

1. (9 points) Let $f(x) = \sqrt{3} \cos x + \sin x + 1$, where $x \in \mathbb{R}$.
 - (a) (3 points) Write $f(x)$ in the form $R \sin(x + \alpha) + 1$, where $R > 0$ and $\alpha \in (-\frac{\pi}{2}, \frac{\pi}{2})$.
 - (b) (1 point) State the range of f .
 - (c) (2 points) Sketch the graph of f .
 - (d) (3 points) Solve the equation $f(x) = -1$.

2. (18 points)

(a) (2 points) Sketch the graph of $f(x) = \sec x$ for $x \in [0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$.

(b) (3 points) Write down:

- i. the coordinates of the maximum and minimum points of the graph,
- ii. the equation of the asymptote of the graph,
- iii. the range of $f(x)$.

(c) (2 points) On a separate diagram sketch the graph of $f^{-1}(x) = \arccsc(x)$.

(d) (1 point) Write down the domain of $\arccsc(x)$.

(e) (5 points) Assume $x > 1$.

- i. Use an appropriate right triangle to show that

$$\sin(\arccsc(x)) = \frac{\sqrt{x^2 - 1}}{x}$$

- ii. Find similar expressions for $\cos(\arccsc(x))$ and $\tan(\arccsc(x))$.

(f) (3 points) Calculate the exact value of

$$\tan(\arccsc(2) + \arccsc(3))$$

(g) (2 points) Find the exact value of

$$\tan(\arccsc(-2) + \arccsc(-3))$$

3. (5 points) If $\alpha \in (\pi, \frac{3\pi}{2})$ and $\cos \alpha = -\frac{2}{5}$, find the value of:
- (a) $\sin 2\alpha$,
 - (b) $\tan 3\alpha$.