Arithmetic sequences

You need to be able solve the following problem relating to arithmetic sequences:

- Find any specific term, given a common difference and any other term.
- Find the common difference and any specific term, given two terms of an arithemtic sequence.
- Find the number of the term, given its value, some other term and common difference.

Example 1

Find u_{12} of an arithemtic sequence given that $u_7 = 14$ and $d = 3$.

First we will do it the long way. We start by finding u_1:

$$u_7 = u_1 + 6d$$

substituting the values we get:

$$14 = u_1 + 6 \cdot 3$$

so we get that $u_1 = -4$.

Now we can use the general formula again:

$$u_{12} = u_1 + 11d = -4 + 11 \cdot 3 = 29$$

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Useful formula

We have a formula for arithmetic sequences:

$$u_n = u_1 + (n-1)d$$

But this is just a special case of a more general formula for arithmetic sequences:

$$u_n = u_k + (n-k)d$$

So in an arithmetic sequence we have for instance

$$u_9 = u_4 + (9-4)d = u_4 + 5d$$
$$u_{17} = u_{13} + (17-13)d = u_{13} + 4d$$
$$u_{100} = u_{80} + 20d$$
$$u_{123} = u_{23} + 100d$$

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$$\begin{aligned} u_9 &= u_4 + (9-4)d = u_4 + 5d \\ u_{17} &= u_{13} + (17-13)d = u_{13} + 4d \\ u_{100} &= u_{80} + 20d \\ u_{123} &= u_{23} + 100d \end{aligned}$$

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Example 2

In an arithmetic sequence $u_6 = 11$ and $u_{11} = 36$. Find the common difference d and u_{20}.

Again we will do it two ways. You should know both ways, because you may be asked to do some intermediate steps of one of the methods on the exam.

We will set up a system of equations:

$$\begin{cases} u_6 = u_1 + 5d \\ u_{11} = u_1 + 10d \end{cases} \rightarrow \begin{cases} 11 = u_1 + 5d \\ 36 = u_1 + 10d \end{cases}$$

Solving this (by hand or using GDC) we get $u_1 = -14$ and $d = 5$.

Now $u_{20} = u_1 + 19d = -14 + 19 \cdot 5 = 81$.

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The second method uses the fact that

$$u_n = u_k + (n-k)d$$

So:

$$u_{11} = u_6 + 5d$$

Hence we have:

$$36 = 11 + 5d$$

So $d = 5$. Now $u_{20} = u_{11} + 9d = 36 + 9 \cdot 5 = 81$.

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Example 3

In a finite arithmetic sequence the first term is 2, the common difference is -3 and the last term is -37. Find the number of terms of this sequence.

We will denote the last terms u_n, so $u_n = -37$. Now n is the number of the last term, so if we knew what n was, we would know how many terms there are.

We use:

$$u_n = u_1 + (n-1)d$$

Substituting the values we get:

$$-37 = 2 + (n-1)(-3)$$

Solving this gives $n = 14$, so the sequence has 14 terms.

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Test

The short test may include problems similar to the above.