Formulae for probability problems

We will introduce some formulae that you can use when solving probability questions. These formulae are under sections 3.6 and 3.7 in the formula booklet. You can find the formula booklet online (it's in the first post on the website).

Basic formulae

Recall that we defined the probability of an element being in A as

$$P(A) = \frac{n(A)}{n(U)}$$

Basic formulae

Recall that we defined the probability of an element being in A as

$$P(A) = \frac{n(A)}{n(U)}$$

It easily follows that the probability of an element **not** being in A, ie being in A' is

$$P(A') = \frac{n(A')}{n(U)} = \frac{n(U) - n(A)}{n(U)} = 1 - \frac{n(A)}{n(U)} = 1 - P(A)$$

so we have that $P(A') = 1 - P(A)$.

Basic formulae

Recall that we defined the probability of an element being in A as

$$P(A) = \frac{n(A)}{n(U)}$$

It easily follows that the probability of an element **not** being in A, ie being in A' is

$$P(A') = \frac{n(A')}{n(U)} = \frac{n(U) - n(A)}{n(U)} = 1 - \frac{n(A)}{n(U)} = 1 - P(A)$$

so we have that $P(A') = 1 - P(A)$.

Note that this formula works both ways, so we also have $P(A) = 1 - P(A')$.

Basic formulae

Another important formula is

$$P(A\cup B)=\frac{n(A\cup B)}{n(U)}=\frac{n(A)+n(B)-n(A\cap B)}{n(U)}=$$
$$=\frac{n(A)}{n(U)}+\frac{n(B)}{n(U)}-\frac{n(A\cap B)}{n(U)}=P(A)+P(B)-P(A\cap B)$$

So we have $P(A\cup B)=P(A)+P(B)-P(A\cap B)$.

Basic formulae

Another important formula is

$$P(A \cup B) = \frac{n(A \cup B)}{n(U)} = \frac{n(A) + n(B) - n(A \cap B)}{n(U)} =$$
$$= \frac{n(A)}{n(U)} + \frac{n(B)}{n(U)} - \frac{n(A \cap B)}{n(U)} = P(A) + P(B) - P(A \cap B)$$

So we have $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Again remember that you can rearrange this formula to get $P(A \cap B) = P(A) + P(B) - P(A \cup B)$.

Conditional probability

This one is a little bit less obvious.

Conditional probability

This one is a little bit less obvious. Recall that when we calculate conditional probability A given B, then we want to know what is the probability of an element from B to be also in A.

Conditional probability

This one is a little bit less obvious. Recall that when we calculate conditional probability A given B, then we want to know what is the probability of an element from B to be also in A. We denote the conditional probability A given B as $P(A|B)$. We have:

$$P(A|B) = \frac{n(A \cap B)}{n(B)} = \frac{P(A \cap B)}{P(B)}$$

Conditional probability

This one is a little bit less obvious. Recall that when we calculate conditional probability A given B, then we want to know what is the probability of an element from B to be also in A. We denote the conditional probability A given B as $P(A|B)$. We have:

$$P(A|B) = \frac{n(A \cap B)}{n(B)} = \frac{P(A \cap B)}{P(B)}$$

Note that if we want to calculate probability of B given A we have:

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

numerator is of course the same $P(A \cap B) = P(B \cap A)$, but the denominator is different.

Example 1

If $P(A) = 0.2$, $P(B') = 0.6$ and $P(A \cup B) = 0.5$. Find:
i. $P(A \cap B)$,
ii. $P(A|B)$.

Example 1

If $P(A) = 0.2$, $P(B') = 0.6$ and $P(A \cup B) = 0.5$. Find:
i. $P(A \cap B)$,
ii. $P(A|B)$.

We start by finding $P(B)$. We have $P(B) = 1 - P(B') = 1 - 0.6 = 0.4$

Example 1

If $P(A) = 0.2$, $P(B') = 0.6$ and $P(A \cup B) = 0.5$. Find:
i. $P(A \cap B)$,
ii. $P(A|B)$.

We start by finding $P(B)$. We have $P(B) = 1 - P(B') = 1 - 0.6 = 0.4$
Now we can find $P(A \cap B)$.

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.2 + 0.4 - 0.5 = 0.1$$

Example 1

If $P(A) = 0.2$, $P(B') = 0.6$ and $P(A \cup B) = 0.5$. Find:
i. $P(A \cap B)$,
ii. $P(A|B)$.

We start by finding $P(B)$. We have $P(B) = 1 - P(B') = 1 - 0.6 = 0.4$
Now we can find $P(A \cap B)$.

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.2 + 0.4 - 0.5 = 0.1$$

Finally we can find $P(A|B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.1}{0.4} = 0.25$$

Example 2

If $P(A') = 0.65$, $P(B') = 0.2$ and $P(A \cup B) = 0.9$. Find:
i. $P(A \cap B)$,
ii. $P(B|A)$.

Example 2

If $P(A') = 0.65$, $P(B') = 0.2$ and $P(A \cup B) = 0.9$. Find:
i. $P(A \cap B)$,
ii. $P(B|A)$.

We start by finding $P(A)$ and $P(B)$. We have
$P(A) = 1 - P(A') = 1 - 0.65 = 0.35$
$P(B) = 1 - P(B') = 1 - 0.2 = 0.8$

Example 2

If $P(A') = 0.65$, $P(B') = 0.2$ and $P(A \cup B) = 0.9$. Find:
i. $P(A \cap B)$,
ii. $P(B|A)$.

We start by finding $P(A)$ and $P(B)$. We have
$P(A) = 1 - P(A') = 1 - 0.65 = 0.35$
$P(B) = 1 - P(B') = 1 - 0.2 = 0.8$
Now we can find $P(A \cap B)$.

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.35 + 0.8 - 0.9 = 0.25$$

Example 2

If $P(A') = 0.65$, $P(B') = 0.2$ and $P(A \cup B) = 0.9$. Find:
i. $P(A \cap B)$,
ii. $P(B|A)$.

We start by finding $P(A)$ and $P(B)$. We have
$P(A) = 1 - P(A') = 1 - 0.65 = 0.35$
$P(B) = 1 - P(B') = 1 - 0.2 = 0.8$
Now we can find $P(A \cap B)$.

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.35 + 0.8 - 0.9 = 0.25$$

Finally we can find $P(B|A)$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.25}{0.65} = 0.387$$

Example 3

If $P(A') = 0.4$, $P(B) = 0.2$ and $P(A \cup B) = 0.8$. Find:
i. $P(A \cap B)$,
ii. $P(B|A)$.

Example 3

If $P(A') = 0.4$, $P(B) = 0.2$ and $P(A \cup B) = 0.8$. Find:
i. $P(A \cap B)$,
ii. $P(B|A)$.

We start by finding $P(A)$. We have $P(A) = 1 - P(A') = 1 - 0.4 = 0.6$

Example 3

If $P(A') = 0.4$, $P(B) = 0.2$ and $P(A \cup B) = 0.8$. Find:
i. $P(A \cap B)$,
ii. $P(B|A)$.

We start by finding $P(A)$. We have $P(A) = 1 - P(A') = 1 - 0.4 = 0.6$
Now we can find $P(A \cap B)$.

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.6 + 0.2 - 0.8 = 0$$

Example 3

If $P(A') = 0.4$, $P(B) = 0.2$ and $P(A \cup B) = 0.8$. Find:
i. $P(A \cap B)$,
ii. $P(B|A)$.

We start by finding $P(A)$. We have $P(A) = 1 - P(A') = 1 - 0.4 = 0.6$
Now we can find $P(A \cap B)$.

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.6 + 0.2 - 0.8 = 0$$

Finally we can find $P(B|A)$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0}{0.6} = 0$$

The short test on Tuesday will include examples very much like the ones above.