

1. (6 points) Given that the graph of the function

$$f(x) = x^2 - \sqrt{8} \cos \theta x + 3 \cos \theta - 1$$

intersects the x -axis exactly once. Find the:

- (a) the possible values of θ , given that $0 \leq \theta \leq 2\pi$,
- (b) the coordinates of the point of intersection of the graph of f with the x -axis.

2. (7 points) When a person is at rest, the blood pressure, P millimetres of mercury at any time t seconds can be approximately modelled by the equation

$$P(t) = -20 \cos\left(\frac{5\pi}{3}t\right) + 100, \quad t \geq 0$$

- (a) Determine the amplitude and period of P .
- (b) What is the maximum blood pressure reading that can be recorded for this person?
- (c) Sketch the graph of $P(t)$, showing one full cycle.
- (d) Find the first two times when the pressure reaches a reading of 110 mmHg.

3. (8 points) (a) Prove that:

$$\frac{\sin 2x + \sin x}{\cos 2x + \cos x + 1} = \tan x$$

(b) Hence solve the equation:

$$\frac{\sin 2x + \sin x}{\cos 2x + \cos x + 1} = -1$$

for $0 \leq x \leq \pi$.

(c) The equation

$$\sin 2x + \sin x + \cos 2x + \cos x + 1 = 0$$

has two solutions in the interval $0 \leq x \leq \pi$.

- i. state one of the solutions,
- ii. find the other solution.

4. (7 points) If α is an obtuse angle and $\tan \alpha = -\frac{1}{2}$, find the exact value of:

(a) $\cos 2\alpha$,

(b) $\sin 2\alpha$,

(c) $\sin 3\alpha$.

5. (4 points) Solve the inequality $|x + 2| > |3x - 1|$.