Converse, inverse, contrapositive

In this presentation we will take a look at conditional statements of the form $p \to q$ and introduce the converse, the inverse and the contrapositive of such conditional statements.

Definition

Given a conditonal statement $p \to q$:

the **converse** of this statement is $q \to p$,

the **inverse** of this statement is $\neg p \to \neg q$,

the **contrapositive** of this statement is $\neg q \to \neg p$.

Example 1

Consider the statement:

If it rains, then $2 + 2 = 4$.

This statement is of the form $p \to q$, where:

- p it rains
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The **converse** of this statement will be $q \to p$, so it will be:

If $2+2=4$*, then it rains.*

The **inverse** of this statement will be $\neg p \to \neg q$, so it will be:

If it doesn't rain, then $2+2 \neq 4$*.*

The **contrapositive** of this statement will be $\neg q \to \neg p$, so it will be:

If $2+2 \neq 4$*, then it doesn't rain.*

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The **converse** of this statement will be $q \rightarrow p$, so it will be:

If $2 + 2 = 4$, *then it rains.*

The **inverse** of this statement will be $\neg p \rightarrow \neg q$, so it will be:

If it doesn't rain, then $2 + 2 \neq 4$.

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If $2 + 2 \neq 4$, *then it doesn't rain.*

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Example 2

Consider the statement:

If a number is divisible by 4, then it is divisible by 2.

This statement is of the form $p \to q$, where:

p a number is divisible by 4

q a number is divisible by 2

Example 2

Consider the statement:

If a number is divisible by 4, then it is divisible by 2.

This statement is of the form $p \to q$, where:

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The **converse** of this statement will be $q \to p$, so it will be:

If a number is divisible by 2, then it is divisible by 4.

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If a number is not divisible by 4, then it is not divisible by 2.

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Exercise

Construct truth tables for the statement $p \to q$, its converse, its inverse and its contrapositive. Hence decide which pairs of statements are equivalent.