Truth tables

In this presentation we will go through a few examples of truth tables for compound statements and we will introduce the notion of **tautology**.

Example 1

We want to construct the truth table for the proposition:

$$(p \wedge q) \rightarrow (\neg p \vee \neg q)$$

The first observation is that there are two simple statements involved in this proposition, namely p and q. So our table will have four rows.

The second observation is that apart for columns for p and q and our proposition $(p \wedge q) \rightarrow (\neg p \vee \neg q)$, we also need columns for: $p \wedge q$, $\neg p$, $\neg q$ and $\neg p \vee \neg q$.

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Example 2

Now we want to construct the truth table for the proposition:

$$(p \vee q) \vee (\neg r \wedge \neg q)$$

This time we have three simple statements involved in this proposition: p,q and r. So our table will have eight rows.

We need the following columns: p, q, r and then also $p \vee q$, $\neg r$, $\neg q$, $\neg r \wedge \neg q$ and finally column for our proposition $(p \vee q) \vee (\neg r \wedge \neg q)$

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F	T	T	T	F	F	F	T
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Tautology

Definition

A statement is a tautology if it is **always** true, i.e. in the truth table the column for this statement contains only truth (T).

Example 3

Check if the statement $p \to (p \vee q)$ is a tautology.

We need to construct a truth table for this statement and check if the last column contains only Ts.

Example 3

Check if the statement $p \rightarrow (p \vee q)$ is a tautology.

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The truth table will look as follows.

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Remember: a statement is a tautology if it is always true, so it has to have all Ts. If it has at least one F, then it is not a tautology.