1. (6 points) Consider the events A and B, where $P(A) = \frac{2}{5}$, $P(B') = \frac{1}{4}$ and $P(A \cup B) = \frac{7}{8}$.
   (a) Write down the value of $P(B)$.
   (b) Find $P(A \cap B)$.
   (c) Find $P(A|B)$.

2. (6 points) Consider $f(x) = 2kx^2 - 4kx + 1$, for $k \neq 0$. The equation $f(x) = 0$ has two equal roots.
   (a) Find the value of k
   (b) The line $y = m$ intersects the graph of f. Find all possible values of m.

3. (6 points) Let $p = \log 3$, $q = \log 4$ and $r = \log 5$. Express each of the following in terms of p, q and r.

   i. $\log 36$,

   ii. $\log 30$,

   iii. $\log_2 15$,

   iv. $\log_3 0.8$.

4. (5 points) Find the coefficient of x^4 in the expansion of $(2x - \frac{1}{x})^8$.

5. (5 points) Solve the inequality $|x-1| > |2x-1|$.

6. (6 points)
   (a) Express the quadratic $3x^2 - 12x + 7$ in the form $a(x+b)^2 + c$, where $a, b, c \in \mathbb{Z}$.
   (b) Describe the sequence of transformations that transforms the graph of $y = x^2$ into the graph of $y = 3x^2 - 12x + 7$.

7. (7 points) The following diagram shows the triangle ABC.

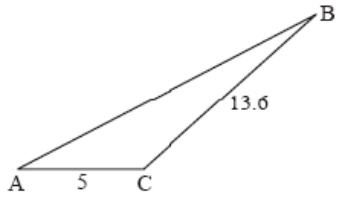

diagram not to scale

The angle at C is obtuse, $AC = 5cm$, $BC = 13.6cm$ and the area is $20cm^2$.

(a) Find $\angle ACB$.

(b) Find the length of AB.

8. (7 points) Solve the equation:

$$5 + 5\sin\theta = 2\cos^2\theta$$

for $0 \leqslant \theta < 2\pi$.

9. (13 points) In a class of 100 boys, 60 boys play football and 74 boys play rugby. Each boy must play at least one sport from football and rugby.

   (a) (3 points) (i) Find the number of boys who play both sports.
   (ii) Write down the number of boys who play only rugby.

   (b) (4 points) One boy is selected at random.
   (i) Find the probability that he plays only one sport.
   (ii) Given that the boy selected plays only one sport, find the probability that he plays rugby.

   Let A be the event that a boy plays football and B be the event that a boy plays rugby.

   (c) (1 point) Explain why A and B are not mutually exclusive.

   (d) (2 points) Show that A and B are not independent.

   Two boys are selected at random.

   (e) (3 points) Find the probability that one of them play only football and the other plays only rugby.

10. (9 points) Let $f(x) = \frac{3x+4}{x-2}$.
    (a) Find the expression for $f^{-1}(x)$.
    (b) Sketch the graph of $f^{-1}(x)$.
    (c) The equation $|f^{-1}(|x|)| = k$ has exactly 2 solutions. Find the possible values of k.

11. (10 points) Let $f(x) = x^2 - 4x + 6$.

(a) Write $f(x)$ in the vertex form.

Let $g(x) = \dfrac{2}{f(x)}$.

(b) Show that the graph of g has no vertical asymptote.

(c) Write down the equation of the horizontal asymptote of g.

(d) Find the range of $g(x)$.

(e) Sketch the graph of $g(|x|)$.