4 The probability that it will snow today is 0.9. If it does snow today then the probability it will snow tomorrow is 0.7. However, if it does not snow today then the probability that it will snow tomorrow is 0.6.
Draw a tree diagram which shows the possible weather conditions for the two days.

a Find the probability of two snowy days.
b Find the probability of exactly one snowy day.
c Given that there is exactly one snowy day, what is the probability that it is today?
d Given that there is at least one snowy day, what is the probability that it is today?

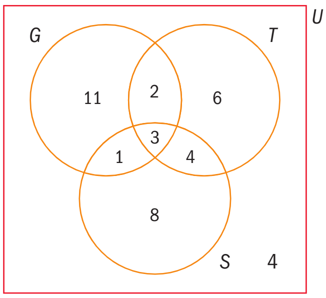
5 There are eight identical discs in a bag, five of which are black and the other three are red. The random experiment is: choose a disc at random from the bag, do not return the disc to the bag, then choose a **second** disc from the bag. Find the probability that the second disc chosen is red.

Review exercise

Paper 1 style questions

EXAM-STYLE QUESTIONS

1 The activities offered by a school are golf (G), tennis (T), and swimming (S). The Venn diagram shows the numbers of people involved in each activity.



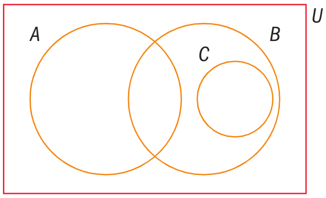
a Write down the number of people who
  i play tennis only
  ii play both tennis and golf
  iii play at least two sports
  iv do not play tennis.
b Copy the diagram and shade the part of the Venn diagram that represents the set $G' \cap S$.

2 A group of 40 children are surveyed to find out which of the three sports volleyball (A), basketball (B) or cricket (C) they play. The results are as follows:
7 children do not play any of these sports
2 children play all three sports
5 play volleyball and basketball
3 play cricket and basketball
10 play cricket and volleyball
15 play basketball
20 play volleyball.

a Draw a Venn diagram to illustrate the relationship between the three sports played.
b On your Venn diagram indicate the number of children that belong to each **disjoint** region.
c Find the number of children that play cricket only.

EXAM-STYLE QUESTIONS

3 The following Venn diagram shows the sets U, A, B and C.



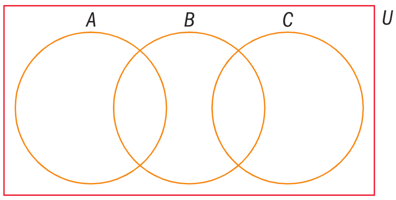
State whether the following statements are true or false for the information illustrated in the Venn diagram.

a $A \cup C = \varnothing$ **b** $C \subset (C \cup B)$

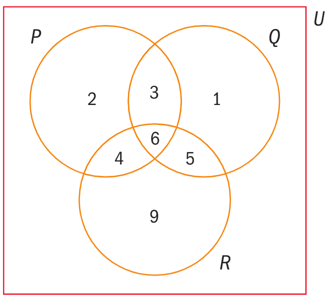
c $C \cap (A \cup B) = \varnothing$ **d** $C \subset A'$

e $C \cap B = C$ **f** $(A \cup B)' = A' \cap B'$

4 **a** Copy this diagram and shade $A \cup (B \cap C')$.



b In the Venn diagram on the right, the number of elements in each region is given. Find $n((P \cup Q) \cap R)$.



c U is the set of positive integers, $\mathbb{Z}^+$.
E is the set of odd numbers.
M is the set of multiples of 5.

i List the first four elements of the set M.

ii List the first three elements of the set $E' \cap M$.

5 $\mathbb{Z}$ is the set of integers, $\mathbb{Q}$ is the set of rational numbers, $\mathbb{R}$ is the set of real numbers.

a Write down an element of $\mathbb{Z}$. **b** Write down an element of $\mathbb{R} \cap \mathbb{Z}'$.

c Write down an element of $\mathbb{Q}$. **d** Write down an element of $\mathbb{Q} \cup \mathbb{Z}'$.

e Write down an element of $\mathbb{Q}'$. **f** Write down an element of $\mathbb{Q}' \cap \mathbb{Z}'$.

6 The table below shows the number of left- and right-handed tennis players in a sample of 60 males and females.

	Left-handed	Right-handed	Total
Male	8	32	40
Female	4	16	20
Total	12	48	60

If a tennis player was selected at random from the group, find the probability that the player is

a female and left-handed **b** male or right-handed

c right-handed, given that the player selected is female.

EXAM-STYLE QUESTIONS

7 A bag contains 3 red, 4 yellow and 8 green sweets.
You Jin chooses one sweet out of the bag at random and eats it.
She then takes out a second sweet.

a Write down the probability that the first sweet chosen was red.

b Given that the first sweet was not red, find the probability that the second sweet is red.

c Find the probability that both the first and second sweets chosen were yellow.

8 Ernest rolls two cubical dice. One of the dice has three red faces and three black faces. The other dice has the faces numbered from 1 to 6. By means of a sample space diagram, or otherwise, find

a the number of different possible combinations he can roll

b the probability that he will roll a black and an even number

c the probability that he will roll a number more than 4.

9 The table below shows the number of words in the extended essays of an IB class.

Number of words	$3100 \le w < 3400$	$3400 \le w < 3700$	$3700 \le w < 4000$	$4000 \le w < 4300$
Frequency	7	20	18	5

a Write down the modal group.

b Write down the probability that a student chosen at random writes an extended essay with a number of words in the range: $4000 \le w < 4300$.

The maximum word count for an extended essay is 4000 words.
Find the probability that a student chosen at random:

c does not write an extended essay that is on or over the word count

d writes an extended essay with the number of words in the range $3400 \le w < 3700$ given that it is not on or over the word count.

Paper 2 style questions

EXAM-STYLE QUESTIONS

1 Let $U = \{x \mid 8 \le x < 13, x \in \mathbb{N}\}$.
P, Q and R are the subsets of U such that
P = {multiples of four}
Q = {factors of 24}
R = {square numbers}

a List the elements of U.

b **i** Draw a Venn diagram to show the relationship between sets P, Q and R.
ii Write the elements of U in the appropriate places on the Venn diagram.

c List the elements of:
i $P \cap R$ **ii** $P' \cap Q \cap R$

d Describe in words the set $P \cup Q$.

e Shade the region on your Venn diagram that represents $(P \cup R) \cap Q'$.

EXAM-STYLE QUESTIONS

2 In a club with 70 members, everyone attends either on Tuesday for Drama (D) or on Thursday for Sports (S), or on both days for Drama and Sports.
One week it is found that 48 members attend for Drama, 44 members attend for Sports, and x members attend for both Drama and Sports.

a **i** Draw and **label fully** a Venn diagram to illustrate this information.
ii Find the number of members who attend for both Drama and Sports.
iii Describe, in words, the set represented by $(D \cap S)'$.
iv What is the probability that a member selected at random attends for Drama only or Sports only?

The club has 40 female members, 10 of whom attend for both Drama and Sports.

b What is the probability that a member of the club selected at random
i is female and attends for Drama only or Sports only?
ii is male and attends for both Drama and Sports?

3 On a particular day 50 children are asked to make a note of what they drank that day.
They are given three choices: water (P), fruit juice (Q) or coffee (R).
2 children drank only water.
4 children drank only coffee.
12 children drank only fruit juice.
3 children drank all three.
4 children drank water and coffee only.
5 children drank coffee and fruit juice only.
15 children drank water and fruit juice only.

a Represent the above information on a Venn diagram.
b How many children drank none of the above?
c A child is chosen at random. Find the probability that the child drank
i fruit juice
ii water or fruit juice but not coffee
iii no fruit juice, given that the child did drink water.
d Two children are chosen at random. Find the probability that both children drank all three choices.

EXAM-STYLE QUESTIONS

4 The sets P, Q, and R are subsets of U. They are defined as follows:

U = {positive integers less than 13}

P = {prime numbers}

Q = {factors of 18}

R = {multiples of 3}

a List the elements (if any) of

i P **ii** Q **iii** R **iv** $P \cap Q \cap R$

b i Draw a Venn diagram showing the relationship between the sets U, P, Q and R.

ii Write the elements of sets U, P, Q and R in the appropriate places on the Venn diagram.

c From the Venn diagram, list the elements of

i $P \cup (Q \cap R)$ **ii** $(P \cup R)'$ **iii** $(P \cup Q)' \cap R'$

d Find the probability that a number chosen at random from the universal set U will be

i a prime number

ii a prime number, but **not** a factor of 18

iii a factor of 18 or a multiple of 3, but **not** a prime number

iv a prime number, given that it is a factor of 18.

5 There are two biscuit tins on a shelf. The **red** tin contains four chocolate biscuits and six plain biscuits. The **blue** tin contains one chocolate biscuit and nine plain biscuits.

A child reaches into the **red** tin and randomly selects a biscuit. The child returns that biscuit to the tin, shakes the tin, and then selects another biscuit.

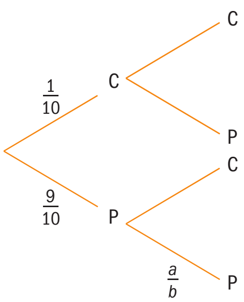
a Draw a tree diagram that shows the possible outcomes. Place the appropriate probability on each branch of the tree diagram.

b Find the probability that

i both biscuits chosen are chocolate

ii one of the biscuits is plain and the other biscuit is chocolate.

c A second child chooses a biscuit from the **blue** tin. The child eats the biscuit and chooses another one from the **blue** tin. The tree diagram on the right represents the possible outcomes for this event.



i Write down the value of a and of b.

ii Find the probability that both biscuits are chocolate.

iii What is the probability that *at least* one of the biscuits is plain?

d Suppose that, before the two children arrived, their brother randomly selected one of the biscuit tins and took out one biscuit. Calculate the probability that this biscuit was chocolate.

EXAM-STYLE QUESTION

6 The data in the table below refer to a sample of 60 randomly chosen plants.

Growth rate	Classification by environment			
	desert	temperate	waterlogged	total
high	4	7	13	24
low	9	11	16	36
total	13	18	29	60

a **i** Find the probability of a plant being in a desert environment.

ii Find the probability of a plant having a low growth rate and being in a waterlogged environment.

iii Find the probability of a plant not being in a temperate environment.

b A plant is chosen at random from the above group.
Find the probability that the chosen plant has

i a high growth rate or is in a waterlogged environment, but not both

ii a low growth rate, given that it is in a desert environment.

c The 60 plants in the above group were then classified according to leaf type. It was found that 15 of the plants had type A leaves, 36 had type B leaves and 9 had type C leaves.
Two plants were randomly selected from this group. Find the probability that

i both plants had type B leaves

ii neither of the plants had type A leaves.

CHAPTER 8 SUMMARY

Basic set theory

- A **set** is simply a collection of objects. The objects are called the **elements** of the set.
- The number of elements in a set A is denoted as $n(A)$.

Venn diagrams

- The **universal set** (denoted U), must be stated to make a set well defined.
- If every element in a given set, M, is also an element of another set, N, then M is a **subset** of N, denoted $M \subseteq N$
- A **proper subset** of a given set is one that is **not identical** to the original set.
  If M is a proper subset of N (denoted $M \subset N$) then
  1 **every** element of M also lies in N and
  2 there are some elements in N that do not lie in M.
- The empty set $\varnothing$ is a subset of every set.
- Every set is a subset of itself.
- The **intersection** of set M and set N (denoted $M \cap N$) is the set of all elements that are in **both** M **and** N.

Continued on next page

- The **union** of set M and set N (denoted $M \cup N$) is the set of all elements that are in **either** M **or** N or **both**.
- The **complement** of set M, denoted as M', is the set of all the elements in the universal set that **do not** lie in M.
- The complement of the universal set, U', is the empty set, $\varnothing$.

Basic probability theory

- If all of the equally likely possible outcomes of a random experiment can be listed as U, the universal set, and an event A is defined and represented by a set A, then:

  $$\mathrm{P}(A) = \frac{n(A)}{n(U)}$$

  There are three consequences of this law:

  1 $\mathrm{P}(U) = \frac{n(U)}{n(U)} = 1$ (the probability of a **certain** event is 1)

  2 $\mathrm{P}(\varnothing) = \frac{n(\varnothing)}{n(U)} = 0$ (the probability of an **impossible** event is 0)

  3 $0 \le \mathrm{P}(A) \le 1$ (the probability of any event **always** lies between 0 and 1)
- For complementary events, $\mathrm{P}(A') = 1 - \mathrm{P}(A)$
- For combined events, $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

Conditional probability

- The conditional probability that A occurs given that B has occurred is written as $\mathrm{P}(A \mid B)$ and is defined as:

  $$\mathrm{P}(A \mid B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$$

Two special cases: mutually exclusive and independent events

- Events A and B are mutually exclusive if and only if $\mathrm{P}(A \cap B) = 0$.
- A and B are independent if and only if $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$.