1. A continuous random variable X has probability density function

$$f(x) = \begin{cases} 0, & x < 0 \\ ae^{-ax}, & x \geq 0. \end{cases}$$

It is known that $P(X < 1) = 1 - \frac{1}{\sqrt{2}}$.

(a) Show that $a = \frac{1}{2}\ln 2$.

(6)

(b) Find the median of X.

(5)

(c) Calculate the probability that $X < 3$ given that $X > 1$.

(9)

(Total 20 marks)

2. After a shop opens at 09:00 the number of customers arriving in any interval of duration t minutes follows a Poisson distribution with mean $\frac{t}{10}$.

(a) (i) Find the probability that exactly five customers arrive before 10:00.

(ii) Given that exactly five customers arrive before 10:00, find the probability that exactly two customers arrive before 09:30.

(7)

(b) Let the second customer arrive at T minutes after 09:00.

(i) Show that, for $t > 0$,

$$P(T > t) = \left(1 + \frac{t}{10}\right)e^{-\frac{t}{10}}.$$

(ii) Hence find in simplified form the probability density function of T.

(iii) Evaluate $E(T)$.
(You may assume that, for $n \in \mathbb{Z}^+$ and $a > 0$, $\lim_{t \to \infty} t^n e^{-at} = 0$.)

(12)

(Total 19 marks)