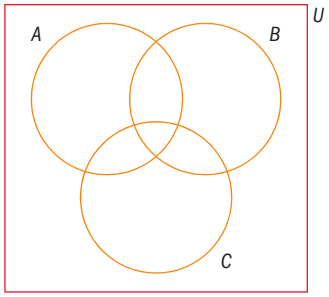
8.3 Extending to three sets

This Venn diagram shows a general three-set problem.



Use the same notation for three sets. But take great care using brackets to describe the sets.

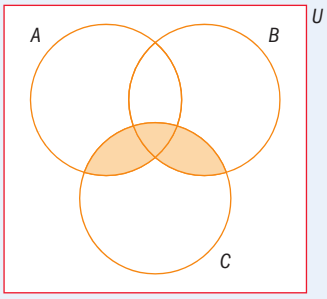
Example 8

Shade the region on a Venn diagram that shows the sets:

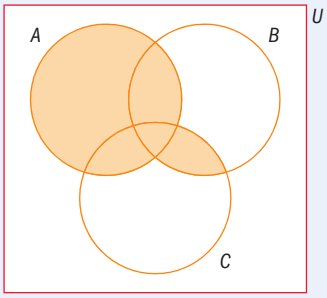
a $(A \cup B) \cap C$ **b** $A \cup (B \cap C)$

Answers

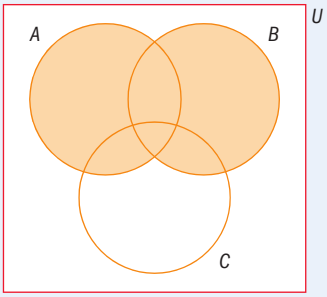
a $(A \cup B) \cap C$



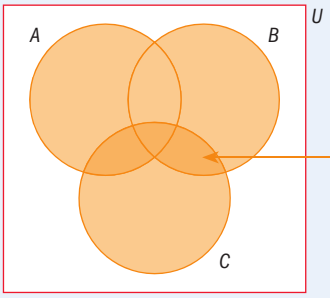
b $A \cup (B \cap C)$



First shade the region in the brackets $(A \cup B)$:

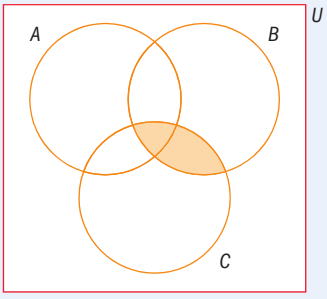


Then shade the other region, C:



The dark region is the intersection

First shade the region in the brackets $(B \cap C)$:



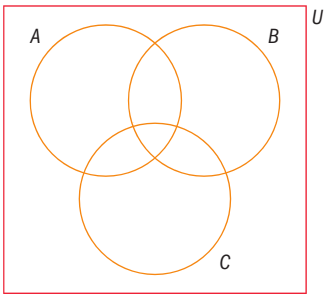
Then shade the other region A.

The union is *all* the shaded areas

Note that the statement $A \cup B \cap C$ has no mathematical meaning. The brackets are required to remove the ambiguity.

Exercise 8F

1 Shade the region on a three-set Venn diagram that shows each set:



a **i** $(A \cup B) \cup C$ **ii** $A \cup (B \cup C)$

b **i** $(A \cap B) \cap C$ **ii** $A \cap (B \cap C)$

c **i** $(A \cup C) \cap B$ **ii** $A \cup (C \cap B)$

d **i** $C \cap (A \cup B)$ **ii** $B \cup (C \cap A)$

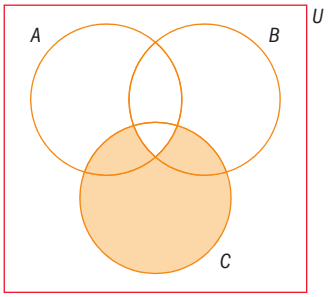
e **i** $(A \cup B) \cup C'$ **ii** $A \cup (B \cup C')$

f **i** $(A \cap B') \cap C$ **ii** $A \cap (B' \cap C)$

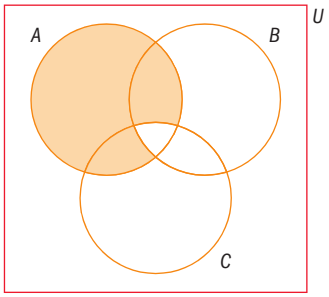
g **i** $(A \cup C) \cap B'$ **ii** $A \cup (C \cap B')$

2 Use set notation to name the shaded region in each Venn diagram.

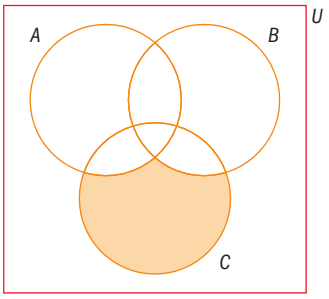
a



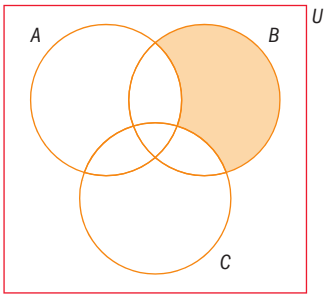
b



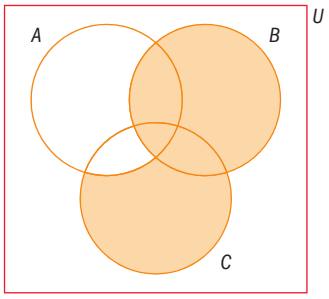
c



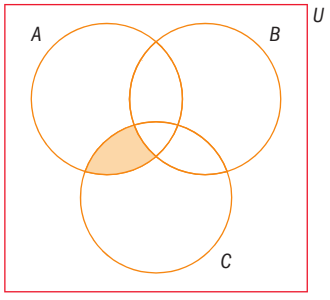
d



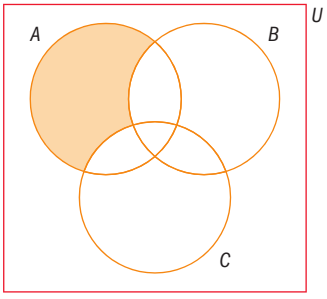
e



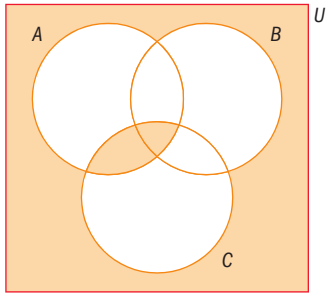
f



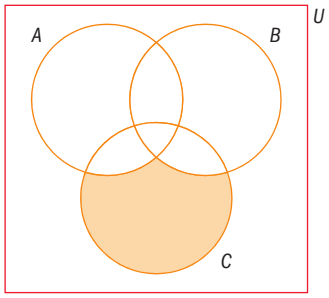
g



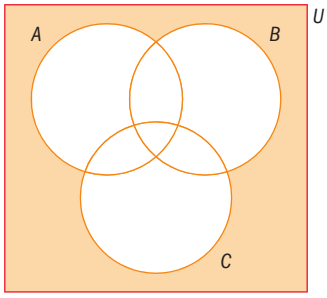
h



i

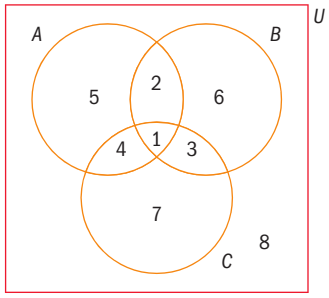


j



3 In this Venn diagram, $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$.
List the elements of:

a $A \cap B \cap C$
b $A' \cap B \cap C$
c $A \cap B' \cap C$
d $A \cap B \cap C'$
e $A' \cap B' \cap C$
f $A' \cap B \cap C'$
g $A \cap B' \cap C'$
h $A' \cap B' \cap C'$



4 For the Venn diagram in question 3, list the elements of:

a $A \cap (B \cup C)$ **b** $A' \cap (B \cup C)$
c $(A \cup B') \cap C$ **d** $(A \cup B) \cap C'$
e $(A' \cup B') \cap C$ **f** $(A' \cup B) \cap C'$
g $B \cap (A' \cup C')$ **h** $B' \cap (A' \cup C)$

8.4 Problem-solving using Venn diagrams

Here is the problem from the first investigation in this chapter:

Investigation – a contradiction?

A teacher asks her class how many of them study Chemistry.
She finds that there are 15. She then asks how many study Biology and finds that there are 13.
Later, she remembers that there are 26 students in the class.
But 15 + 13 = 28. Has she miscounted?

We can represent this problem on a Venn diagram.

Let B be the set of students studying Biology, and C be the set of students studying Chemistry. Then $n(B) = 13$, $n(C) = 15$ and $n(U) = 26$.

The teacher asks another question and finds out that 5 of the students study neither Biology nor Chemistry, so $n(B' \cap C') = 5$.