Truth tables

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In this presentation we will go through a few examples of truth tables for compound statements and we will introduce the notion of **tautology**.

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We want to construct the truth table for the proposition:

 $(p \wedge q)
ightarrow (\neg p \vee \neg q)$

The first observation is that there are two simple statements involved in this proposition, namely *p* and *q*. So our table will have four rows.

The second observation is that apart for columns for p and q and our proposition $(p \land q) \rightarrow (\neg p \lor \neg q)$, we also need columns for: $p \land q, \neg p, \neg q$ and $\neg p \lor \neg q$.

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р	q	$p \wedge q$	$\neg p$	$\neg q$	$ eg p \lor eg q$	$(p \wedge q) o (eg p \vee eg q)$
Т	Т		F	E	F	
Т	F		F			
F	Т	E	T			
F	F	F	Т	Т	Т	

$$p$$
 q $p \land q$ $\neg p$ $\neg q$ $\neg p \lor \neg q$ $(p \land q) \rightarrow (\neg p \lor \neg q)$ TTTTTFFFTFFFFFFF

$$p$$
 q $p \land q$ $\neg p$ $\neg q$ $\neg p \lor \neg q$ $(p \land q) \rightarrow (\neg p \lor \neg q)$ TTTFTFFFFFFTFFFT

$$p$$
 q $p \land q$ $\neg p$ $\neg q$ $\neg p \lor \neg q$ $(p \land q) \rightarrow (\neg p \lor \neg q)$ TTTFFTFFTFFFTFTFFFTFFFTF

$$p$$
 q $p \land q$ $\neg p \lor \neg q$ $\neg p \lor \neg q$ $(p \land q) \rightarrow (\neg p \lor \neg q)$ TTTFFFTFFTTFFFFTTFFTFTTFFFFTTTFFFTTT

$$p$$
 q $p \land q$ $\neg p \lor \neg q$ $\neg p \lor \neg q$ $(p \land q) \rightarrow (\neg p \lor \neg q)$ TTTFFFTFFTTTFFFTTTFTFTTTFFFTTTFFFTTT

$$p$$
 q $p \land q$ $\neg p \lor \neg q$ $\neg p \lor \neg q$ $(p \land q) \rightarrow (\neg p \lor \neg q)$ TTTFFFTFFTTTFFFTTTFTFTTTFFFTTTFFFTTT

$$p$$
 q $p \land q$ $\neg p \lor \neg q$ $\neg p \lor \neg q$ $(p \land q) \rightarrow (\neg p \lor \neg q)$ TTTFFFTFFTTTFFFTTTFTFTTTFFFTTTFFFTTT

Now we want to construct the truth table for the proposition:

 $(p \lor q) \lor (\neg r \land \neg q)$

This time we have three simple statements involved in this proposition: *p*,*q* and *r*. So our table will have eight rows.

We need the following columns: p, q, r and then also $p \lor q$, $\neg r$, $\neg q$, $\neg r \land \neg q$ and finally column for our proposition $(p \lor q) \lor (\neg r \land \neg q)$

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The truth table will look as follows. Again try to complete a column and then move to the next slide to check your answers.

р	q	r	$p \lor q$	$\neg r$	$ \neg q$	$\neg r \land \neg q$	$(p \lor q) \lor (\neg r \land \neg q)$
Т	Т	Т		F	F	F	T
Т	Т	F	T			F	
Т	F	Т		F	T		
Т	F	F		Т	T	T	
F	Т	Т		F	E		
F	Т	F		Т	E		
F	F	Т	E	F	T		
F	F	F	E			Т	

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р	q	r	$p \lor q$	$\neg r$	$\neg q$	$\neg r \land \neg q$	$(p \lor q) \lor (\neg r \land \neg q)$
Т	Т	Т	Т	F		F	T
Т	Т	F	Т			F	
Т	F	Т	Т	F	T		
Т	F	F	Т	Т	T	Т	
F	Т	Т	Т	F	F	F	
F	Т	F	Т	T	F	F	
F	F	Т	F	E	T	F	
F	F	F	F	Т		Т	

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р	q	r	$p \lor q$	$\neg r$	$\neg q$	$\neg r \land \neg q$	$(p \lor q) \lor (\neg r \land \neg q)$
Т	Т	Т	Т	F		F	Т
Т	Т	F	Т	Т		F	
Т	F	Т	Т	F	T	F	
Т	F	F	Т	Т	T	T	
F	Т	Т	Т	F	F	F	
F	Т	F	Т	Т	F	F	
F	F	Т	F	F	T		F
F	F	F	F	Т		Т	

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Т	Т	Т	Т	F	F	F	Т
Т	Т	F	Т	Т	F	F	
Т	F	Т	Т	F	Т	F	
Т	F	F	Т	Т	Т	T	
F	Т	Т	Т	F	F		Т
F	Т	F	Т	Т	F		Т
F	F	Т	F	F	Т		F
F	F	F	F	Т	Т	Т	

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Т	Т	Т	Т	F	F	F	Т
Т	Т	F	Т	Т	F	F	
Т	F	Т	Т	F	Т	F	
Т	F	F	Т	Т	Т	Т	
F	Т	Т	Т	F	F	F	
F	Т	F	Т	Т	F	F	
F	F	Т	F	F	Т	F	
F	F	F	F	Т	T	Т	

The truth table will look as follows. Again try to complete a column and then move to the next slide to check your answers.

р	q	r	$p \lor q$	$\neg r$	$\neg q$	$\neg r \land \neg q$	$(p \lor q) \lor (\neg r \land \neg q)$
Т	Т	Т	Т	F	F	F	Т
Т	Т	F	Т	Т	F	F	
Т	F	Т	Т	F	Т	F	T
Т	F	F	Т	Т	Т	Т	
F	Т	Т	Т	F	F	F	T
F	Т	F	Т	Т	F	F	Т
F	F	Т	F	F	Т	F	F
F	F	F	F	Т	Т	Т	

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Т	Т	Т	Т	F	F	F	Т
Т	Т	F	Т	Т	F	F	Т
Т	F	Т	Т	F	Т	F	
Т	F	F	Т	Т	Т	Т	
F	Т	Т	Т	F	F	F	
F	Т	F	Т	Т	F	F	T
F	F	Т	F	F	Т	F	F
F	F	F	F	Т	Т	Т	

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Т	Т	Т	Т	F	F	F	Т
Т	Т	F	Т	Т	F	F	Т
Т	F	Т	Т	F	Т	F	Т
Т	F	F	Т	Т	Т	Т	
F	Т	Т	Т	F	F	F	
F	Т	F	Т	Т	F	F	
F	F	Т	F	F	Т	F	F
F	F	F	F	Т	Т	Т	

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Т	Т	Т	Т	F	F	F	Т
Т	Т	F	Т	Т	F	F	Т
Т	F	Т	Т	F	Т	F	Т
Т	F	F	Т	Т	Т	Т	Т
F	Т	Т	Т	F	F	F	
F	Т	F	Т	Т	F	F	
F	F	Т	F	F	Т	F	
F	F	F	F	Т	Т	Т	

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Т	Т	Т	Т	F	F	F	Т
Т	Т	F	Т	Т	F	F	Т
Т	F	Т	Т	F	Т	F	Т
Т	F	F	Т	Т	Т	Т	Т
F	Т	Т	Т	F	F	F	Т
F	Т	F	Т	Т	F	F	
F	F	Т	F	F	Т	F	F
F	F	F	F	Т	Т	Т	

The truth table will look as follows. Again try to complete a column and then move to the next slide to check your answers.

р	q	r	$p \lor q$	$\neg r$	$\neg q$	$\neg r \land \neg q$	$(p \lor q) \lor (\neg r \land \neg q)$
Т	Т	Т	Т	F	F	F	Т
Т	Т	F	Т	Т	F	F	Т
Т	F	Т	Т	F	Т	F	Т
Т	F	F	Т	Т	Т	Т	Т
F	Т	Т	Т	F	F	F	Т
F	Т	F	Т	Т	F	F	Т
F	F	Т	F	F	Т	F	F
F	F	F	F	Т	Т	Т	

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Т	Т	Т	Т	F	F	F	Т
Т	Т	F	Т	Т	F	F	Т
Т	F	Т	Т	F	Т	F	Т
Т	F	F	Т	Т	Т	Т	Т
F	Т	Т	Т	F	F	F	Т
F	Т	F	Т	Т	F	F	Т
F	F	Т	F	F	Т	F	F
F	F	F	F	Т	Т	Т	

The truth table will look as follows. Again try to complete a column and then move to the next slide to check your answers.

р	q	r	$p \lor q$	$\neg r$	$ \neg q$	$\neg r \land \neg q$	$(p \lor q) \lor (\neg r \land \neg q)$
Т	Т	Т	Т	F	F	F	Т
Т	Т	F	Т	Т	F	F	Т
Т	F	Т	Т	F	Т	F	Т
Т	F	F	Т	Т	Т	Т	Т
F	Т	Т	Т	F	F	F	Т
F	Т	F	Т	Т	F	F	Т
F	F	Т	F	F	Т	F	F
F	F	F	F	Т	T	Т	Т



Definition

A statement is a tautology if it is **always** true, i.e. in the truth table the column for this statement contains only truth (T).

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Check if the statement $p ightarrow (p \lor q)$ is a tautology.

We need to construct a truth table for this statement and check if the last column contains only Ts.

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Check if the statement $p
ightarrow (p \lor q)$ is a tautology.

We need to construct a truth table for this statement and check if the last column contains only Ts.

р	q	$p \lor q$	p ightarrow (p ee q)
Т	Т		T
Т	F		
F	Т		T
F	F	F	Т

The statement $ho
ightarrow (
ho \lor q)$ is always true, so it is a tautology.

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р	q	$p \lor q$	p ightarrow (p ee q)
Т	Т	Т	T
Т	F	Т	
F	Т	Т	T
F	F	F	

The statement ho o (
ho ee q) is always true, so it is a tautology.

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р	q	$p \lor q$	p ightarrow (p ee q)
Т	Т	Т	Т
Т	F	Т	T
F	Т	Т	T
F	F	F	

The statement $ho
ightarrow (
ho \lor q)$ is always true, so it is a tautology.

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р	q	$p \lor q$	p ightarrow (p ee q)
Т	Т	Т	Т
Т	F	Т	Т
F	Т	Т	Т
F	F	F	

The statement $ho
ightarrow (
ho \lor q)$ is always true, so it is a tautology.

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р	q	$p \lor q$	p ightarrow (p ee q)
Т	Т	Т	Т
Т	F	Т	Т
F	Т	Т	Т
F	F	F	

The statement $ho
ightarrow (
ho \lor q)$ is always true, so it is a tautology.

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р	q	$p \lor q$	p ightarrow (p ee q)
Т	Т	Т	Т
Т	F	Т	Т
F	Т	Т	Т
F	F	F	Т

The statement $ho
ightarrow (
ho \lor q)$ is always true, so it is a tautology.

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р	q	$p \lor q$	p ightarrow (p ee q)
Т	Т	Т	Т
Т	F	Т	Т
F	Т	Т	Т
F	F	F	Т

The statement $p \to (p \lor q)$ is always true, so it is a tautology.

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Check if the statement $p ightarrow (p \wedge q)$ is a tautology.

Again we need to construct a truth table for this statement and check if the last column contains only Ts.

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Check if the statement $p
ightarrow (p \land q)$ is a tautology.

Again we need to construct a truth table for this statement and check if the last column contains only Ts.

р	q	$p \wedge q$	$p ightarrow (p \wedge q)$
Т	Т		T
Т	F	F	
F	Т	F	
F	F	F	

The statement $p \rightarrow (p \lor q)$ is not always true (the second row shows F), so it is **not** a tautology.

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р	q	$p \wedge q$	$ \hspace{.1cm} p ightarrow (p \wedge q)$
Т	Т	Т	Т
Т	F	F	
F	Т	F	
F	F	F	

The statement $p \rightarrow (p \lor q)$ is not always true (the second row shows F), so it is **not** a tautology.

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р	q	$p \wedge q$	$p ightarrow (p \wedge q)$
Т	Т	Т	Т
Т	F	F	F
F	Т	F	
F	F	F	

The statement $p \rightarrow (p \lor q)$ is not always true (the second row shows F), so it is **not** a tautology.

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р	q	$p \wedge q$	$p ightarrow (p \wedge q)$
Т	Т	Т	Т
Т	F	F	F
F	Т	F	
F	F	F	

The statement $p \rightarrow (p \lor q)$ is not always true (the second row shows F), so it is **not** a tautology.

3

р	q	$p \wedge q$	$p ightarrow (p \wedge q)$
Т	Т	Т	Т
Т	F	F	F
F	Т	F	Т
F	F	F	T

The statement $p \rightarrow (p \lor q)$ is not always true (the second row shows F), so it is **not** a tautology.

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р	q	$p \wedge q$	$p ightarrow (p \wedge q)$
Т	Т	Т	Т
Т	F	F	F
F	Т	F	Т
F	F	F	Т

The statement $p \rightarrow (p \lor q)$ is not always true (the second row shows F), so it is **not** a tautology.

3

р	q	$p \wedge q$	$p ightarrow (p \wedge q)$
Т	Т	Т	Т
Т	F	F	F
F	Т	F	Т
F	F	F	Т

The statement $p \rightarrow (p \lor q)$ is not always true (the second row shows F), so it is **not** a tautology.

Remember: a statement is a tautology if it is always true, so it has to have all Ts. If it has at least one F, then it is not a tautology.

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