b **i** Implication: True Converse: False
Inverse: False Contrapositive: True
ii Implication: True Converse: True
Inverse: True Contrapositive: True
iii Implication: False Converse: True
Inverse: True Contrapositive: False

6 **a** **i** No weak students are in Year 11.
ii No Year 11 students are weak.
b **i** If $x \in W$ then $x \notin E$. **ii** If $x \in E$ then $x \notin W$.
c They are contrapositives.

EXERCISE 8F.1

1 **a** $p \Leftrightarrow q$
$\underline{\neg q}$
$\neg p$

b $(p \Leftrightarrow q) \wedge \neg q \Rightarrow \neg p$

c

p	q	$p \Leftrightarrow q$	$\neg q$	$(p \Leftrightarrow q) \wedge \neg q$	$\neg p$	$(p \Leftrightarrow q) \wedge \neg q \Rightarrow \neg p$
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	F	F	F	T	T
F	F	T	T	T	T	T

We have a tautology, $\therefore$ argument is valid.

2 **a** **i** $(p \Rightarrow q) \wedge \neg q \Rightarrow \neg p$ **ii** $(p \vee q) \wedge \neg p \Rightarrow q$
iii $(p \vee q) \Rightarrow p$ **iv** $(p \Rightarrow q) \wedge \neg p \Rightarrow \neg q$
v $(p \Rightarrow q) \wedge (q \Rightarrow p) \Rightarrow p$

b **i**

p	q	$p \Rightarrow q$	$\neg q$	$(p \Rightarrow q) \wedge \neg q$	$\neg p$	$(p \Rightarrow q) \wedge \neg q \Rightarrow \neg p$
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

$\therefore$ argument is valid.

ii

p	q	$p \vee q$	$\neg p$	$(p \vee q) \wedge \neg p$	$(p \vee q) \wedge \neg p \Rightarrow q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

$\therefore$ argument is valid.

iii

p	q	$p \vee q$	$(p \vee q) \Rightarrow p$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	T

$\therefore$ argument is not valid.

iv

p	q	$p \Rightarrow q$	$\neg p$	$(p \Rightarrow q) \wedge \neg p$	$\neg q$	$(p \Rightarrow q) \wedge \neg p \Rightarrow \neg q$
T	T	T	F	F	F	T
T	F	F	F	F	T	T
F	T	T	T	T	F	F
F	F	T	T	T	T	T

$\therefore$ argument is not valid.

v

p	q	$p \Rightarrow q$	$q \Rightarrow p$	$(p \Rightarrow q) \wedge (q \Rightarrow p)$	$(p \Rightarrow q) \wedge (q \Rightarrow p) \Rightarrow p$
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	F	F	T
F	F	T	T	T	F

$\therefore$ argument is not valid.

3 **a** valid **b** not valid **c** valid **d** not valid

5 **b** Don has visited Australia and New Zealand.

6 **a** valid **b** not valid **c** valid
d not valid **e** valid **f** not valid

EXERCISE 8F.2

1 **a** It is sunny and I am warm. Hence, I feel happy.
b It is sunny and I am not warm. Hence, I do not feel happy.
c I am warm and I feel happy. Hence, it is sunny.

2 **B** **3** **b** p, q, r are all true.

4 **a** p: I do not like the subject. q: I do not work hard.
r: I fail.
b $(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge \neg r \Rightarrow \neg p$
c Argument is valid, $\therefore$ conclusion is a result of valid reasoning.

5 not valid (he can be tall and fast, but not on the team)

REVIEW SET 8A

1 **a** proposition, true **b** not a proposition
c proposition, indeterminate **d** not a proposition
e not a proposition **f** proposition, true
g not a proposition **h** proposition, false
i proposition, indeterminate **j** proposition, true

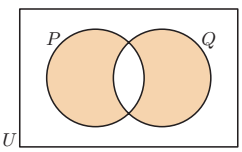
2 **a** x is not an even number.
b x is an even number or is divisible by 3.
c x is an even number or is divisible by 3, but not both.
d If x is an even number, then x is divisible by 3.
e x is not an even number and is divisible by 3.
f x is not an even number or x is divisible by 3, but not both.
g If x is an even number then x is not divisible by 3.
h If x is not an even number then x is not divisible by 3.

3 **a** $p \Rightarrow q$, 7 **b** $\neg p$, 4 **c** $q \wedge \neg p$, 14
d $p \veebar q$, 2 **e** $\neg p \wedge \neg q$, 6
Note: There are other numbers that satisfy these statements.

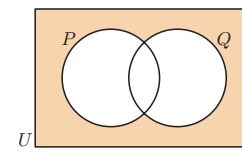
4 **a** Implication: $p \Rightarrow q$ — If I love swimming, then I live near the sea.
Inverse: $\neg p \Rightarrow \neg q$ — If I do not love swimming, then I do not live near the sea.
Converse: $q \Rightarrow p$ — If I live near the sea, then I love swimming.
Contrapositive: $\neg q \Rightarrow \neg p$ — If I do not live near the sea, then I do not love swimming.

b Implication: $p \Rightarrow q$ — If I like food, then I eat a lot.
Inverse: $\neg p \Rightarrow \neg q$ — If I do not like food, then I do not eat a lot.
Converse: $q \Rightarrow p$ — If I eat a lot, then I like food.
Contrapositive: $\neg q \Rightarrow \neg p$ — If I do not eat a lot, then I do not like food.

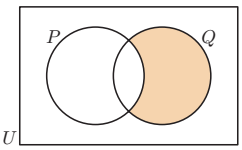
5 **a**



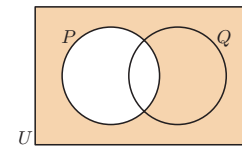
b



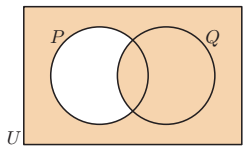
c



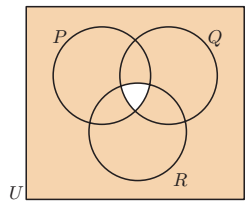
d



e



f



6 **a** $\{1, 2, 3, 4, 6, 12\}$ **b** $\{1, 3, 5, 7, 9\}$ **c** $\{1, 3\}$
d $\{1, 2, 3, 4, 5, 6, 7, 9, 12\}$

7 **a** invalid **b** invalid **c** invalid

REVIEW SET 8B

1 **a** $P = \{20, 24, 28\}$, $Q = \{1, 2, 3, 4, 6, 8, 12, 24\}$
$R = \{20, 22, 24, 26, 28\}$

b **i** $\{24\}$ **ii** $\{24\}$ **iii** $\{20, 24, 28\}$
iv $P \cap Q \cap R = \{24\}$

2 **a** Eddy is not good at football.
b The maths class includes 10 or less boys.
c The writing is legible. **d** Ali does not own a new car.

3 **a** If a creature is a bird, then it has two legs.
b If a creature is a snake, then it is not a mammal.
c If a polygon is a rectangle, then it does not have five sides.
d If this equation has solutions, then they are not real solutions.

4 **a** It is neither. **b** x is zero or positive.

5 **a** $\neg(p \vee q)$ **b** $p \wedge \neg q$ **c** $p \wedge q \wedge r$

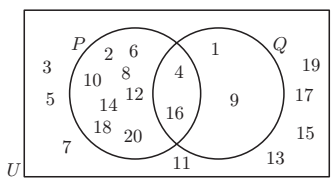
6 **a** logically equivalent **b** logically equivalent
c not logically equivalent **d** logically equivalent

7 **a** p: The sun is shining. q: I will wear my shorts.
$(p \Rightarrow q) \wedge p \Rightarrow q$
The argument is valid.
b p: Marty is a teacher. q: Marty works hard.
$(p \Rightarrow q) \wedge \neg p \Rightarrow \neg q$
The argument is not valid.

REVIEW SET 8C

1 **a** $x > 3$ for $x \in \mathbb{Z}$ **b** $x \in \{\text{brush, hairclip, bobby pin}\}$
c x is a woman, but is not tall.

2 **a**



b **i** $\{4, 16\}$ **ii** $\{1, 3, 4, 5, 7, 9, 11, 13, 15, 16, 17, 19\}$
iii $\{3, 4, 5, 7, 11, 13, 15, 16, 17, 19\}$

3 Inverse: If a parallelogram is not a rhombus, then its diagonals are not equal in length.
Converse: If the diagonals of a parallelogram are equal in length, then the paralellogram is a rhombus.
Contrapositive: If the diagonals of a parallelogram are not equal in length, then the parallelogram is not a rhombus.

4 **a** $\neg p \Rightarrow \neg q$ **b** $\neg p \Rightarrow q$ **c** $q \wedge \neg p$ **d** $\neg p \vee q$

5 **a** If the plane leaves from gate 5, then it leaves this morning and it does not leave from gate 2.
b $\neg r \Leftrightarrow q \vee p$

6 **a**

p	q	$p \Rightarrow q$	$(p \Rightarrow q) \wedge q$	$(p \Rightarrow q) \wedge q \Rightarrow p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

$\therefore$ it is neither

b

p	q	$p \wedge q$	$p \vee q$	$\neg(p \vee q)$	$(p \wedge q) \wedge \neg(p \vee q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

$\therefore$ logical contradiction

c

p	q	$\neg p$	$\neg p \Leftrightarrow q$
T	T	F	F
T	F	F	T
F	T	T	T
F	F	T	F

$\therefore$ it is neither

d

p	q	$\neg q$	$p \vee \neg q$	$(p \vee \neg q) \Rightarrow q$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

$\therefore$ it is neither

e

p	q	r	$\neg p$	$\neg p \vee q$	$(\neg p \vee q) \Rightarrow r$
T	T	T	F	T	T
T	T	F	F	T	F
T	F	T	F	F	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	T	T
F	F	F	T	T	F

$\therefore$ it is neither

f

p	q	$p \wedge q$	$(p \wedge q) \Rightarrow q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

$\therefore$ tautology

7 **a** p: Fred is a dog. q: Fred has fur.
r: Fred has a cold nose.
$(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge p \Rightarrow r$
The argument is valid.
b p: Viv is a judge. q: Viv wears a robe.
r: Viv wears a wig.
$(p \Rightarrow q \vee r) \wedge (\neg r \wedge \neg p) \Rightarrow \neg q$
Argument is not valid.

EXERCISE 9A.1

1 **a** 0.78 **b** 0.22 **2** **a** 0.487 **b** 0.051 **c** 0.731
3 **a** 43 days **b** **i** 0.0465 **ii** 0.186 **iii** 0.465
4 **a** 0.0895 **b** 0.126

EXERCISE 9A.2

1 **a** 0.265 **b** 0.861 **c** 0.222
2 **a** 0.146 **b** 0.435 **c** 0.565
3 **a** **i** 0.189 **ii** 0.55 **b** 0.381 **c** 0.545

EXERCISE 9B

1 **a** {A, B, C, D} **b** {BB, BG, GB, GG}