## Tautology, contradiction, equivalence

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In this presentation we will practice checking if a compound proposition is **tautology/contradiction** and if two statements are **equivalent**.

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# Check if the following proposition is a tautology, contradiction or neither. $(p ightarrow q) ightarrow (\neg p \lor q)$

We need to construct the truth table for this proposition. There are 2 simple sentences involved *p* and *q*, so the table will have four rows.

We will need columns for  $p, q, p \rightarrow q, \neg p, \neg p \lor q$  and finally  $(p \rightarrow q) \rightarrow (\neg p \lor q)$ .

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$$p$$
 $q$  $p \rightarrow q$  $\neg p$  $\neg p \lor q$  $(p \rightarrow q) \rightarrow (\neg p \lor q)$  $T$  $T$  $T$  $F$  $T$  $F$  $F$  $F$  $F$  $T$  $F$  $F$  $F$  $F$ 

The last columns has Ts only, so the statement is always true i.e. it is a tautology.

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Check if the following compound statement is a tautology, contradiction or neither.

 $(p \land q) \lor (r \to \neg q)$ 

We have three simple statements involved in this proposition: *p*,*q* and *r*. So our table will have eight rows.

We need the following columns: p, q, r and then also  $p \land q$ ,  $\neg q$ ,  $r \rightarrow \neg q$ and the statement we want to check  $(p \land q) \lor (r \rightarrow \neg q)$ 

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The truth table will look as follows. Again try to complete a column and then move to the next slide to check your answers.

p	q	r	$p \wedge q$	$\neg q$	$r \rightarrow \neg q$	$(p \wedge q) \lor (r  o  eg q)$
Т	Т	Т	Т	F	F	Т
Т	Т	F	Т			
Т	F	Т	F			
Т	F	F	F	T		Т
F	Т	Т	F	E		
F	Т	F	E	E		Т
F	F	Т	E	T		Т
F	F	F	F	Т		Т

The truth table will look as follows. Again try to complete a column and then move to the next slide to check your answers.

р	q	r	$p \wedge q$	$\neg q$	$r  ightarrow \neg q$	$(p \wedge q) \lor (r  o  eg q)$
Т	Т	Т	Т	F	F	Т
Т	Т	F	Т	E		
Т	F	Т	F	T		Т
Т	F	F	F	T		Т
F	Т	Т	F	E		
F	Т	F	F	E		T
F	F	Т	F	T		Т
F	F	F	F	Т		Т

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р	q	r	$p \wedge q$	$\neg q$	$r  ightarrow \neg q$	$(p \wedge q) \lor (r  o  eg q)$
Т	Т	Т	Т	F	F	Т
Т	Т	F	Т	F	T	
Т	F	Т	F	Т	Т	T
Т	F	F	F	Т	Т	
F	Т	Т	F	F	F	
F	Т	F	F	F	Т	T
F	F	Т	F	Т	Т	Т
F	F	F	F	Т	Т	Т

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p	q	r	$p \wedge q$	$\neg q$	$r  ightarrow \neg q$	$(p \wedge q) \lor (r  o  eg q)$
Т	Т	Т	Т	F	F	Т
Т	Т	F	Т	F	Т	
Т	F	Т	F	Т	Т	
Т	F	F	F	Т	Т	Т
F	Т	Т	F	F	F	
F	Т	F	F	F	Т	Т
F	F	Т	F	Т	Т	Т
F	F	F	F	Т	Т	

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Т	Т	Т	Т	F	F	Т
Т	Т	F	Т	F	Т	
Т	F	Т	F	Т	Т	
Т	F	F	F	Т	Т	Т
F	Т	Т	F	F	F	F
F	Т	F	F	F	Т	Т
F	F	Т	F	Т	Т	Т
F	F	F	F	Т	Т	Т

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Т	Т	Т	Т	F	F	Т
Т	Т	F	Т	F	Т	Т
Т	F	Т	F	Т	Т	
Т	F	F	F	Т	Т	Т
F	Т	Т	F	F	F	
F	Т	F	F	F	Т	Т
F	F	Т	F	Т	Т	Т
F	F	F	F	Т	Т	

The truth table will look as follows. Again try to complete a column and then move to the next slide to check your answers.

p	q	r	$p \wedge q$	$\neg q$	$r  ightarrow \neg q$	$(p \wedge q) \lor (r  o  eg q)$
Т	Т	Т	Т	F	F	Т
Т	Т	F	Т	F	Т	Т
Т	F	Т	F	Т	Т	Т
Т	F	F	F	Т	Т	Т
F	Т	Т	F	F	F	
F	Т	F	F	F	Т	Т
F	F	Т	F	Т	Т	Т
F	F	F	F	Т	Т	

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p	q	r	$p \wedge q$	$ \neg q$	$r  ightarrow \neg q$	$(p \wedge q) \lor (r  o  eg q)$
Т	Т	Т	Т	F	F	Т
Т	Т	F	Т	F	Т	Т
Т	F	Т	F	Т	Т	Т
Т	F	F	F	Т	Т	Т
F	Т	Т	F	F	F	
F	Т	F	F	F	Т	
F	F	Т	F	Т	Т	Т
F	F	F	F	Т	Т	Т

The truth table will look as follows. Again try to complete a column and then move to the next slide to check your answers.

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Т	Т	Т	Т	F	F	Т
Т		F		1	Т	Т
Т	F	Т	F	Т	Т	Т
Т	F	F	F	Т	Т	Т
F	Т	Т	F	F	F	F
F	Т	F	F	F	Т	
F	F	Т	F	Т	Т	Т
F	F	F	F	Т	Т	Т

The truth table will look as follows. Again try to complete a column and then move to the next slide to check your answers.

p	q	r	$p \wedge q$	$ \neg q$	$r  ightarrow \neg q$	$(p \wedge q) \lor (r  o  eg q)$
Т	Т	Т	Т		F	Т
Т	Т	F	Т	F	Т	Т
Т	F	Т	F	Т	Т	Т
Т	F	F	F	Т	Т	Т
F	Т	Т	F	F	F	F
F	Т	F	F	F	Т	Т
F	F	Т	F	Т	Т	Т
F	F	F	F	Т	Т	Т

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p	q	r	$p \wedge q$	$ \neg q$	$r  ightarrow \neg q$	$(p \wedge q) \lor (r  o  eg q)$
Т	Т	Т	Т		F	Т
Т	Т	F	Т	F	Т	Т
Т	F	Т	F	Т	Т	Т
Т			F		Т	Т
F	Т	Т	F	F	F	F
F			F		Т	Т
F	F	Т	F	Т	Т	Т
F	F	F	F	Т	Т	Т

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Т	Т	Т	Т	F	F	Т
Т	Т	F	Т	F	Т	Т
Т	F	Т	F	Т	Т	Т
Т			F		Т	Т
F	Т	Т	F	F	F	F
F	Т	F	F	F	Т	Т
F	F	Т	F F	Т	Т	Т
F	F	F	F	Т	Т	Т



The compound statement  $(p \land q) \lor (r \rightarrow \neg q)$  is neither a tautology nor a contradiction.

Check if the statement  $(p \lor \neg p) \rightarrow (q \land \neg q)$  is a tautology, a contradiction or neither.

We have 2 simple statements involved: p and q, so we will have 4 rows. We need columns for p, q,  $\neg p$ ,  $p \lor \neg p$ ,  $\neg q$ ,  $q \land \neg q$  and  $(p \lor \neg p) \rightarrow (q \land \neg q)$ 

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$$p$$
 $q$  $\neg p$  $p \lor \neg p$  $\neg q$  $q \land \neg q$  $(p \lor \neg p) \rightarrow (q \land \neg q)$ TTFTTFFTFFTFTTFFT

$$p$$
 $q$  $\neg p$  $p \lor \neg p$  $\neg q$  $q \land \neg q$  $(p \lor \neg p) \rightarrow (q \land \neg q)$ TTFTFFTFFTTFFFTTFFFTTFFFFTTFFFTTF

$$p$$
 $q$  $\neg p$  $p \lor \neg p$  $\neg q$  $q \land \neg q$  $(p \lor \neg p) \rightarrow (q \land \neg q)$ TTFTFFTFFTTFFFTTFFFTTFFFFTTFFFTTF

$$p$$
 $q$  $\neg p$  $p \lor \neg p$  $\neg q$  $q \land \neg q$  $(p \lor \neg p) \rightarrow (q \land \neg q)$ TTFFFFTFFFFFTFFFFFFTTFFFFTTFFFTTF

$$p$$
 $q$  $\neg p$  $p \lor \neg p$  $\neg q$  $q \land \neg q$  $(p \lor \neg p) \rightarrow (q \land \neg q)$ TTFFFFTFFFFFFTTFFFTTFFFFTTFFFTTF

$$p$$
 $q$  $\neg p$  $p \lor \neg p$  $\neg q$  $q \land \neg q$  $(p \lor \neg p) \rightarrow (q \land \neg q)$ TTFFFFTFFFFFFTTFFFTTFFFFTTFFFTTF

## Example 3

The statement  $(p \lor \neg p) \rightarrow (q \land \neg q)$  is always false, so it is a contradiction.

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## Check if the statements $\neg(p \land q)$ and $\neg p \lor \neg q$ are equivalent.

We need to construct truth table for both statements. We will try to do it in one table.

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р	q	$p \wedge q$	$ eg(p \wedge q)$	$\neg p$	eg q	$ eg p \lor  eg q$
Т	Т		F	F	F	F
Т	F	F	Т	-		
F	Т	E	Т	Т		
F	F	F	Т	T		Т

$$p$$
 $q$  $p \land q$  $\neg (p \land q)$  $\neg p$  $\neg q$  $\neg p \lor \neg q$ TTTTTTFFTTFTFTTFFFT

$$p$$
 $q$  $p \land q$  $\neg (p \land q)$  $\neg p$  $\neg q$  $\neg p \lor \neg q$ TTTFTFFTFTFTFFFTFFFT

$$p$$
 $q$  $p \land q$  $\neg (p \land q)$  $\neg p$  $\neg q$  $\neg p \lor \neg q$ TTTFFITFFTFIFTFTTIFFFTTIFFFTTI

$$p$$
 $q$  $p \land q$  $\neg (p \land q)$  $\neg p$  $\neg q$  $\neg p \lor \neg q$ TTTFFFTFFTFTFTFTTFFFFTTFFFFTTT

$$p$$
 $q$  $p \land q$  $\neg (p \land q)$  $\neg p$  $\neg q$  $\neg p \lor \neg q$ TTTFFFFTFFTFTTFTFTTFTFTFTTTFFFFTTT

## Example 4

The columns for  $\neg(p \land q)$  and  $\neg p \lor \neg q$  are identical, so the two statements are equivalent.