If and only if

In this presentation we will take a look at conditional statements in more detail. In particular we will look at statements such as

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we will also analyse truth value of such statements.

Implication

Recall that the statement such $p \to q$ is false in only one scenario, namely if p is true and q is false.

Consider a statement:

If a number is divisible by 3, then it is divisible by 6.

This is a statement of the form $p \to q$, where:

p *a number is divisible by 3,*

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If we were to show that the statement

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is false, we would need to find a number, for which p is true and q is false.

9 is such a number, because 9 is a number divisible by 3, but 9 is number which is not divisible by 6.

We will call 9 a **counterexample**, because 9 shows that our statement was false.

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Examples

Find counterexamples, which show that the following statements are false.

a) *If a number is greater than 100, then it is greater than 200.*

Number 101 is a counter example. There are of course many more counterexamples to this statement.

b) *If $x^2 = 4$, then $x = 2$.*

Number -2 is a counter example, because $(-2)^2 = 4$ so the first statement is true, but $-2 \neq 2$, so the second statement is false.

c) *If a number is divisible by 2, then it is divisible by 4.*

Number 6 is a counter example, because 6 is divisible by 2, so the first statement is true, but 6 is not divisible by 4, so the second statement is false.

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Examples

d) *If a number is a factor of 30, then it is a factor of 10.*

Number 15 is a counter example. 15 is a factor of 30 so the first statement is true, but 15 is not a factor of 10, so the second statement is false.

e) *If a quadrilateral has all right angles, then it is a square.*

Number rectangle with unequal sides is a counter example, because it has all right angles, so the first statement is true, but it is not a square, so the second statement is false.

f) *If a triangle is not a right triangle, then it is equilateral.*

A triangle with sides 2,3 and 4 is a counter examples. It is not a right triangle (because $2^2 + 3^2 \neq 4^2$), so the first statement is true, but it is not an equilateral triangle (because its sides are not equal), so the second statement is false.

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Sometimes a conditional statement may be written in different order. Consider the following two simple statements:

- p *it rains*
- q *my dog barks*

The statement:

My dog barks, if it rains.

has the logical form $p \to q$, because it is the same as:

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Now compare the previous statement to:

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This statement has the logical form $q \to p$.
In other words it means the same as

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Let's analyse this in more detail.

If you have a statement:

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and you see that it rains, then you would expect your dog to be barking.

So the fact that it rains implies the fact that your dog barks.

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However, in case of the statement:

My dog barks, only if it rains.

if you hear your dog barking, you would expect to see that it rains. Why?

Because your dog barks **only** if it rains.

So the fact that the dog is barking implies that it rains.

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