Mutually exclusive and independent events

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An example would be rolling a dice once and A - scoring a 6, B - scoring an odd number. Of course A and B exclude each other, so $P(A \cap B) = 0$

Independent events

Two events are called independent if occurence of one event does not influence the occurence of the other. In terms of probability we write A and B are independent if

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An example would be rolling a dice twice and B - scoring a 6 on the first roll, A - scoring a 6 on the second roll. The first roll does not influence the outcome of the second roll, so we have $P(A|B) = P(A) = \frac{1}{6}$.

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and by multiplying by $P(B)$ we get:

$$P(A \cap B) = P(A)P(B)$$

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Note, this is very important, **you can only use these formulae if you are told that the events are mutually exclusive/independent or if you want to check if they are**. A common mistake is to apply the second formula for events that are not independent.

Example 1

Given that $P(A) = 0.4$ and $P(B) = 0.5$ find $P(A \cap B)$ if

i. $P(A \cup B) = 0.8$,

ii. A and B are mutually exclusive,

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Example 3

Two events A and B are such that $P(A) = 0.6$, $P(B) = 0.25$ and $P(A \cup B) = 0.8$. Check if A and B are

i. mutually exclusive,

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Two events A and B are such that $P(A) = 0.6$, $P(B) = 0.25$ and $P(A \cup B) = 0.8$. Check if A and B are

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We first calculate $P(A \cap B)$. We use the formula that can always be applied:

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If A and B were independent then $P(A \cap B)$ would be equal to $P(A) \cdot P(B) = 0.6 \cdot 0.25 = 0.15$, but $P(A \cap B) \neq 0.15$, so these events **are not** independent.

Example 4

Two events A and B are such that $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cup B) = 0.7$. Check if A and B are

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$P(A \cap B) = 0$, so these events **are** mutually exclusive.

If A and B were independent then $P(A \cap B)$ would be equal to $P(A) \cdot P(B) = 0.4 \cdot 0.3 = 0.12$, but $P(A \cap B) \neq 0.12$, so these events **are not** independent.

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Two events A and B are such that $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cup B) = 0.8$. Check if A and B are

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If A and B were independent then $P(A \cap B)$ would be equal to $P(A) \cdot P(B) = 0.6 \cdot 0.5 = 0.3$ and indeed $P(A \cap B) = 0.3$, so these events **are** independent.

The short test on Monday will include similar examples.