I PROBLEM SOLVING WITH SIMULTANEOUS EQUATIONS

Many problems can be described using a pair of linear equations. We saw an example of this in the investigation on page **108** in which Kobeng was importing rackets.

You should follow these steps to solve problems involving simultaneous equations:

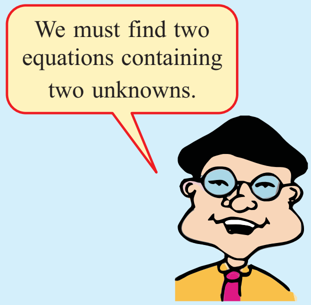
Step 1: Decide on the two unknowns, for example x and y. Do not forget the units.
Step 2: Write down **two** equations connecting x and y.
Step 3: Solve the equations simultaneously.
Step 4: Check your solutions with the original data given.
Step 5: Give your answer in sentence form.

Example 24 **Self Tutor**

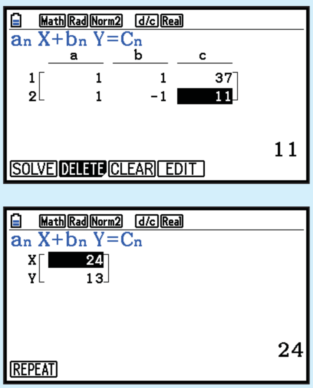
Find two numbers which have a sum of 37 and a difference of 11.

Let x and y be the unknown numbers, where $x > y$.

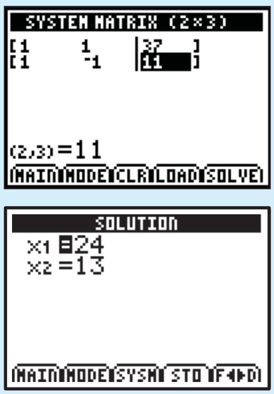
Then $x + y = 37$ .... (1)
{'sum' means add}
and $x - y = 11$ .... (2)
{'difference' means subtract}



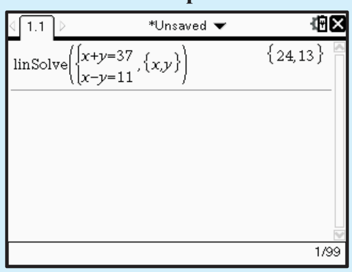
Casio fx-CG20



TI-84 Plus



TI-*n*spire



The solution is $x = 24$, $y = 13$.

$\therefore$ the numbers are 24 and 13.

EXERCISE 4I

1 Two numbers have a sum of 58 and a difference of 22. Find the numbers.

2 The larger of two numbers is one more than double the smaller, and their sum is 82. Find the two numbers.

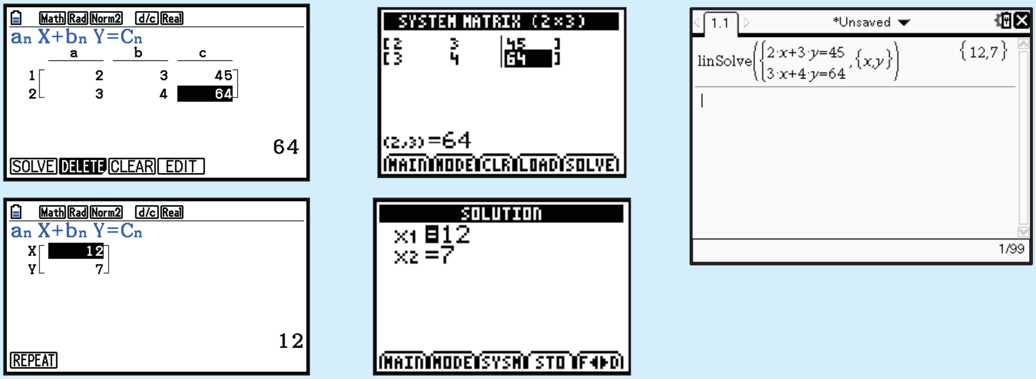
Example 25 **Self Tutor**

Two adults' tickets and three children's tickets to a baseball match cost \$45, while three adults' and four children's tickets cost \$64. Find the cost of each type of ticket.

Let \$$x$ be the cost of an adult's ticket and \$$y$ be the cost of a child's ticket.

So, $2x + 3y = 45$ and $3x + 4y = 64$

Casio fx-CG20 **TI-84 Plus** **TI-*n*spire**



The solution is $x = 12$, $y = 7$.
So, an adult's ticket costs \$12 and a child's ticket costs \$7.

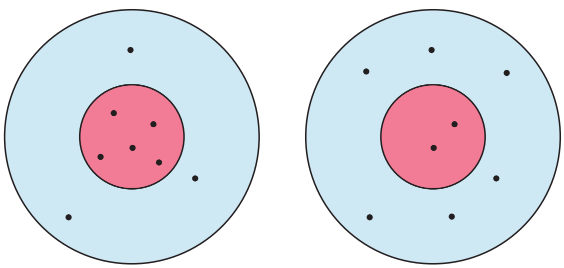
3 A hairdresser has 13 small and 14 large cans of hairspray, giving a total of 9 L of hairspray. At this time last year she had 4 small and 12 large cans, totalling 6 L of hairspray. How much spray is in each size can?

4 A violinist is learning a waltz and a sonatina. One day she practices for 33 minutes by playing the waltz 4 times and the sonatina 3 times. The next day she plays the waltz 6 times and the sonatina only once, for a total of 25 minutes. Determine the length of each piece.

5 A shop sells two lengths of extension cable. Tomasz buys 2 short cables and 5 long cables for a total length of 26 m. Alicja buys 24.3 m of cabling by getting 3 short and 4 long cables. Find the two different lengths of the extension cables.

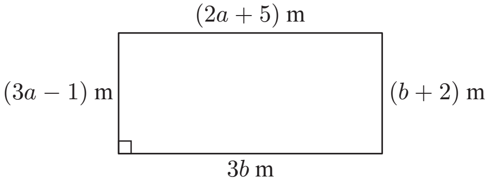
6 In an archery competition, competitors fire 8 arrows at a target. They are awarded points based on which region of the target is hit. The results for two of the competitors are shown opposite. How many points are awarded for hitting the:

a red **b** blue region?

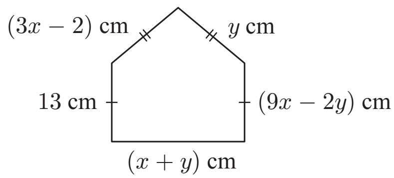


68 points 56 points

7 **a** Find the length of the longest side of this rectangle:



b Find the length of wire required to construct this pentagon:



8 A hardware store sells 3 litre paint cans for £15 and 5 litre paint cans for £20. In one day the store sells 71 litres of paint, worth a total of £320. How many paint cans did the store sell?

9 A piano teacher charges \$30 for a one hour lesson, and \$50 for a two hour lesson. She works for 25 hours in one week, and earns \$690. Determine how many two hour lessons she gave.

10 Kristen can run at 15 km h^{-1} and walk at 5 km h^{-1}. She completed a 42 km marathon in 4 hours. What distance did Kristen run during the marathon?

J QUADRATIC EQUATIONS

A **quadratic equation** in x is an equation which can be written in the form $ax^2+bx+c=0$ where a, b, and c are constants and $a \neq 0$.

The solutions of the equation are the values of x which make the equation true. We call these the **roots** of the equation, and they are also the **zeros** of the quadratic expression ax^2+bx+c.

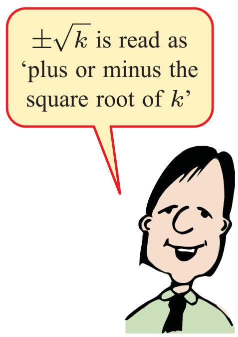
SOLUTION OF $x^2 = k$

Just as for linear equations, we can perform operations on both sides of a quadratic equation so as to maintain the balance.

Many quadratic equations can hence be rearranged into the form $x^2 = k$.

If k is positive then $\sqrt{k}$ exists such that $(\sqrt{k})^2 = k$ and $(-\sqrt{k})^2 = k$.

Thus the solutions are $x = \pm\sqrt{k}$.



$$\text{If } x^2 = k \text{ then } \begin{cases} x = \pm\sqrt{k} & \text{if } k > 0 \\ x = 0 & \text{if } k = 0 \\ \text{there are no real solutions if } k < 0. \end{cases}$$

Example 26 Self Tutor

Solve for x: **a** $3x^2 - 1 = 8$ **b** $5 - 2x^2 = 11$

a $3x^2 - 1 = 8$
$\therefore\ 3x^2 = 9$ {+1 to both sides}
$\therefore\ x^2 = 3$ {÷ both sides by 3}
$\therefore\ x = \pm\sqrt{3}$

b $5 - 2x^2 = 11$
$\therefore\ -2x^2 = 6$ {−5 from both sides}
$\therefore\ x^2 = -3$ {÷ both sides by −2}
which has no real solutions as x^2 cannot be negative.