Self-assessment answers: 3 Polynomials

1. (a) $y = a(x + 1)(x - 2)(x - 5)$

$y = -10$ when $x = 0 \Rightarrow -10 = a(1)(-2)(-5) \Rightarrow a = -1$

$\therefore y = -(x + 1)(x - 2)(x - 5)$

(b) $y = a(x + 2)(x - 1)^2$

$y = 6$ when $x = 0 \Rightarrow 6 = a(1)(2) \Rightarrow a = 3$

$\therefore y = 3(x - 1)^2(x + 2)$ *[6 marks]*

2. When $x = 2$: $3(2)^3 - a(2)^2 + 4(2) + b = 0 \Rightarrow 4a - b = 32$

When $x = -1$: $3(-1)^3 - a(-1)^2 + 4(-1) + b = 3 \Rightarrow a - b = -10$

Solving simultaneous equations: $a = 14$, $b = 24$ *[4 marks]*

3. $a_3x^3 + a_2x^2 + 5x + 12 = 0$

$$\Rightarrow a_3\left(x^3 + \frac{a_2}{a_3}x^2 + \frac{5}{a_3}x + \frac{12}{a_3}\right) = 0$$

Suppose the three real roots are b, c and d.

Then $\left(x^3 + \frac{a_2}{a_3}x^2 + \frac{5}{a_3}x + \frac{12}{a_3}\right) = (x - b)(x - c)(x - d)$

But $bcd = -\frac{12}{a_3} = -16$ (product of the roots) $\Rightarrow a_3 = \frac{3}{4}$

And $b + c + d = -\frac{a_2}{a_3} = 5$(sum of the roots) $\Rightarrow a_2 = -5a_3 = -\frac{15}{4}$ *[3 marks]*

4. The discriminant is zero: $(-3)^2 - 4(k)(6) = 0 \Rightarrow k = \frac{9}{24}$ *[3 marks]*

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5. $f(-2) = 2(-8) + 3(4) - 12(-2) - 20 = -16 + 12 + 24 - 20 = 0$, so $(x + 2)$ is a factor.

$2x^3 + 3x^2 - 12x - 20 = (x + 2)(2x^2 + cx - 10)$

$= 2x^3 + (c + 4)x^2 + (2c - 10)x - 20$

Comparing coefficients: $c + 4 = 3 \quad \Rightarrow \quad c = -1$

So $f(x) = (x + 2)(2x^2 - x - 10) = (x + 2)(2x - 5)(x + 2) = (x + 2)^2(2x - 5)$ *[6 marks]*

6. The discriminant is: $(m + 3)^2 - 4(m + 1) = m^2 + 2m + 5 = (m + 1)^2 + 4 > 0$ for all m.

Hence the graph always has two x-intercepts. *[5 marks]*

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