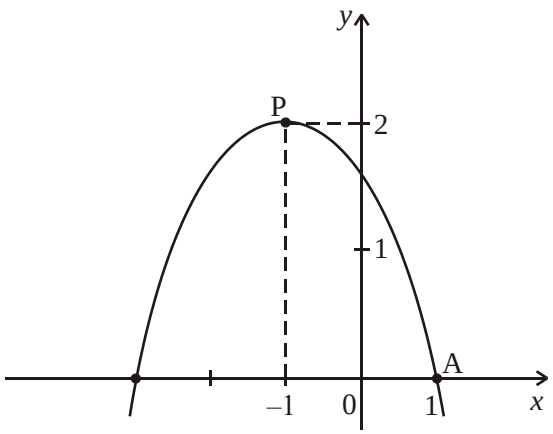
1. The diagram shows part of the graph of $y = a(x - h)^2 + k$. The graph has its vertex at P, and passes through the point A with coordinates (1, 0).



(a) Write down the value of

(i) h;

(ii) k.

(b) Calculate the value of a.

Working:

Answers:

(a) (i) .....

(ii) .....

(b) .....

(Total 6 marks)

2. Consider the function $f(x) = 2x^2 - 8x + 5$.

(a) Express $f(x)$ in the form $a(x-p)^2 + q$, where $a, p, q \in \mathbb{Z}$.

(b) Find the minimum value of $f(x)$.

Working:

Answers:

(a) .....

(b) .....

(Total 6 marks)

3. Let $f(x) = \frac{1}{x}$, $x \neq 0$.

(a) Sketch the graph of f.

(2)

The graph of f is transformed to the graph of g by a translation of $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

(b) Find an expression for $g(x)$.

(2)

(c) (i) Find the intercepts of g.

(ii) Write down the equations of the asymptotes of g.

(iii) Sketch the graph of g.

(10)
(Total 14 marks)

4. Let $f(x) = 2x + 1$ and $g(x) = 3x^2 - 4$.

Find

(a) $f^{-1}(x)$;

(b) $(g \circ f)(-2)$;

(c) $(f \circ g)(x)$.

Working:

Answers:

(a) ……..

(b) ……..

(c) ……..

(Total 6 marks)

5. Consider the functions $f(x) = 2x$ and $g(x) = \frac{1}{x-3}$, $x \neq 3$.

(a) Calculate $(f \circ g)(4)$.

(b) Find $g^{-1}(x)$.

(c) Write down the domain of g^{-1}.

Working:

Answers:

(a) .....

(b) .....

(c) .....

(Total 6 marks)

6. Consider the line L with equation $y + 2x = 3$. The line L_1 is parallel to L and passes through the point $(6, -4)$.

(a) Find the gradient of L_1.

(b) Find the equation of L_1 in the form $y = mx + b$.

(c) Find the x-coordinate of the point where line L_1 crosses the x-axis.

Working:

Answers:

(a) .....

(b) .....

(c) .....

(Total 6 marks)

7. (a) Expand $(x - 2)^4$ and simplify your result.

(3)

(b) Find the term in x^3 in $(3x + 4)(x - 2)^4$.

(3)
(Total 6 marks)

8. Determine the constant term in the expansion of $\left(x-\frac{2}{x^2}\right)^9$.

Working:

Answer:

.....

(Total 4 marks)

9. Find the coefficient of x^5 in the expansion of $(3x-2)^8$.

Working:

Answer:

.....

(Total 4 marks)

10. Find the coefficient of a^5b^7 in the expansion of $(a + b)^{12}$.

Working:

Answer:

.....

(Total 4 marks)

11. Use the binomial theorem to complete this expansion.

$$(3x + 2y)^4 = 81x^4 + 216x^3y + ...$$

Working:

Answer:

.....

(Total 4 marks)

12. Find the coefficient of x^3 in the expansion of $(2 - x)^5$.

Working:

Answer:

.....

(Total 6 marks)

13. Complete the following expansion.

$(2 + ax)^4 = 16 + 32ax + \ldots$

Working:

Answer:

.....

(Total 6 marks)

14. Consider the expansion of $\left(3x^2 - \frac{1}{x}\right)^9$.

(a) How many terms are there in this expansion?

(b) Find the constant term in this expansion.

Working:

Answers:

(a) .....

(b) .....

(Total 6 marks)

15. Given that $\left(3+\sqrt{7}\right)^3 = p + q\sqrt{7}$ where p and q are integers, find

(a) p;

(b) q.

Working:

Answers:

(a) .....

(b) .....

(Total 6 marks)

16. Find the term containing x^3 in the expansion of $(2 - 3x)^8$.

Working:

Answer:

…….....

(Total 6 marks)

17. Find the term in x^3 in the expansion of $\left(\frac{2}{3}x-3\right)^8$.

(Total 5 marks)

18. When the expression $(2 + ax)^{10}$ is expanded, the coefficient of the term in x^3 is 414 720. Find the value of a.

Working:

Answer:

……..

(Total 6 marks)

19. One of the terms of the expansion of $(x + 2y)^{10}$ is $ax^8 y^2$. Find the value of a.

(Total 6 marks)

20. The fifth term in the expansion of the binomial $(a + b)^n$ is given by $\binom{10}{4}p^6(2q)^4$.

(a) Write down the value of n.

(1)

(b) Write down a and b, in terms of p and/or q.

(2)

(c) Write down an expression for the sixth term in the expansion.

(3)

(Total 6 marks)

21. Consider the expansion of $(x^2 - 2)^5$.

(a) Write down the number of terms in this expansion.

(b) The first four terms of the expansion in descending powers of x are

$$x^{10} - 10x^8 + 40x^6 + Ax^4 + ...$$

Find the value of A.

Working:

Answers:

(a) .....

(b) .....

(Total 6 marks)

22. Find the term in x^4 in the expansion of $\left(3x^2 - \frac{2}{x}\right)^5$.

(Total 6 marks)

23. Consider the expansion of the expression $(x^3 - 3x)^6$.

(a) Write down the number of terms in this expansion.

(b) Find the term in x^{12}.

(Total 6 marks)

24. (a) Expand $(2 + x)^4$ and simplify your result.

(3)

(b) Hence, find the term in x^2 in $(2 + x)^4\left(1+\frac{1}{x^2}\right)$.

(3)
(Total 6 marks)

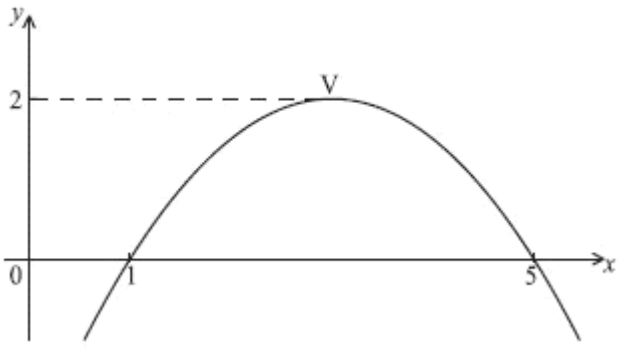
25. Let $g(x) = 3x - 2$, $h(x) = \frac{5x}{x-4}$, $x \neq 4$.

(a) Find an expression for $(h \circ g)(x)$. Simplify your answer.

(b) Solve the equation $(h \circ g)(x) = 0$.

(Total 6 marks)

26. Part of the graph of the function $y = d\,(x - m)^2 + p$ is given in the diagram below. The x-intercepts are (1, 0) and (5, 0). The vertex is V(m, 2).



(a) Write down the value of

(i) m;

(ii) p.

(b) Find d.

(Total 6 marks)

27. Consider the expansion of $(x + 2)^{11}$.

(a) Write down the number of terms in this expansion.

(1)

(b) Find the term containing x^2.

(4)
(Total 5 marks)

28. Let $f(x) = a\,(x - 4)^2 + 8$.

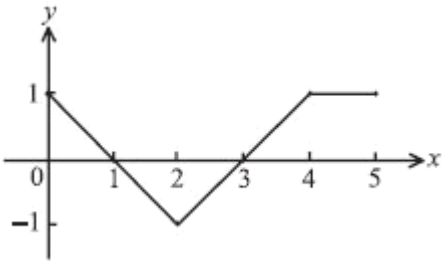
(a) Write down the coordinates of the vertex of the curve of f.

(b) Given that $f(7) = -10$, find the value of a.

(c) Hence find the y-intercept of the curve of f.

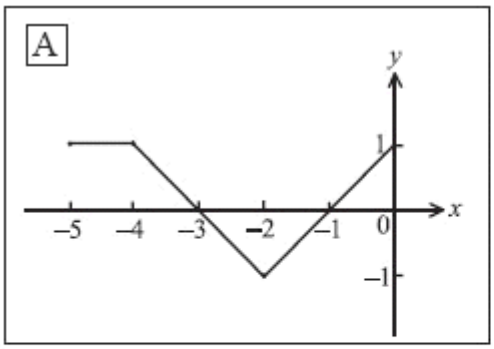
(Total 6 marks)

29. The following diagram shows part of the graph of $f(x)$.

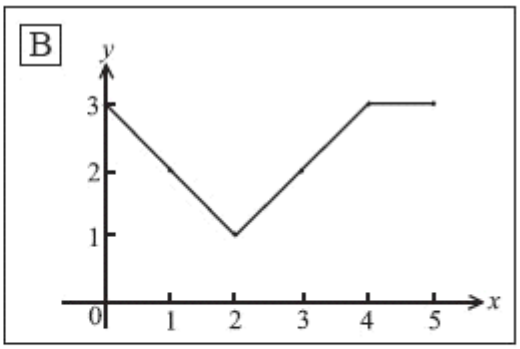


Consider the five graphs in the diagrams labelled A, B, C, D, E below.

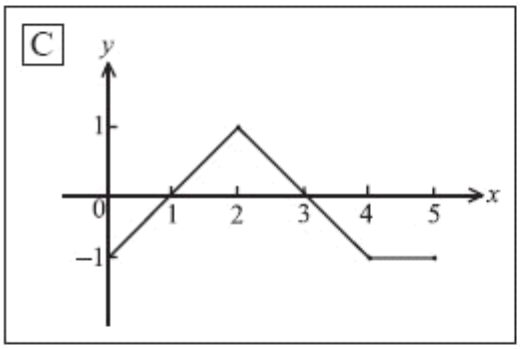
A



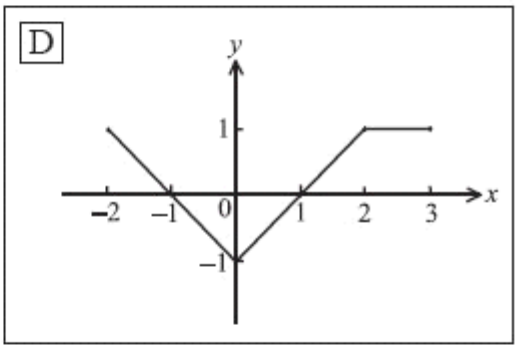
B



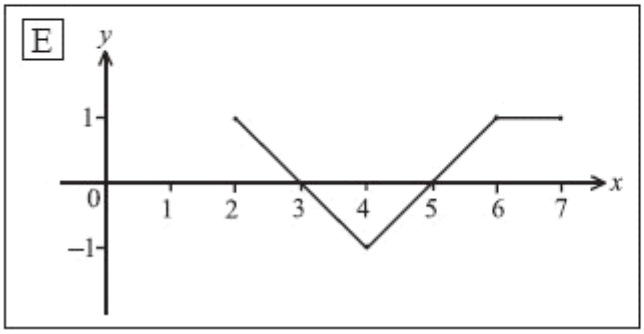
C



D



E



(a) Which diagram is the graph of $f(x + 2)$?

(b) Which diagram is the graph of $-f(x)$?

(c) Which diagram is the graph of $f(-x)$

(Total 6 marks)

30. Consider the function $f(x) = \frac{16}{x-10} + 8, x \neq 10$.

(a) Write down the **equation** of

(i) the vertical asymptote;

(ii) the horizontal asymptote.

(2)

(b) Find the

(i) y-intercept;

(ii) x-intercept.

(2)

(c) Sketch the graph of f, clearly showing the above information.

(4)

(d) Let $g(x) = \frac{16}{x}, x \neq 0$.

The graph of g is transformed into the graph of f using two transformations.

The first is a translation with vector $\begin{pmatrix} 10 \\ 0 \end{pmatrix}$. Give a full geometric description of the second transformation.

(2)

(Total 10 marks)

31. Consider two different quadratic functions of the form $f(x) = 4x^2 - qx + 25$. The graph of each function has its vertex on the x-axis.

(a) Find both values of q.

(b) For the greater value of q, solve $f(x) = 0$.

(c) Find the coordinates of the point of intersection of the two graphs.

(Total 6 marks)

32. Let $f(x) = 2^x$, and $g(x) = \frac{x}{x-2}$, $(x \neq 2)$.

Find

(a) $(g \circ f)(3)$;

(b) $g^{-1}(5)$.

Working:

Answers:

(a) .....

(b) .....

(Total 6 marks)

33. The function f is given by $f(x) = x^2 - 6x + 13$, for $x \geq 3$.

(a) Write $f(x)$ in the form $(x - a)^2 + b$.

(b) Find the inverse function f^{-1}.

(c) State the domain of f^{-1}.

Working:

Answers:

(a) .....

(b) .....

(c) .....

(Total 6 marks)

34. Let $f(x) = 7 - 2x$ and $g(x) = x + 3$.

(a) Find $(g \circ f)(x)$.

(2)

(b) Write down $g^{-1}(x)$.

(1)

(c) Find $(f \circ g^{-1})(5)$.

(2)

(Total 5 marks)

35. The quadratic function f is defined by $f(x) = 3x^2 - 12x + 11$.

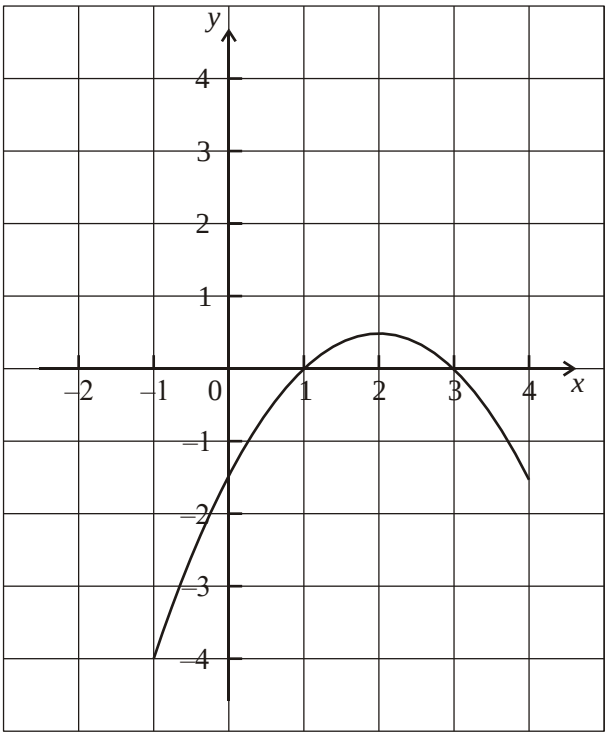
(a) Write f in the form $f(x) = 3(x - h)^2 - k$.

(3)

(b) The graph of f is translated 3 units in the positive x-direction and 5 units in the positive y-direction. Find the function g for the translated graph, giving your answer in the form $g(x) = 3(x - p)^2 + q$.

(3)
(Total 6 marks)

36. Part of the graph of a function f is shown in the diagram below.



(a) On the same diagram sketch the graph of $y = -f(x)$.

(2)

(b) Let $g(x) = f(x + 3)$.

(i) Find $g(-3)$.

(ii) Describe **fully** the transformation that maps the graph of f to the graph of g.

(4)
(Total 6 marks)

37. Find the **exact** value of x in each of the following equations.

(a) $5^{x+1} = 625$

(b) $\log_{\mathbf{a}} (3x + 5) = 2$

(Total 6 marks)

38. Consider $f(x) = 2kx^2 - 4kx + 1$, for $k \neq 0$. The equation $f(x) = 0$ has two equal roots.

(a) Find the value of k.

(5)

(b) The line $y = p$ intersects the graph of f. Find all possible values of p.

(2)
(Total 7 marks)

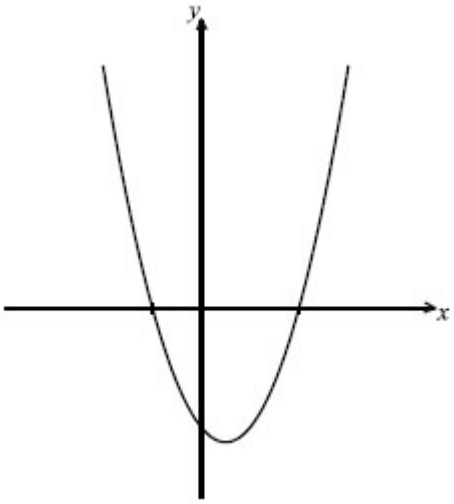
39. Let $f(x) = x^3 - 4$ and $g(x) = 2x$.

(a) Find $(g \circ f)(-2)$.

(b) Find $f^{-1}(x)$.

(Total 6 marks)

40. The following diagram shows part of the graph of f, where $f(x) = x^2 - x - 2$.



(a) Find both x-intercepts.

(4)

(b) Find the x-coordinate of the vertex.

(2)

(Total 6 marks)

41. (a) Express $y = 2x^2 - 12x + 23$ in the form $y = 2(x - c)^2 + d$.

The graph of $y = x^2$ is transformed into the graph of $y = 2x^2 - 12x + 23$ by the transformations

a vertical stretch with scale factor k **followed by**
a horizontal translation of p units **followed by**
a vertical translation of q units.

(b) Write down the value of

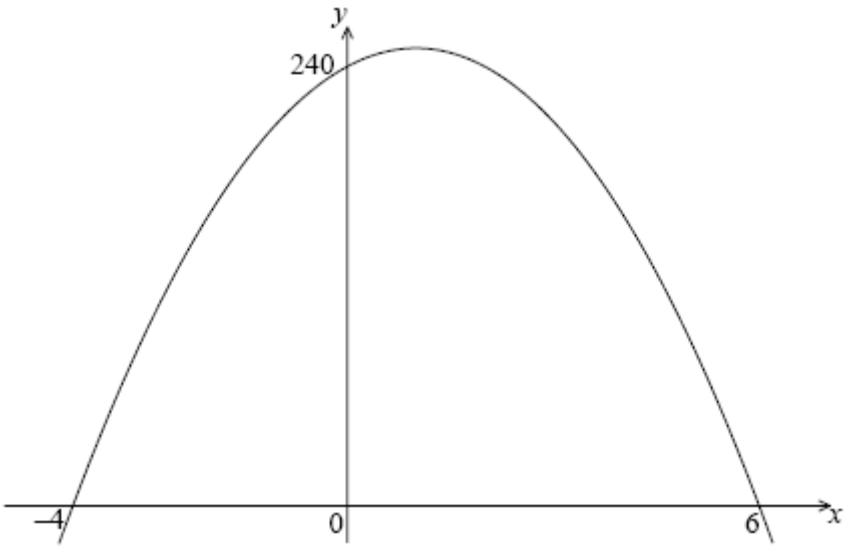
(i) k;

(ii) p;

(iii) q.

(Total 6 marks)

42. The following diagram shows part of the graph of a quadratic function f.



The x-intercepts are at $(–4, 0)$ and $(6, 0)$ and the y-intercept is at $(0, 240)$.

(a) Write down $f(x)$ in the form $f(x) = –10(x – p)(x – q)$.

(2)

(b) Find another expression for $f(x)$ in the form $f(x) = –10(x – h)^2 + k$.

(4)

(c) Show that $f(x)$ can also be written in the form $f(x) = 240 + 20x – 10x^2$.

(2)

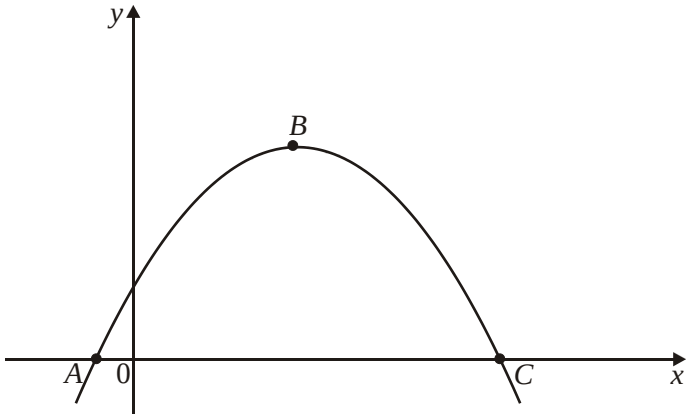
A particle moves along a straight line so that its velocity, v m s$^{–1}$, at time t seconds is given by $v = 240 + 20t – 10t^2$, for $0 \leq t \leq 6$.

(d) (i) Find the value of t when the speed of the particle is greatest.

(ii) Find the acceleration of the particle when its speed is zero.

(7)
(Total 15 marks)

43. The diagram shows the parabola $y = (7 - x)(1 + x)$. The points A and C are the x-intercepts and the point B is the maximum point.



Find the coordinates of A, B and C.

Working:

Answer:

.....

(Total 4 marks)

44. Let $f(x) = 3x^2$. The graph of f is translated 1 unit to the right and 2 units down. The graph of g is the image of the graph of f after this translation.

(a) Write down the coordinates of the vertex of the graph of g.

(2)

(b) Express g in the form $g(x) = 3(x - p)^2 + q$.

(2)

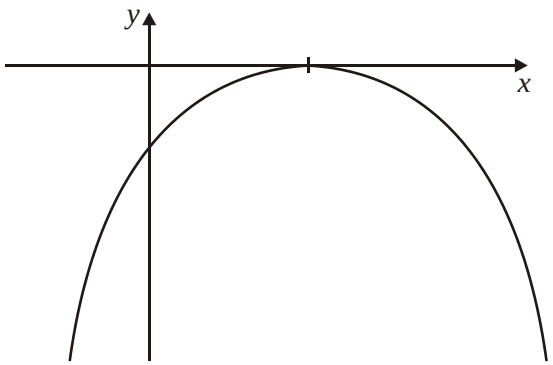
The graph of h is the reflection of the graph of g in the x-axis.

(c) Write down the coordinates of the vertex of the graph of h.

(2)

(Total 6 marks)

45. The diagram shows the graph of the function $y = ax^2 + bx + c$.



Complete the table below to show whether each expression is positive, negative or zero.

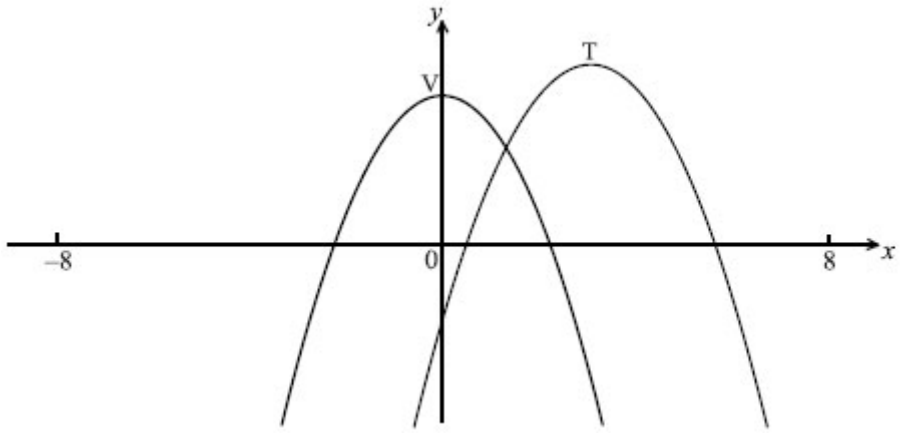
Expression	positive	negative	zero
a			
c			
$b^2 - 4ac$			
b			

Working:

(Total 4 marks)

46. The following diagram shows part of the graph of $f(x) = 5 - x^2$ with vertex V (0, 5).

Its image $y = g(x)$ after a translation with vector $\begin{pmatrix} h \\ k \end{pmatrix}$ has vertex T (3, 6).



(a) Write down the value of

(i) h;

(ii) k.

(2)

(b) Write down an expression for $g(x)$.

(2)

(c) On the same diagram, sketch the graph of $y = g(-x)$.

(2)
(Total 6 marks)

47. The quadratic equation $4x^2 + 4kx + 9 = 0$, $k > 0$ has exactly one solution for x.
Find the value of k.

Working:

Answer:

.....

(Total 4 marks)

48. Let $f(x) = 3x$, $g(x) = 2x - 5$ and $h(x) = (f \circ g)(x)$.

(a) Find $h(x)$.

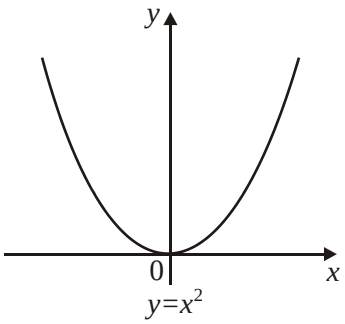
(2)

(b) Find $h^{-1}(x)$.

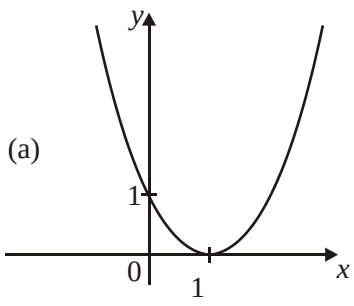
(3)
(Total 5 marks)

49. The diagrams show how the graph of $y = x^2$ is transformed to the graph of $y = f(x)$ in three steps.

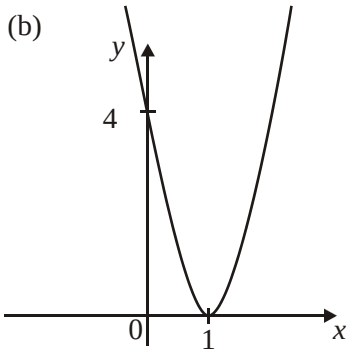
For each diagram give the equation of the curve.



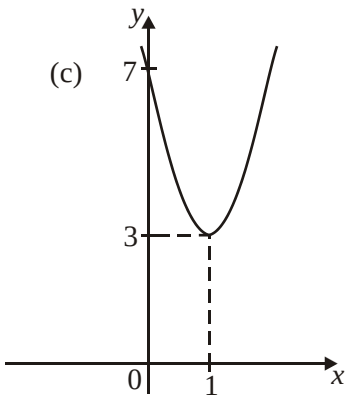
(a)



(b)



(c)



Working:

Answers:

(a) .....

(b) .....

(c) .....

(Total 4 marks)

50. The functions f and g are defined by $f : x \mapsto 3x$, $g : x \mapsto x + 2$.

(a) Find an expression for $(f \circ g)(x)$.

(2)

(b) Find $f^{-1}(18) + g^{-1}(18)$.

(4)
(Total 6 marks)

51. Two functions f, g are defined as follows:

$$f : x \to 3x + 5$$
$$g : x \to 2(1 - x)$$

Find

(a) $f^{-1}(2)$;

(b) $(g \circ f)(-4)$.

Working:

Answers:

(a) .....

(b) .....

(Total 4 marks)

52. Solve the following equations.

(a) $\log_x 49 = 2$

(3)

(b) $\log_2 8 = x$

(2)

(c) $\log_{25} x = -\frac{1}{2}$

(3)

(d) $\log_2 x + \log_2(x - 7) = 3$

(5)
(Total 13 marks)

53. Let $f(x) = 2x^2 - 12x + 5$.

(a) Express $f(x)$ in the form $f(x) = 2(x - h)^2 - k$.

(3)

(b) Write down the vertex of the graph of f.

(2)

(c) Write down the equation of the axis of symmetry of the graph of f.

(1)

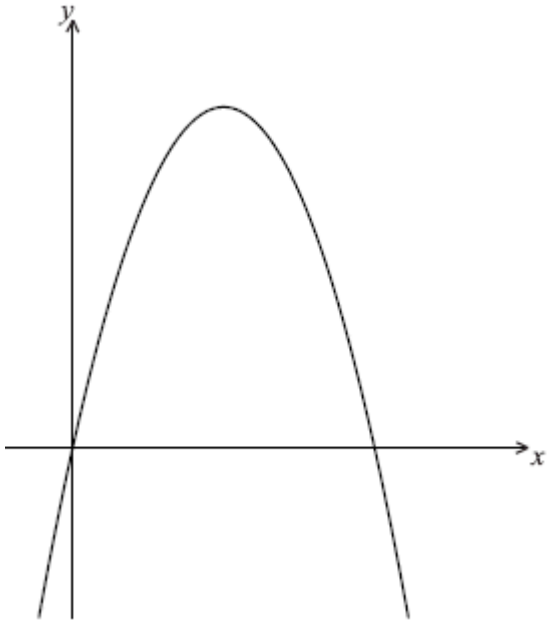
(d) Find the y-intercept of the graph of f.

(2)

(e) The x-intercepts of f can be written as $\frac{p \pm \sqrt{q}}{r}$, where $p, q, r \in \mathbb{Z}$.
Find the value of p, of q, and of r.

(7)
(Total 15 marks)

54. Let $f(x) = 8x - 2x^2$. Part of the graph of f is shown below.



(a) Find the x-intercepts of the graph.

(4)

(b) (i) Write down the equation of the axis of symmetry.

(ii) Find the y-coordinate of the vertex.

(3)
(Total 7 marks)

55. Let $f(x) = 2x^2 + 4x - 6$.

(a) Express $f(x)$ in the form $f(x) = 2(x - h)^2 + k$.

(3)

(b) Write down the equation of the axis of symmetry of the graph of f.

(1)

(c) Express $f(x)$ in the form $f(x) = 2(x - p)(x - q)$.

(2)
(Total 6 marks)

56. Consider $f(x) = \sqrt{x-5}$.

(a) Find

(i) $f(11)$;

(ii) $f(86)$;

(iii) $f(5)$.

(3)

(b) Find the values of x for which f is undefined.

(2)

(c) Let $g(x) = x^2$. Find $(g \circ f)(x)$.

(2)
(Total 7 marks)

57. Let $f(x) = \log_3 \sqrt{x}$, for $x > 0$.

(a) Show that $f^{-1}(x) = 3^{2x}$.

(2)

(b) Write down the range of f^{-1}.

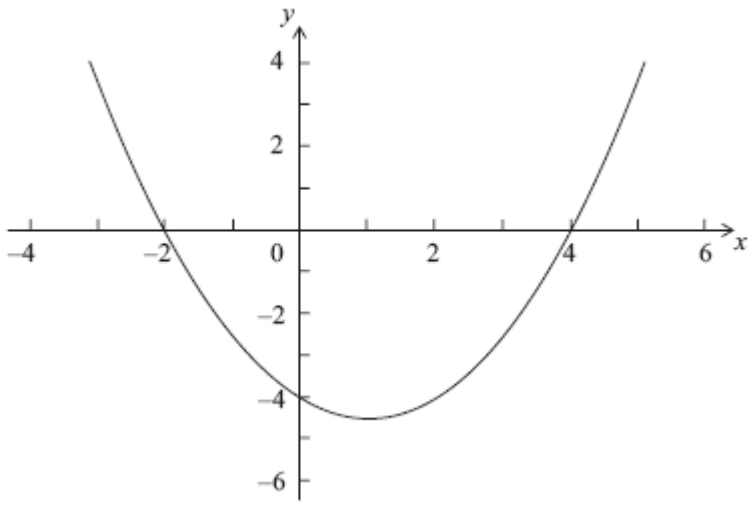
(1)

Let $g(x) = \log_3 x$, for $x > 0$.

(c) Find the value of $(f^{-1} \circ g)(2)$, giving your answer as an integer.

(4)
(Total 7 marks)

58. Let $f(x) = p(x - q)(x - r)$. Part of the graph of f is shown below.



The graph passes through the points (–2, 0), (0, –4) and (4, 0).

(a) Write down the value of q and of r.

(2)

(b) Write down the **equation** of the axis of symmetry.

(1)

(c) Find the value of p.

(3)

(Total 6 marks)

59. Let $f(x) = 3(x + 1)^2 - 12$.

(a) Show that $f(x) = 3x^2 + 6x - 9$.

(2)

(b) For the graph of f

(i) write down the coordinates of the vertex;

(ii) write down the **equation** of the axis of symmetry;

(iii) write down the y-intercept;

(iv) find both x-intercepts.

(8)

(c) **Hence** sketch the graph of f.

(2)

(d) Let $g(x) = x^2$. The graph of f may be obtained from the graph of g by the two transformations:

a stretch of scale factor t in the y-direction

followed by

a translation of $\begin{pmatrix} p \\ q \end{pmatrix}$.

Find $\begin{pmatrix} p \\ q \end{pmatrix}$ and the value of t.

(3)
(Total 15 marks)

60. (a) Express $f(x) = x^2 - 6x + 14$ in the form $f(x) = (x - h)^2 + k$, where h and k are to be determined.

(b) Hence, or otherwise, write down the coordinates of the vertex of the parabola with equation $y - x^2 - 6x + 14$.

Working:

Answers:

(a) .....

(b) .....

(Total 4 marks)

61. Consider the function $f : x \mapsto \sqrt{x+1},\ x \geq -1$

(a) Determine the inverse function f^{-1}.

(b) What is the domain of f^{-1}?

Working:

Answers:

(a) .....

(b) .....

(Total 4 marks)

62. Solve $\log_2 x + \log_2(x - 2) = 3$, for $x > 2$.

(Total 7 marks)

63. Let $f(x) = x^2 + 4$ and $g(x) = x - 1$.

(a) Find $(f \circ g)(x)$.

(2)

The vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ translates the graph of $(f \circ g)$ to the graph of h.

(b) Find the coordinates of the vertex of the graph of h.

(3)

(c) Show that $h(x) = x^2 - 8x + 19$.

(2)

(d) The line $y = 2x - 6$ is a tangent to the graph of h at the point P. Find the x-coordinate of P.

(5)

(Total 12 marks)

64. The quadratic equation $kx^2 + (k - 3)x + 1 = 0$ has two equal real roots.

(a) Find the possible values of k.

(5)

(b) **Write down** the values of k for which $x^2 + (k - 3)x + k = 0$ has two equal real roots.

(2)

(Total 7 marks)

65. Let $f(x) = x^2$ and $g(x) = 2x - 3$.

(a) Find $g^{-1}(x)$.

(2)

(b) Find $(f \circ g)(4)$.

(3)
(Total 5 marks)

66. Let $f(x) = x^2$ and $g(x) = 2(x - 1)^2$.

(a) The graph of g can be obtained from the graph of f using two transformations. Give a full geometric description of each of the two transformations.

(2)

(b) The graph of g is translated by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ to give the graph of h.

The point $(-1, 1)$ on the graph of f is translated to the point P on the graph of h. Find the coordinates of P.

(4)
(Total 6 marks)

67. Solve the equation $\log_9 81 + \log_9 \frac{1}{9} + \log_9 3 = \log_9 x$.

Working:

Answer:

.....

(Total 4 marks)