

1. Find all values of x that satisfy the inequality $\frac{2x}{|x-1|} < 1$.

(Total 5 marks)

2. Consider the functions given below.

$$f(x) = 2x + 3$$

$$g(x) = \frac{1}{x}, x \neq 0$$

- (a) (i) Find $(g \circ f)(x)$ and write down the domain of the function.
(ii) Find $(f \circ g)(x)$ and write down the domain of the function.

(2)

- (b) Find the coordinates of the point where the graph of $y = f(x)$ and the graph of $y = (g^{-1} \circ f \circ g)(x)$ intersect.

(4)

(Total 6 marks)

3. Use mathematical induction to prove that $5^n + 9^n + 2$ is divisible by 4, for $n \in \mathbb{Z}^+$.

(Total 9 marks)

4. The quadratic function $f(x) = p + qx - x^2$ has a maximum value of 5 when $x = 3$.

- (a) Find the value of p and the value of q .

(4)

- (b) The graph of $f(x)$ is translated 3 units in the positive direction parallel to the x -axis. Determine the equation of the new graph.

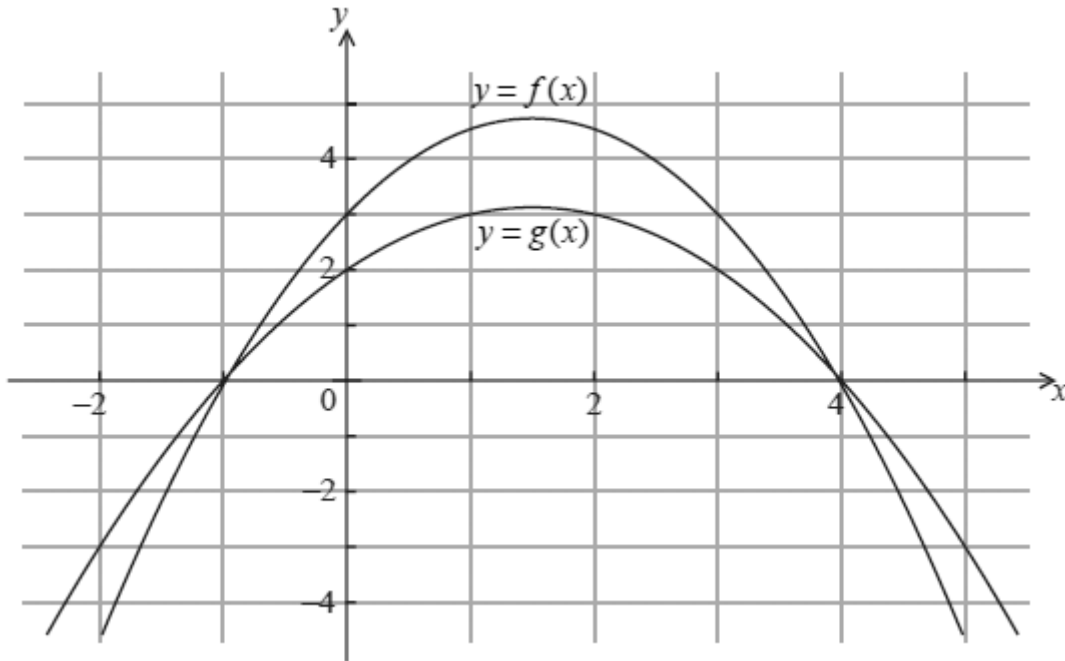
(2)

(Total 6 marks)

5. Solve the equation $\log_3(x + 17) - 2 = \log_3 2x$.

(Total 5 marks)

6. Shown below are the graphs of $y = f(x)$ and $y = g(x)$.



If $(f \circ g)(x) = 3$, find all possible values of x .

(Total 4 marks)

7. When $\left(1 + \frac{x}{2}\right)^n$, $n \in \mathbb{N}$, is expanded in ascending powers of x , the coefficient of x^3 is 70.

(a) Find the value of n .

(5)

(b) Hence, find the coefficient of x^2 .

(1)

(Total 6 marks)

8. Solve the equation $2^{2x+2} - 10 \times 2^x + 4 = 0$, $x \in \mathbb{R}$.

(Total 6 marks)

9. Let $f(x) = \frac{x+4}{x+1}$, $x \neq -1$ and $g(x) = \frac{x-2}{x-4}$, $x \neq 4$. Find the set of values of x such that $f(x) \leq g(x)$.

(Total 6 marks)

10. Solve the equation $4^{x-1} = 2^x + 8$.

(Total 5 marks)

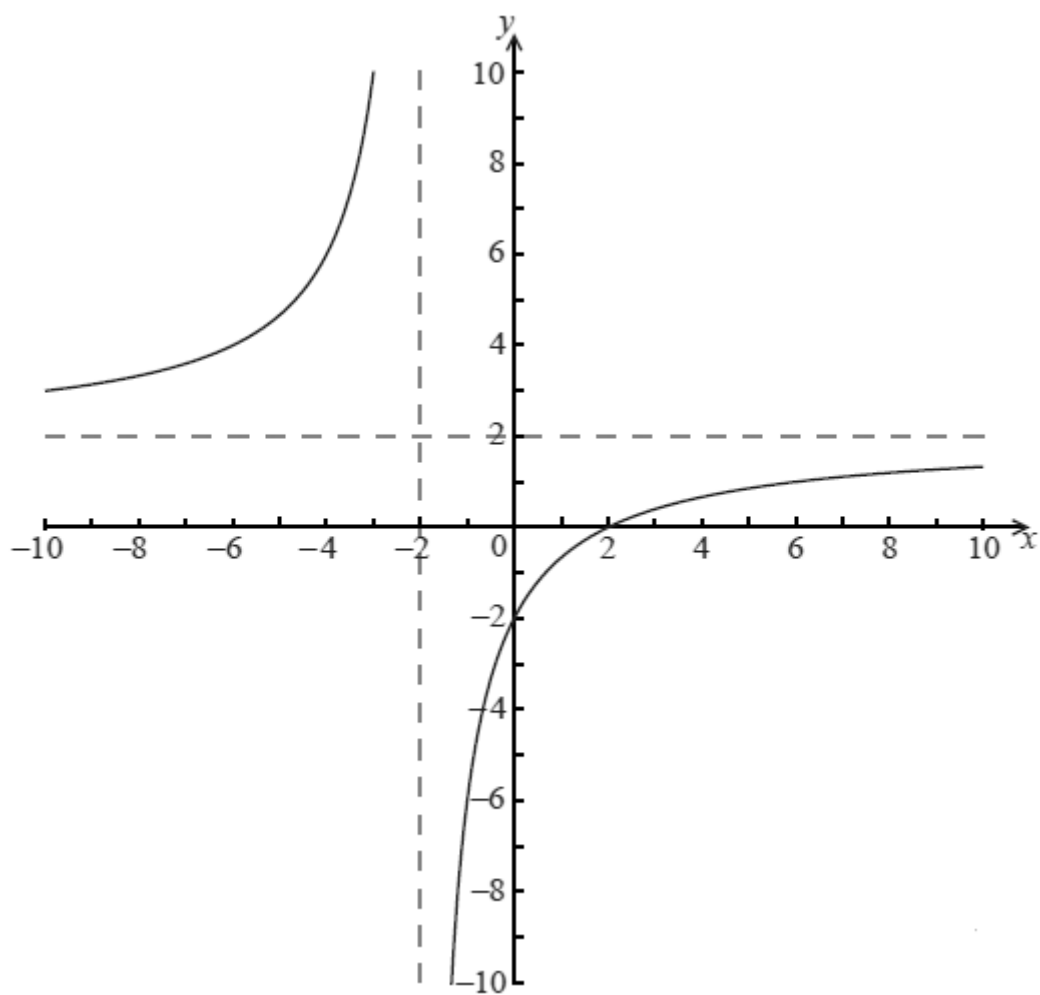
11. Let $g(x) = \log_5 |2\log_3 x|$. Find the product of the zeros of g .

(Total 5 marks)

12. Expand and simplify $\left(x^2 - \frac{2}{x}\right)^4$.

(Total 4 marks)

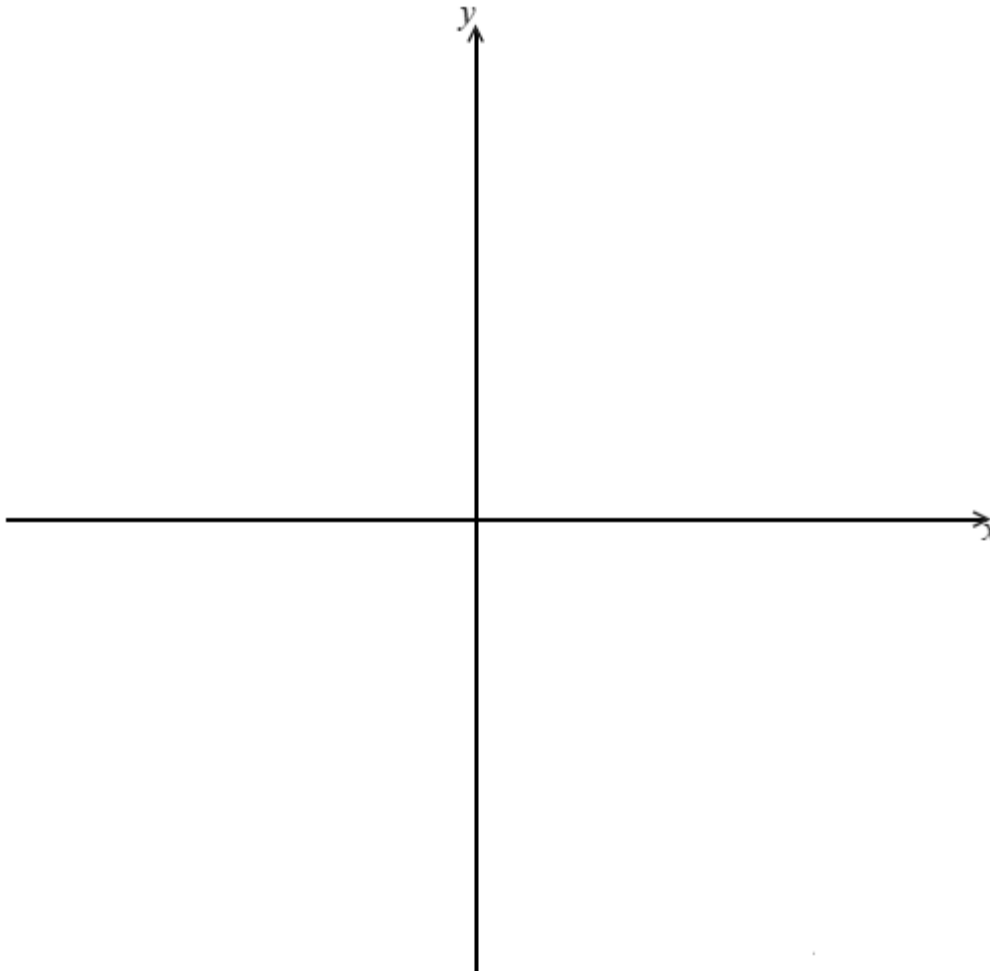
13. The graph of $y = \frac{a+x}{b+cx}$ is drawn below.



- (a) Find the value of a , the value of b and the value of c .

(4)

- (b) Using the values of a , b and c found in part (a), sketch the graph of $y = \frac{b+cx}{a+x}$ on the axes below, showing clearly all intercepts and asymptotes.



(4)
(Total 8 marks)

14. (a) Write down the quadratic expression $2x^2 + x - 3$ as the product of two linear factors. (1)

- (b) Hence, or otherwise, find the coefficient of x in the expansion of $(2x^2 + x - 3)^8$. (4)
(Total 5 marks)

15. Find the set of values of x for which $|x - 1| > |2x - 1|$.

(Total 4 marks)

16. Prove by mathematical induction that, for $n \in \mathbb{Z}^+$,

$$1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \dots + n\left(\frac{1}{2}\right)^{n-1} = 4 - \frac{n+2}{2^{n-1}}.$$

(Total 8 marks)

17. A function f is defined by $f(x) = \frac{2x-3}{x-1}$, $x \neq 1$.

(a) Find an expression for $f^{-1}(x)$.

(3)

(b) Solve the equation $|f^{-1}(x)| = 1 + f^{-1}(x)$.

(3)

(Total 6 marks)

18. (a) Express the quadratic $3x^2 - 6x + 5$ in the form $a(x + b)^2 + c$, where $a, b, c \in \mathbb{Z}$.

(3)

(b) Describe a sequence of transformations that transforms the graph of $y = x^2$ to the graph of $y = 3x^2 - 6x + 5$.

(3)

(Total 6 marks)