Exercise 8C

1 Consider these sets:

$M = \{x \mid 2 \leq x < 5, x \in \mathbb{Z}\}$

$N = \{x \mid 0 < x \leq 5, x \in \mathbb{Z}\}$

$P = \{x \mid -2 \leq x < 6, x \in \mathbb{Z}^+\}$

$S = \{(x, y) \mid x + y = 5, x \in \mathbb{Z}^+, y \in \mathbb{Z}^+\}$

$T = \{(x, y) \mid x + y = 5, x \in \mathbb{Z}, y \in \mathbb{Z}\}$

$V = \{p \mid p \text{ is a prime number and a multiple of } 4\}$

$W = \{x \mid x \text{ is a factor of } 20\}$

$X = \{x \mid x < 200, x \in \mathbb{R}\}$

State whether each statment is true or false:

a $N \subset M$ **b** $S \subset T$ **c** $P \subset M$ **d** $W \subset X$
e $M \subset P$ **f** $P \subset N$ **g** $\varnothing \subset T$ **h** $V \subset W$

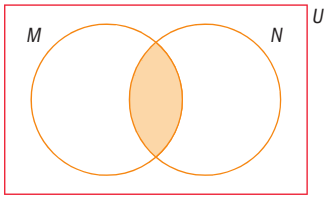
2 **a** List all the subsets of
i $\{a\}$ **ii** $\{a, b\}$ **iii** $\{a, b, c\}$ **iv** $\{a, b, c, d\}$
b How many subsets does a set with n members have?
c How many subsets does $\{a, b, c, d, e, f\}$ have?
d A set has 128 subsets. How many elements are there in this?

3 **a** List all the proper subsets of
i $\{a\}$ **ii** $\{a, b\}$ **iii** $\{a, b, c\}$ **iv** $\{a, b, c, d\}$
b How many proper subsets does a set with n members have?
c How many proper subsets has $\{a, b, c, d, e, f\}$?
d A set has 254 subsets. How many elements are there in this?

Intersection

→ The **intersection** of set M and set N (denoted $M \cap N$) is the set of all elements that are in **both** M **and** N.

$M \cap N$ is the shaded region on the Venn diagram:



Example 4

Given the sets:
$A = \{1, 2, 3, 4, 5\}$
$B = \{x \mid 0 < x \leq 5, x \in \mathbb{Z}\}$
$C = \{p \mid p$ is a prime number and a multiple of 10$\}$
$D = \{4, 5, 6, 7\}$
$E = \{x \mid x$ is a square number less than 50$\}$
write down the sets
a $A \cap D$ **b** $A \cap B$ **c** $D \cap E$ **d** $C \cap D$

Answers

a $A \cap D = \{4, 5\}$.

b $A \cap B = \{1, 2, 3, 4, 5\}$.

c The element 4 lies in both sets, hence $D \cap E = \{4\}$.

d $C \cap D = \varnothing$.

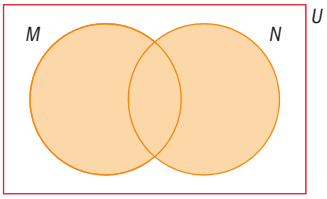
First, list the elements of each set:
$A = \{1, 2, 3, 4, 5\}$
$B = \{1, 2, 3, 4, 5\}$
$C = \varnothing$
$D = \{4, 5, 6, 7\}$
$E = \{1, 4, 9, 16, 25, 36, 49\}$
Compare the sets
$A = \{1, 2, 3, 4, 5\}$ *and*
$D = \{4, 5, 6, 7\}$.
Sets A and B are identical.
$D = \{4, 5, 6, 7\}$ *and*
$E = \{1, 4, 9, 16, 25, 36, 49\}$.
C does not contain any elements; hence there is no element that lies in both sets.

Is it always true that for any set X:
$\varnothing \cap X = \varnothing$ and
$X \cap X = X$?

Union

➔ The **union** of set M and set N (denoted $M \cup N$) is the set of all elements that are in **either** M **or** N or **both**.

$M \cup N$ is the shaded region on the Venn diagram:



$M \cup N$ includes those elements that are in **both** M and N.
This is important!

Example 5

Given the sets:
$A = \{1, 2, 3, 4, 5\}$
$B = \{1, 2, 3, 4, 5\}$
$C = \varnothing$
$D = \{4, 5, 6, 7\}$
$E = \{1, 4, 9, 16, 25, 36, 49\}$
Write down the sets
a $A \cup D$ **b** $A \cup B$ **c** $C \cup D$

Answers

a $A \cup D = \{1, 2, 3, 4, 5, 6, 7\}$

b $A \cup B = \{1, 2, 3, 4, 5\}$

c $C \cup D = \{4, 5, 6, 7\}$

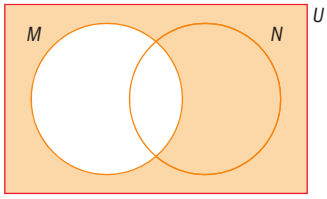
$A = \{1, 2, 3, 4, 5\}$ and $D = \{4, 5, 6, 7\}$.
*To write down $A \cup D$ list **every** element of each set, but **only once**.*
A and B are identical.
$C = \varnothing$ and $D = \{4, 5, 6, 7\}$.
$C \cup D = D$, since there are no extra elements to list from C.

Is it always true that for any set X:
$\varnothing \cup X = X$ and
$X \cup X = X$?

Complement

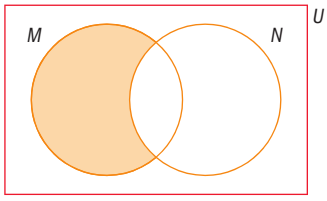
→ The **complement** of set M, denoted as M', is the set of all the elements in the universal set that **do not** lie in M.

M' is the shaded part of this Venn diagram:



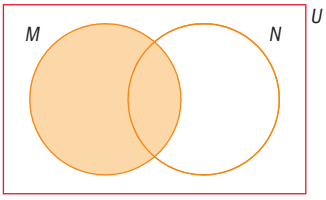
→ The complement of the universal set, U', *is the empty set*, $\varnothing$.

We can use Venn diagrams to represent different combinations of set complement, intersection and union. For example, $M \cap N'$ is shown here:

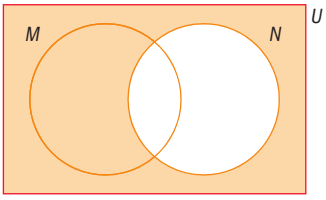


To see this in more detail, look at at the separate diagrams of M and N':

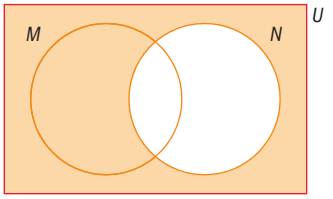
M



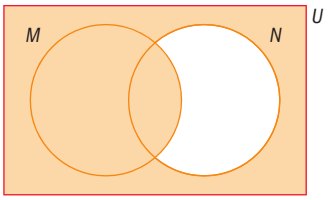
N'



Combining these for the intersection $M \cap N'$ gives shading only in the area common to both diagrams.



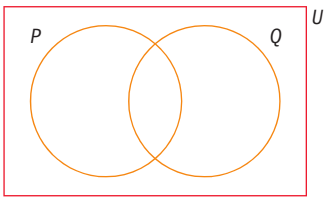
This diagram shows the set $M \cup N'$. Since it is the region that satisfies **either** M **or** N', it includes the shading from both diagrams.



Exercise 8D

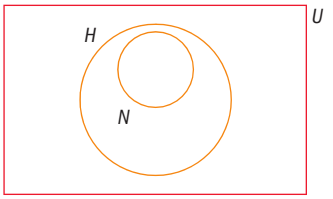
1 Copy the Venn diagram for sets P and Q. Shade the region that represents

a $P \cup Q'$ **b** $P \cap Q'$ **c** $P' \cup Q'$
d $P' \cap Q'$ **e** $(P \cup Q)'$ **f** $(P \cap Q)'$



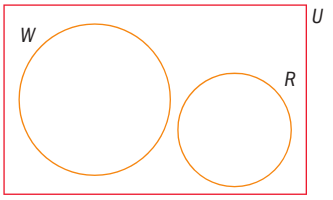
2 Copy the Venn diagram for sets H and N. Shade the region that represents

a H' **b** $H \cap N'$ **c** N'
d $H' \cup N'$ **e** $H' \cap N'$ **f** $H \cup N'$



3 Copy the Venn diagram for sets W and R. Shade the region that represents

a W' **b** $W' \cap R'$ **c** $W' \cap R$
d $W' \cup R'$ **e** $(W \cup R)'$ **f** $(W' \cap R)'$



4 U is defined as the set of all integers. Consider the following sets:

$A = \{1, 2, 3, 4, 5\}$
$B = \{x \mid 0 \leq x < 5, x \in \mathbb{Z}\}$
$C = \{p \mid p \text{ is an even prime number}\}$
$D = \{4, 5, 6, 7\}$
$E = \{x \mid x \text{ is a square number less than } 50\}$

Write down the sets:

a $A \cap B$ **b** $B \cap E$ **c** $C \cap D$ **d** $C \cap E$ **e** $B \cap D$
f $A \cup B$ **g** $B \cup A$ **h** $C \cup D$ **i** $C \cup A$ **j** $B \cup D$

Decide whether each statment is true or false.

k $A \subset B$ **l** $B \subset A$ **m** $C \subset A$ **n** $C \subset D$ **o** $(C \cap D) \subset E$

Venn diagrams can show individual set elements as well.

Example 6

$U = \{4, 5, 6, 7, 8, 9, 10\}$, $F = \{4, 5, 6, 7\}$ and $G = \{6, 7, 8, 9\}$.

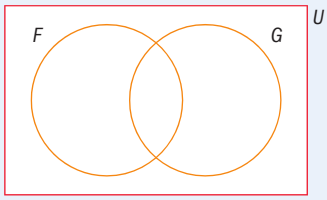
a Draw a Venn diagram for F, G and U.

b Write down these sets:

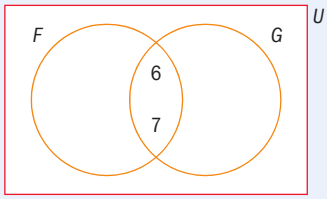
i F' **ii** $F \cap G'$ **iii** $(F \cap G)'$

Answers

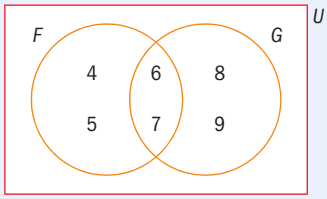
Sketch the empty Venn diagram.



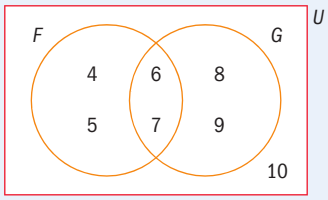
$F \cap G = \{6, 7\}$. *Add 6 and 7 to the diagram.*



Add the remaining elements of F and G.



a



Add the remaining elements of U.

b **i** $F' = \{8, 9, 10\}$

ii $F \cap G' = \{4, 5\}$

iii $(F \cap G)' = \{4, 5, 8, 9, 10\}$

Use the diagram to write down the elements of these sets.

Note that $F \cap G' \neq (F \cap G)'$. You must be very precise when using brackets.

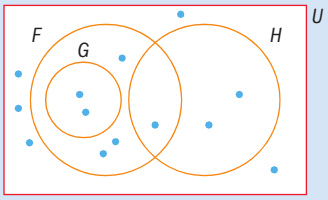
You can use Venn diagrams to work out the **number of elements** in each set without writing them all down.

Example 7

In this Venn diagram, each dot represents an element.

Write down:

a $n(G)$
b $n(F)$
c $n(G \cap F)$
d $n(H')$
e $n(F \cap H)$
f $n(G \cap H)$



Is each statement true or false?

g $n(F \cup H) = n(F) + n(H)$
h $n(G \cup H) = n(G) + n(H)$

Answers

a $n(G) = 2$
b $n(F) = 6$
c $n(G \cap F) = 2$
d $n(H') = 10$
e $n(F \cap H) = 1$
f $n(G \cap H) = 0$
g The statement is false.
h The statement is true.

Count the dots in each set.

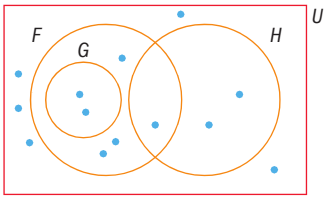
$n(F \cup H) = 8$, $n(F) = 6$, $n(H) = 3$
$n(G \cup H) = 5$, $n(G) = 2$, $n(H) = 3$.

The statements in **e** and **f** help you decide whether statements **g** and **h** are true or false.

Exercise 8E

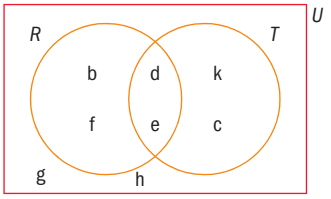
1 Is each statement true or false?

a $F \subset G$
b $n(F \cup G) = 6$
c $n(G') = 8$
d $n(F \cup H) = 6$
e $H \cup F = G'$
f $F' \subset H$
g $n(G' \cap H) = 5$
h $n(F' \cap G) = 5$



2 List the elements of

a U
b R
c R'
d T
e T'



3 List the elements of

a A
b A'
c $A \cup B'$
d $A \cap B'$
e $A' \cup B'$