1. (5 points) A surveying team is trying to measure the height of a hill. They measure the angle of elevation from an observation point to the top of the hill to be $23°$. They then move a distance of 250m on level ground directly away from the hill and measure the angle of elevation to be $19°$. Find the height of the hill, correct to the nearest metre.

2. (5 points) Express $\sin x - \sqrt{3}\cos x$ in the form $R\sin(x+\alpha)$ with $R > 0$ and $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ and hence solve the equation:

$$\sin x - \sqrt{3}\cos x = -1$$

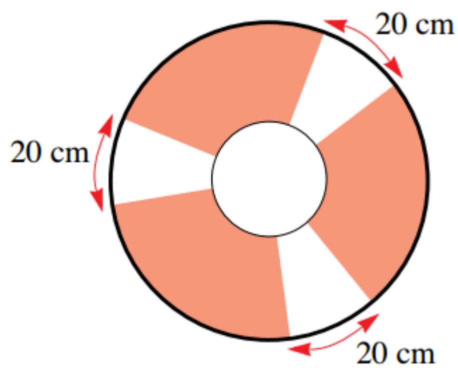
for $0 \leqslant x \leqslant 2\pi$.

3. (8 points) Let $f(x) = 3x^2 - 2x + 5$ for $-1 \leqslant x \leqslant 1$.
   (a) (2 points) Write $f(x)$ in the vertex form and hence write down the coordinates of the vertex of f.
   (b) (2 points) Write down the range of f.

   Let $g(\theta) = 3\cos 2\theta - 4\cos\theta + 13$ for $0 \leqslant \theta \leqslant 2\pi$.
   (c) (2 points) Show that $g(\theta) = 6\cos^2\theta - 4\cos\theta + 10$.
   (d) (2 points) Hence find the range of g.

4. (5 points) From a point X, 200m South of a cliff, the angle of elevation of the top of the cliff is $30°$. From a point Y, due East of the cliff, the angle of elevation of the top of the cliff is $20°$. How fart apart are points X and Y.

5. (5 points) The diagram shows a disc of radius $40cm$ with parts of it shaded. The smaller circle (having the same centre as the disc) has a radius of $10cm$. What area of the disc has been shaded?



6. (9 points) The rabbit population, $R(t)$ thousands, in a northern region of South Australia is modelled by the equation

$$R(t) = 12 + 3\cos\left(\frac{\pi}{6}t\right)$$

for $0 \leqslant t \leqslant 24$, where t is measured in months after the first of January.

(a) (1 point) What is the largest rabbit population predicted by this model?

(b) (2 points) How long is it between the times when the population reaches consecutive peaks?

(c) (3 points) Sketch the graph of $R(t)$ for $0 \leqslant t \leqslant 24$.

(d) (3 points) Find the longest time span for which $R(t) \geqslant 13.5$.

7. (9 points) Let $\tan\left(\dfrac{\theta}{2}\right) = t$.

   (a) (1 point) Use double angle formula to write down $\tan\theta$ in terms of t.

   (b) (3 points) Use an appropriate right triangle to express $\sin\theta$ and $\cos\theta$ in terms of t.

   (c) (5 points) Hence, by using the t-substitution, solve the equation

$$\sin\theta + \cos\theta = 1$$

   in the interval $-\pi \leqslant \theta \leqslant \pi$.

8. (7 points) Consider the equation

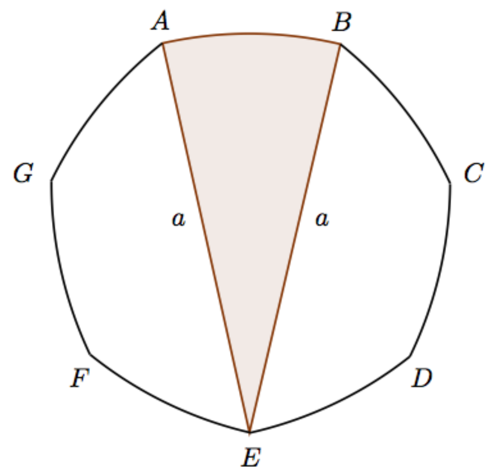
$$(u^2 - 1)^2 + (v^2 - 1)^2 + 2(u - v)^2 = 0$$

(a) (2 points) By considering the sign of the expressions on the left hand side of the equation, write down two possible pairs of solutions to this equation.

(b) (5 points) By expanding the left hand side of the equation and letting $u = \sin x$ and $v = \cos y$ find all values of x and y that satisfy:

$$\sin^4 x + \cos^4 y + 2 = 4 \sin x \cos y$$

9. (9 points) The diagram shows a British 50 pence coin.



The seven arcs $AB, BC, ...FG, GA$ are of equal length and each is formed from a circle of radius a having its centre at the vertex diametrically opposite the mid-point of the arc.

(a) (2 points) Calculate the size of the angle AEB by referring to appropriate theorems concerning angles in a circle.

(b) (2 points) Express, in terms of a, the area of the sector ABE.

(c) (5 points) Show that the area of the face of the coins is

$$\frac{a^2}{2}\left(\pi - 7\tan\frac{\pi}{14}\right)$$