Imię i nazwisko:

1. (5 points) In a group of 25 high school students, 14 students like maths and 9 students like physics. The number of students who like both subjects is half the number of students who don't like either of the two subjects.
   (a) Represent the above information on the Venn diagram.
   (b) What percentage of students like both subjects?
   (c) What percentage of students like exactly one of the two subjects?

2. (5 points) In a certain town there are two schools that offer the IB Diploma programme. Last year the IB graduates at school A had a mean score of 32 points, while the graduates of a school B had a mean score of 40 points. State for each of the following statements if they must be true:

   i. The mean score of all the graduates of an IB school at that town was 36 points.

   ii. The median score was higher at school B.

   iii. The range of scores in the whole town was at least 8.

   Justify your answers.

3. (10 points)

(a) (6 points) If $m = \log_x 4$ and $n = \log_x 3$ find an expression, in terms of m and n, for:

i. $\log_4 3$,

ii. $\log_x 12$,

iii. $\log_x(\frac{3x}{2})$,

iv. $\log_2 9x$.

(b) (4 points) Solve the equation:

$$4^{x+1} + 15 \times 2^x = 4$$

4. (10 points) Consider the expansion of $\left(ax + \dfrac{b}{x^2}\right)^7$

   (a) Suppose $a = 2$, $b = 1$.

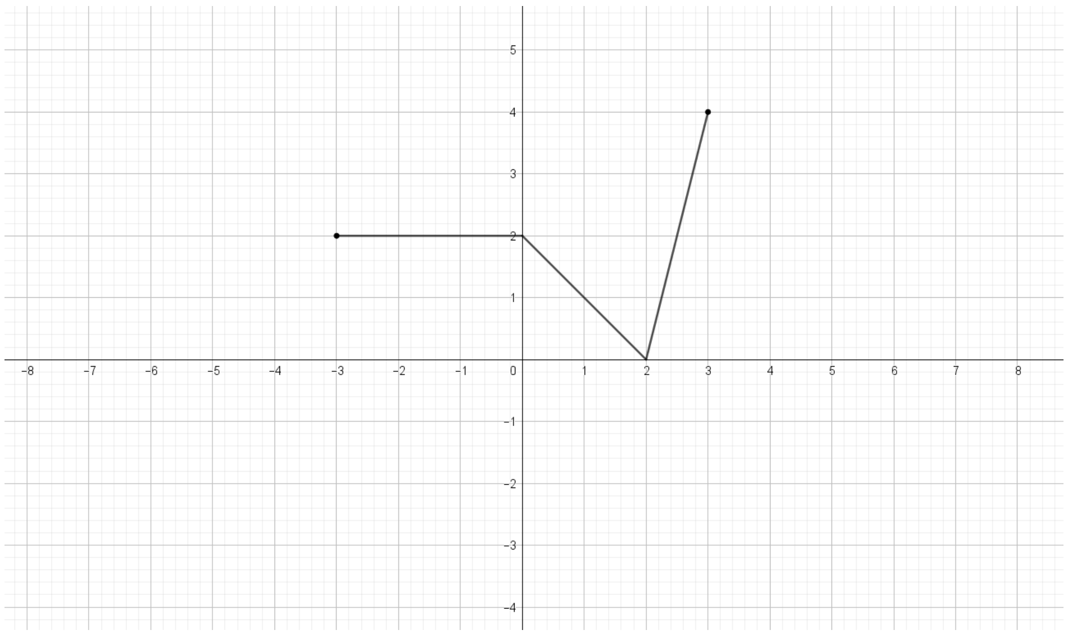
      i. Write down the number of terms in this expansion.

      ii. Write down the general term of the expansion.

      iii. Find the coefficient of x.

   (b) Now suppose a and b are unknown. The coefficient of x^4 is 21 and the coefficient of x is 189. Find two equations for a and b and hence find the values of these constants.

5. (10 points) The graph of the function $f(x)$ is shown below.



(a) State the domain and range of f.

(b) Let $g(x) = f(\frac{1}{2}x) + 1$. Draw the graph of g on the same diagram.

(c) State the domain and range of g.

Let $h(x) = \dfrac{x+2}{2x-1}$.

(d) Find the expression for $h^{-1}(x)$.

(e) Find $(h \circ f)(0)$ and $(f \circ h)(0)$.

(f) Solve $(f \circ h)(x) = 0$.

6. (10 points) Let $f(x) = kx^2 + 12x + 5$, where k is a constant.

   (a) Find the set of values of k for which the graph of f intersects the x-axis twice.

   Let $k = 2$.

   (b) Write $f(x)$ in the vertex form and state the coordinates of the vertex.

   (c) Sketch the graph of f on the set of axes below, clearly indicating the x and y intercepts.

   Let $g(x) = x + 1$.

   (d) Solve $f(x) = g(x)$.

   (e) Add the graph of $g(x)$ to your diagram. Mark the points of intersections of the graphs of the two functions and state the coordinates of these points.

   (f) Write down the set of values of x for which $g(x) > f(x)$.

7. (8 points) Consider the points $A(-1,-2)$, $B(3,1)$ and $C(1,2)$.

   (a) Find the gradient of the line through A and C.

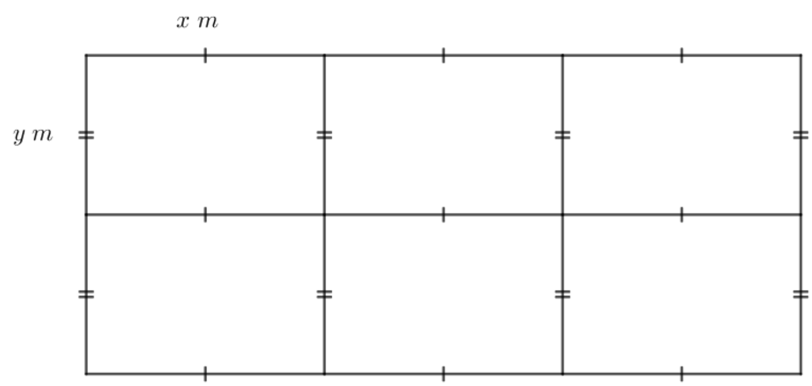
   (b) Find the gradient of the line through B and C.

   (c) Hence show that the triangle ABC is a right triangle.

   (d) Find the area of $\triangle ABC$.

   (e) Let M be the mid-point of AB. Show that the triangles AMC and CMB are isosceles.

8. (6 points) 600 metres of fencing is used to construct 6 rectangular animal pens as shown.



(a) Find the formula for the area of each pen in terms of x.

(b) Find the dimensions of each pen so that it has the maximum possible area.

(c) Find the total area of the whole enclosure for which each pen is of maximum area.

9. (10 points) Consider the function $f(x) = \dfrac{2x+4}{x-3}$.

(a) Write down the equations of horizontal and vertical asymptotes of f.

The graph of f has been translated by the vector $\begin{pmatrix} 1 \\ -5 \end{pmatrix}$ to form a graph of g.

(b) Write down the equation of $g(x)$ in the form $\dfrac{ax+b}{cx+d}$.

(c) Write down the equations of horizontal and vertical asymptotes of g.

(d) Sketch both f and g on the set of axes below.

(e) Mark the points where the graphs intersect.

(f) Solve $f(x) = g(x)$ algebraically.

10. (6 points) Prove, using mathematical induction, that:

$$1+2+3+...+n=\frac{1}{2}n(n+1)$$