

**Self-assessment answers: 10 Trigonometric equations and identities**

1.  $3x = \arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$  or  $\frac{5\pi}{6} + 2k\pi$  for  $k \in \mathbb{Z}$ .

$$\Rightarrow 3x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \dots$$

$$\Rightarrow x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}$$

[4 marks]

2. Using the identity  $\sin^2 x = 1 - \cos^2 x$ :

$$2\sin^2 x - \cos x = 1 \Rightarrow 2\cos^2 x + \cos x - 1 = 0$$

$$\Rightarrow (2\cos x - 1)(\cos x + 1) = 0$$

$$\Rightarrow \cos x = \frac{1}{2} \text{ or } -1$$

$$\Rightarrow x = \pm \frac{\pi}{3}, \pm\pi$$

[8 marks]

3.  $\sin x (\sin x - \sqrt{3} \cos x) = 0$

$$\Rightarrow \sin x = 0 \text{ or } \tan x = \sqrt{3}$$

$$\Rightarrow x = 0^\circ, 60^\circ, 240^\circ, 360^\circ$$

[6 marks]

4. (a)  $\frac{1}{\cos x} - \cos x = \frac{1 - \cos^2 x}{\cos x}$

$$= \frac{\sin^2 x}{\cos x}$$

$$= \sin x \tan x$$

$$(b) (i) \sin x - \cos x = \frac{1}{\sin x} - \frac{1}{\cos x}$$

$$\Rightarrow \sin x - \frac{1}{\sin x} = \cos x - \frac{1}{\cos x}$$

$$\Rightarrow \frac{1 - \sin^2 x}{\sin x} = \sin x \tan x \quad (\text{by using your answer from (a)})$$

$$\Rightarrow \cos^2 x = \sin^2 x \tan x$$

$$\Rightarrow \tan^3 x = 1$$

$$\Rightarrow \tan x = 1$$

$$(ii) \tan x = 1 \Rightarrow x = \frac{5\pi}{4}$$

[12 marks]