1. In a particular school, students must choose at least one of three optional subjects: art, psychology or history.

Consider the following propositions

a: I choose art,
p: I choose psychology,
h: I choose history.

(a) Write, in words, the compound proposition

$$\neg h \Rightarrow (p \vee a).$$

(3)

(b) Complete the truth table for $\neg a \Rightarrow p$.

a	p	$\neg a$	$\neg a \Rightarrow p$
T	T	F	
T	F	F	
F	T	T	
F	F	T	

(1)

(c) State whether $\neg a \Rightarrow p$ is a tautology, a contradiction **or** neither. Justify your answer.

(2)

(Total 6 marks)

2. Two propositions p and q are defined as follows.

p: Jones passed this course
q: Smith passed this course

(a) Write in symbolic form

(i) *neither Jones nor Smith passed the course*;

(ii) *it is not the case that Jones and Smith both passed the course.*

(b) Complete the following truth table for the logic statement $\neg p \vee q$.

p	q	$\neg p$	$\neg p \vee q$
T	T		
T	F		
F	T		
F	F		

(Total 4 marks)

3. Complete the Truth Table for the compound proposition $(p \wedge \neg q) \Rightarrow (p \vee q)$.

p	q	$\neg q$	$(p \wedge \neg q)$	$(p \vee q)$	$(p \wedge \neg q) \Rightarrow (p \vee q)$
T	T	F	F		
T	F	T	T		
F	T	F		T	
F	F		F	F	

(Total 8 marks)

4. Consider two propositions p and q.

(a) Complete the truth table below.

p	q	$\neg q$	$p \Rightarrow \neg q$	$\neg p$	$\neg p \Rightarrow q$
T	T				
T	F				
F	T				
F	F				

(4)

(b) Decide whether the compound proposition

$$(p \Rightarrow \neg q) \Leftrightarrow (\neg p \Rightarrow q)$$

is a tautology. State the reason for your decision.

(2)
(Total 6 marks)

5. (a) Complete the truth table shown below.

p	q	$p \wedge q$	$p \vee (p \wedge q)$	$(p \vee (p \wedge q)) \Rightarrow p$
T	T			
T	F			
F	T			
F	F			

(3)

(b) State whether the compound proposition $(p \vee (p \wedge q)) \Rightarrow p$ is a contradiction, a tautology or neither.

(1)

Consider the following propositions.

p: Feng finishes his homework
q: Feng goes to the football match

(c) Write in symbolic form the following proposition.

If Feng does not go to the football match then Feng finishes his homework.

(2)

6. Consider the two propositions p and q.

p: The sun is shining q: I will go swimming

Write in words the compound propositions

(a) $p \Rightarrow q$;

(2)

(b) $\neg p \vee q$.

(2)

The truth table for these compound propositions is given below.

p	q	$p \Rightarrow q$	$\neg p$	$\neg p \vee q$
T	T	T		T
T	F	F		F
F	T	T		T
F	F	T		T

(c) Complete the column for $\neg p$.

(1)

(d) State the relationship between the compound propositions $p \Rightarrow q$ and $\neg p \vee q$.

(1)

(Total 6 marks)

7. Three propositions are given as

p: It is snowing q: The roads are open r: We will go skiing

(a) Write the following compound statement in symbolic form.

"It is snowing and the roads are not open."

(2)

(b) Write the following compound statement in words.

$$(\neg p \wedge q) \Rightarrow r$$

(3)

An incomplete truth table for the compound proposition $(\neg p \wedge q) \Rightarrow r$ is given below.

(c) Copy and complete the truth table **on your answer paper**.

p	q	r	$\neg p$	$\neg p \wedge q$	$(\neg p \wedge q) \Rightarrow r$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

(3)
(Total 8 marks)

8. Consider the following logic propositions:

p: Sean is at school
q: Sean is playing a game on his computer.

(a) Write in words, $p \veebar q$.

(2)

(b) Write in words, the converse of $p \Rightarrow \neg q$.

(2)

(c) Complete the following truth table for $p \Rightarrow \neg q$.

p	q	$\neg q$	$p \Rightarrow \neg q$
T	T		
T	F		
F	T		
F	F		

(2)
(Total 6 marks)

9. (a) (i) Complete the truth table below.

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T			F	F	
T	F			F	T	
F	T			T	F	
F	F			T	T	

(ii) State whether the compound propositions $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are equivalent.

(4)

Consider the following propositions.

p: Amy eats sweets
q: Amy goes swimming.

(b) Write, in symbolic form, the following proposition.

Amy either eats sweets or goes swimming, but not both.

(2)
(Total 6 marks)

10. Let p and q represent the propositions

p: food may be taken into the cinema

q: drinks may be taken into the cinema

(a) Complete the truth table below for the symbolic statement $\neg(p \vee q)$.

p	q	$p \vee q$	$\neg(p \vee q)$
T	T		
T	F		
F	T		
F	F		

(2)

(b) Write down in words the meaning of the symbolic statement $\neg(p \vee q)$.

(2)

(c) Write in symbolic form the compound statement:

"no food and no drinks may be taken into the cinema".

(2)
(Total 6 marks)

11. Let the propositions p, q and r be defined as:

p: Matthew arrives home before six o'clock
q: Matthew cooks dinner
r: Jill washes the dishes

(a) (i) Express the following statement in logical form.

If Matthew arrives home before six o'clock then he will cook dinner.

(1)

(ii) Write the following logic statement in words.

$$\neg q \Rightarrow \neg r$$

(1)

(b) (i) Copy and complete the truth table below.

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$\neg r$	$(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge \neg r$	$\neg p$	$[(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge \neg r] \Rightarrow \neg p$
T	T	T						T
T	T	F						T
T	F	T						T
T	F	F						T
F	T	T						T
F	T	F						T
F	F	T						T
F	F	F						T

(5)

(ii) Explain the significance of the truth table above.

(2)
(Total 9 marks)

12. $[(p \Leftrightarrow q) \wedge p] \Rightarrow q$

(a) Complete the truth table below for the compound statement above.

p	q	$p \Leftrightarrow q$	$(p \Leftrightarrow q) \wedge p$	$[(p \Leftrightarrow q) \wedge p] \Rightarrow q$
T	**T**			
T	**F**			
F	**T**			
F	**F**			

(b) Explain the significance of your result.

(Total 4 marks)

13. You may choose from three courses on a lunchtime menu at a restaurant.

s: you choose a salad,
m: you choose a meat dish (main course),
d: you choose a dessert.

You choose a **two** course meal which **must** include a main course and either a salad or a dessert, but not both.

(a) Write the sentence above using logic symbols.

(2)

(b) Write in words $s \Rightarrow \neg d$.

(2)

(c) Complete the following truth table.

(2)

s	d	$\neg s$	$\neg s \Rightarrow d$
T	T		
T	F		
F	T		
F	F		

(Total 6 marks)