Percentage Error

You have to be able to calculate the percentage error of an approximated or estimated answer.

We will go through a few examples. Calculating the percentage error is just a matter of substituting numbers into a formula. The only part that may cause problems is figuring out which value is the exact value and which is the approximated value.

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Note that the absolute value (the $|\ \ |$ brackets) guarantees that the answer is always positive.

Two examples

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If we calculate the error in both approximations we see that in the first case $\epsilon_1 = 0.62$ and in the second case we get $\epsilon_2 = 2$. The error in the second experiment is larger, but one could (and should) argue that in the second scenario the error is almost insignificant and in the first we got less than a half of the right answer.

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You can see that the percentage error is the absolute error as a percentage of the exact answer.

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$$\epsilon_{\%} = |\frac{v_A - v_E}{v_E}| \cdot 100\% = |\frac{323 - 321}{321}| \cdot 100\% = 0.623\%$$

Percentage error

These answers tell us much more about the relative size of the error. In the first case the error was very large compared to the quantities involved in the problem. In the second case it was very small.

On the next slides we will go through the exercises 1 to 4 from the page I've uploaded on the site.

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$$\epsilon_{\%} = |\frac{v_A - v_E}{v_E}| \cdot 100\% = |\frac{140 - 119.423}{119.423}| \cdot 100\% = 17.2\%$$

The percentage error should be given correct to 3 s.f. (or exactly).

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$$v_A = \frac{8 + 7 + 9}{3} = 8$$

The percentage error is:

$$\epsilon_{\%} = |\frac{v_A - v_E}{v_E}| \cdot 100\% = |\frac{8 - 8\frac{1}{6}}{8\frac{1}{6}}| \cdot 100\% = 2.04\%$$

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$$v_A = 5.3m \cdot 3.5m = 18.55m^2$$

This gives the percentage error of the approximation as:

$$\epsilon_{\%} = |\frac{v_A - v_E}{v_E}| \cdot 100\% = |\frac{18.55 - 18.5832}{18.5832}| \cdot 100\% = 0.179\%$$

Exercise 4

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The perimeter of a circle is given by $2\pi r$, so the perimeter is $2\pi \cdot 5.323 = 33.4$ (3sf).

We have $v_E = 33.4$ and $v_A = 30$, so the percentage error is:

$$\epsilon_{\%} = |\frac{v_A - v_E}{v_E}| \cdot 100\% = |\frac{30 - 33.4}{33.4}| \cdot 100\% = 10\%$$

The final answer is given to 2sf (as the question required)

Exercise 4

Note that in general you should never round exact answers. Exercise 4 may seem different in this regard, but it's not. For part (b) we are not asked to calculate the exact answer, the precision is not specified, so we calculate correct to 3sf. In part (c) we are told to use our answer to part (b) as the exact answer. This is why, we can use the rounded answer.

The short test and the beginning of the class will be similar to the questions above.

In case of any questions you can email me at T.J.Lechowski@gmail.com.