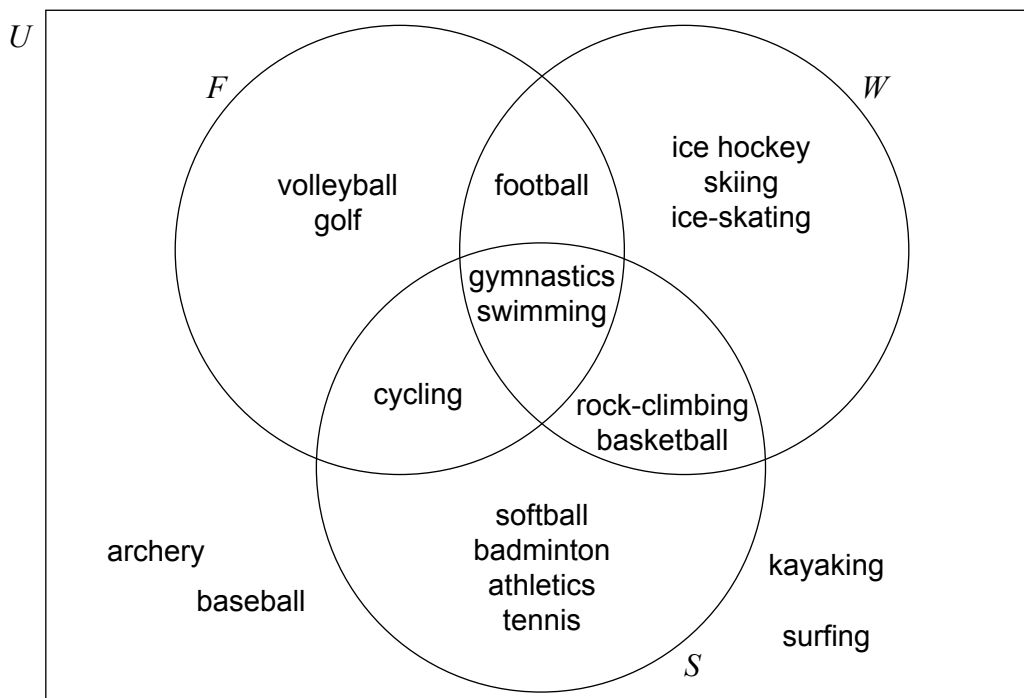Give a reason for your answer.   [4]

Working:

Answers:

(a) ....

(b) ....

(c) (ii) ....

....

....

4. Dune Canyon High School organizes its **school year** into three trimesters: fall/autumn (F), winter (W) and spring (S). The school offers a variety of sporting activities during and outside the school year.

The activities offered by the school are summarized in the following Venn diagram.

(a) Write down the number of sporting activities offered by the school during its **school year**. [1]

(b) Determine whether rock-climbing is offered by the school in the fall/autumn trimester. [1]

(c) Write down

    (i)   the elements of the set $F \cap W'$;

    (ii) $n(W \cap S)$. [2]

(d) Write down, in terms of F, W and S, an expression for the set which contains only archery, baseball, kayaking and surfing. [2]

(This question continues on the following page)

(Question 4 continued)

Working:

Answers:

(a) . . . .

(b) . . . .

(c) (i) . . . .

. . . .

(ii) . . . .

(d) . . . .

5.   The company Snakezen's Ladders makes ladders of different lengths. All the ladders that the company makes have the same design such that:

the first rung is $30\,\text{cm}$ from the base of the ladder,
the second rung is $57\,\text{cm}$ from the base of the ladder,
the distance between the first and second rung is equal to the distance between all adjacent rungs on the ladder.

The ladder in the diagram was made by this company and has eleven equally spaced rungs.

diagram not to scale

$57\,\text{cm}$

$30\,\text{cm}$
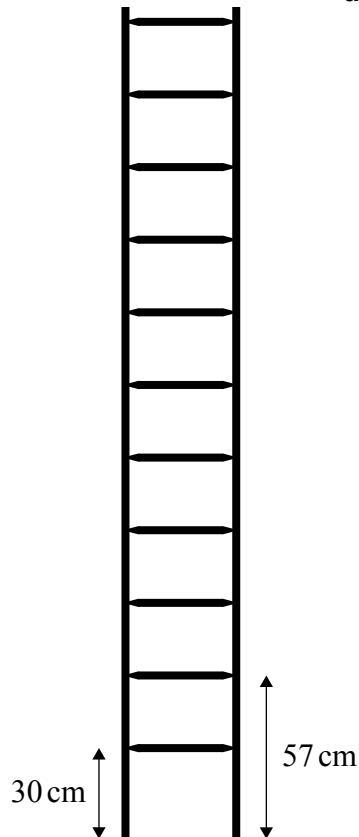
(a)   Find the distance from the base of this ladder to the top rung.   [3]

The company also makes a ladder that is $1050\,\text{cm}$ long.

(b)   Find the maximum number of rungs in this $1050\,\text{cm}$ long ladder.   [3]

(This question continues on the following page)

(Question 5 continued)

Working:

Answers:

(a) ....

(b) ....

Turn over

6. In a school, students in grades 9 to 12 were asked to select their preferred drink. The choices were milk, juice and water. The data obtained are organized in the following table.

	Milk	Juice	Water	**Total**
Grade 9	25	34	15	**74**
Grade 10	31	x	13	**74**
Grade 11	18	35	17	**70**
Grade 12	9	36	26	**71**
Total	**83**	**135**	**71**	**289**

A χ^2 test is carried out at the 5% significance level with hypotheses:

H_0: the preferred drink is independent of the grade
H_1: the preferred drink is not independent of the grade

The χ^2 critical value for this test is 12.6.

(a)   Write down the value of x. [1]

(b)   Write down the number of degrees of freedom for this test. [1]

(c)   Use your graphic display calculator to find the χ^2 statistic for this test. [2]

(d)   State the conclusion for this test. Give a reason for your answer. [2]

Working:

Answers:

(a)   . . . .

(b)   . . . .

(c)   . . . .

(d)   . . . .
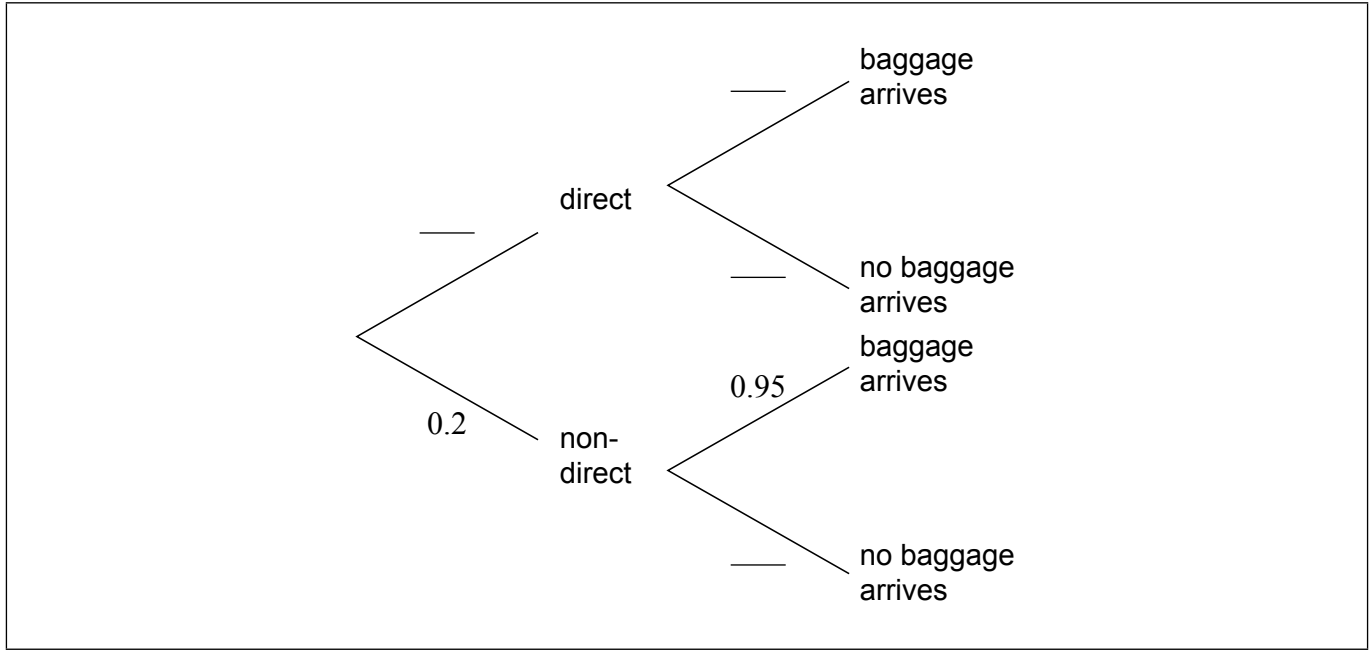
. . . .

7.   Sara regularly flies from Geneva to London. She takes either a direct flight or a non-direct flight that goes via Amsterdam.

If she takes a direct flight, the probability that her baggage does not arrive in London is 0.01.
If she takes a non-direct flight the probability that her baggage arrives in London is 0.95.

The probability that she takes a non-direct flight is 0.2.

(a)   Complete the tree diagram.   [3]

(b)   Find the probability that Sara's baggage arrives in London.   [3]

Working:

Answers:

(b)   . . . .

8. Daniela is going for a holiday to South America. She flies from the US to Argentina stopping in Peru on the way.

In Peru she exchanges 85 United States dollars (USD) for Peruvian nuevo sol (PEN).
The exchange rate is $1\,\text{USD} = 3.25\,\text{PEN}$ and a flat fee of $5\,\text{USD}$ commission is charged.

(a) Calculate the amount of PEN she receives.   [3]

At the end of Daniela's holiday she has 370 Argentinean peso (ARS). She converts this back to USD at a bank that charges a 4% commission on the exchange. The exchange rate is $1\,\text{USD} = 9.60\,\text{ARS}$.

(b) Calculate the amount of USD she receives.   [3]

Working:

Answers:

(a) . . . .

(b) . . . .

9.   A type of candy is packaged in a right circular cone that has volume $100 \, \text{cm}^3$ and vertical height $8 \, \text{cm}$.
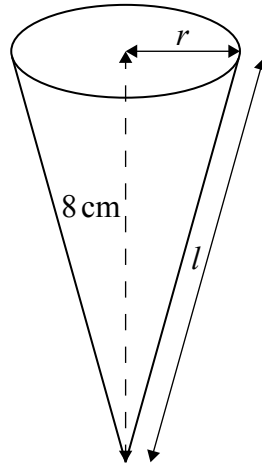
(a)   Find the radius, r, of the circular base of the cone.   [2]

(b)   Find the slant height, l, of the cone.   [2]

(c)   Find the curved surface area of the cone.   [2]

Working:

Answers:

(a)   ....

(b)   ....

(c)   ....

10. The first three terms of a geometric sequence are $u_1 = 486$, $u_2 = 162$, $u_3 = 54$.

    (a)   Find the value of r, the common ratio of the sequence.   [2]

    (b)   Find the value of n for which $u_n = 2$.   [2]

    (c)   Find the sum of the first 30 terms of the sequence.   [2]

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

11. The equation of line L_1 is $y = -\dfrac{2}{3}x - 2$.

    (a)   Write down the gradient of L_1.   [1]

Point P lies on L_1 and has x-coordinate -6.

    (b)   Find the y-coordinate of P.   [2]

The line L_2 is perpendicular to L_1 and intersects L_1 when $x = -6$.

    (c)   Determine the equation of L_2. Give your answer in the form $ax + by + d = 0$,
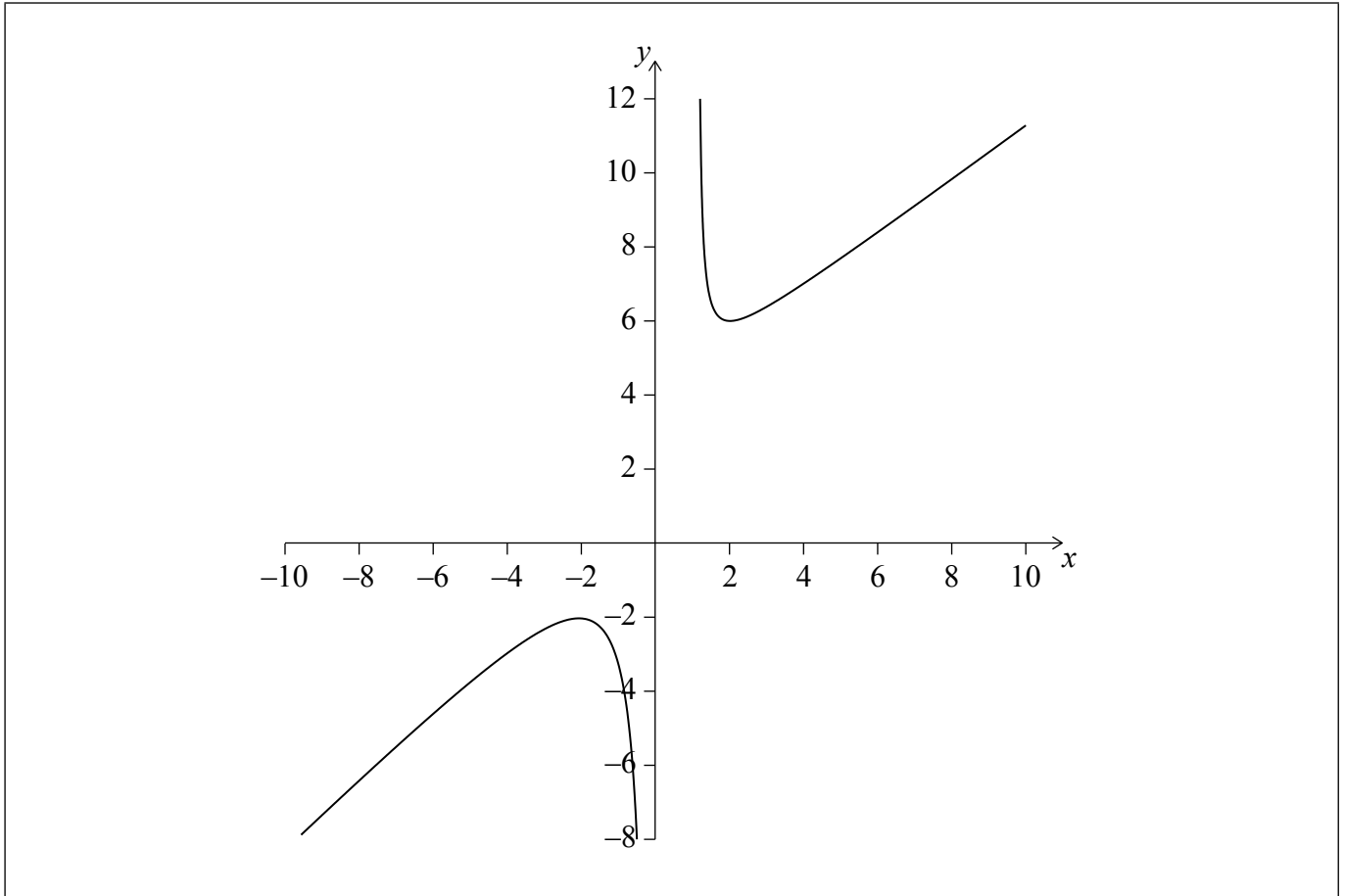   where a, b and d are integers.   [3]

Working:

Answers:

(a) ....

(b) ....

(c) ....

Turn over

12. The function f is of the form $f(x) = ax + b + \dfrac{c}{x}$, where a, b and c are positive integers.

Part of the graph of $y = f(x)$ is shown on the axes below. The graph of the function has its local maximum at $(-2, -2)$ and its local minimum at $(2, 6)$.

(a)   Write down the domain of the function.   [2]

(b)   (i)   Draw the line $y = -6$ on the axes.

(ii)   Write down the number of solutions to $f(x) = -6$.   [2]

(c)   Find the range of values of k for which $f(x) = k$ has no solution.   [2]

(This question continues on the following page)

(Question 12 continued)

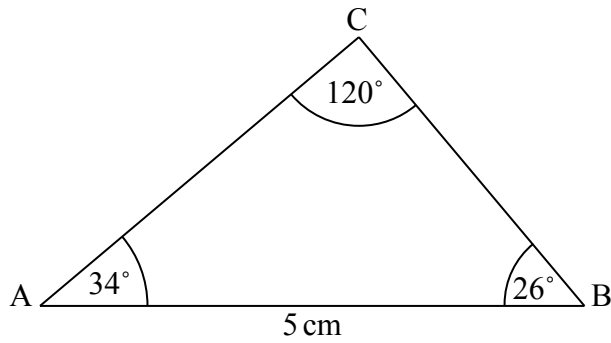**Working:**

Answers:

(a) . . . .

(b) (ii) . . . .

(c) . . . .

Turn over

13. A triangular postage stamp, ABC, is shown in the diagram below, such that $AB = 5\,cm$, $B\hat{A}C = 34°$, $A\hat{B}C = 26°$ and $A\hat{C}B = 120°$.

diagram not to scale

(a) Find the length of BC.   [3]

(b) Find the area of the postage stamp.   [3]

Working:

Answers:

(a) . . . .

(b) . . . .

14. Arthur and Jacob dream of owning a speedboat that costs $35\,300$ euros (EUR).

Arthur invested x EUR in an account that pays a nominal annual interest rate of 3.6%, compounded **monthly**. After 18 years he will have $35\,300$ EUR in the account.

(a) Calculate the value of Arthur's initial investment, x. Give your answer to two decimal places. [3]

Jacob invested 9000 EUR for n years. The investment has a nominal annual interest rate of 3.2% and is compounded **quarterly**. After n years, the investment will be worth $35\,300$ EUR.

(b) Find the value of n. [3]

Working:

Answers:

(a) . . . .

(b) . . . .

15. The graph of a quadratic function has y-intercept 10 and **one** of its x-intercepts is 1.
The x-coordinate of the vertex of the graph is 3.
The equation of the quadratic function is in the form $y = ax^2 + bx + c$.

    (a)   Write down the value of c.   [1]

    (b)   Find the value of a and of b.   [4]

    (c)   Write down the second x-intercept of the function.   [1]

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .