4 If I do not like a subject then I do not work hard. If I do not work hard then I fail. I passed, therefore I must like the subject.

a Identify the propositions p, q, and r. **b** Write the above argument in logical form.

c Is the conclusion a result of valid reasoning?

5 Determine the validity of this argument:

If Jeremy is on the basketball team, then he is tall and fast.
Jeremy is tall and he is not on the basketball team.
Therefore, Jeremy is not fast.

REVIEW SET 8A

1 Which of the following are propositions? If they are propositions, state whether they are true, false, or indeterminate.

a Sheep have four legs. **b** Do giraffes have four legs?

c Alicia is good at Mathematics. **d** I think my favourite team will win.

e Vicki is very clever. **f** There are 7 days in a week.

g Put your shoes on. **h** All cows are brown.

i $a^2 + b^2 = c^2$

j The opposite sides of a parallelogram are equal in length.

2 Consider the propositions p: x is an even number, and q: x is divisible by 3. Write the following in words:

a $\neg p$ **b** $p \vee q$ **c** $p \veebar q$ **d** $p \Rightarrow q$

e $\neg p \wedge q$ **f** $\neg p \veebar q$ **g** $p \Rightarrow \neg q$ **h** $\neg p \Rightarrow \neg q$

3 Consider the propositions p: x is a prime number, and q: x is a multiple of 7.
Write the following in symbolic language:

a If x is a prime number then x is a multiple of 7.

b x is not a prime number.

c x is a multiple of 7 and not a prime number.

d x is either a prime number or a multiple of 7, but not both.

e x is neither a prime number nor a multiple of 7.

In each case, write down a number that satisfies the statement.

4 Write the implication $p \Rightarrow q$, the inverse, converse, and contrapositive of the following propositions in both words and symbols.

a p: I love swimming.
q: I live near the sea.

b p: I like food.
q: I eat a lot.

5 Represent the truth sets of the following on Venn diagrams:

a $p \veebar q$ **b** $\neg(p \vee q)$ **c** $\neg p \wedge q$

d $\neg p$ **e** $\neg p \vee q$ **f** $\neg(p \wedge q \wedge r)$

6 For the propositions p: x is a factor of 12, and q: x is an odd number < 10, list the truth sets of:

a p **b** q **c** $p \wedge q$ **d** $p \vee q$

7 Use truth tables to determine the validity of the following arguments:

a $p \Rightarrow q$, $\neg p$ / $\therefore \neg q$

b $p \vee q$, $\neg q$ / $\therefore \neg p$

c $p \Rightarrow q$, $q \Rightarrow r$ / $\therefore r \vee q$

REVIEW SET 8B

1 Consider the propositions p: x is a multiple of 4, $18 < x < 30$
q: x is a factor of 24,
and r: x is an even number, $18 < x < 30$.

a List the truth sets of p, q, and r.

b List the truth sets of: **i** $p \wedge q$ **ii** $q \wedge r$ **iii** $p \wedge r$ **iv** $p \wedge q \wedge r$

2 Find negations for the following:

a Eddy is good at football.

b The maths class includes more than 10 boys.

c The writing is illegible.

d Ali owns a new car.

3 Write the following statements as implications:

a All birds have two legs.

b Snakes are not mammals.

c No rectangle has five sides.

d This equation has no real solutions.

4 'Positive' and 'negative' are defined as follows:

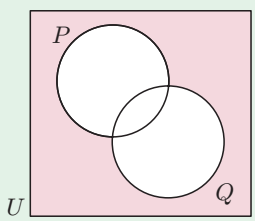
x is positive $\Leftrightarrow$ $x > 0$ $\quad$ x is negative $\Leftrightarrow$ $x < 0$

a Is zero positive or negative?

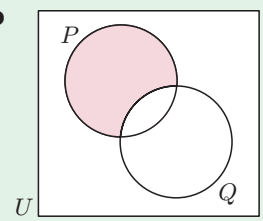
b What is the negation of 'x is negative' when $x \in$ {rational numbers}?

5 Let P, Q, and R be the truth sets of propositions p, q, and r respectively.
Write the following as compound propositions in terms of p, q, and r:

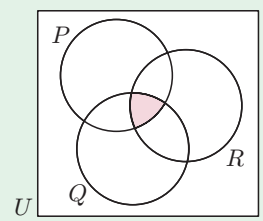
a



b



c



6 Which of the following pairs are logically equivalent?

a $p \Rightarrow q$ and $\neg q \Rightarrow \neg p$

b $\neg(p \wedge q)$ and $\neg p \vee \neg q$

c $p \Leftrightarrow q$ and $(p \wedge q) \wedge \neg q$

d $\neg p \Rightarrow \neg q$ and $q \Rightarrow p$

7 Express the following in logical form. Determine whether or not the argument is valid.

a If the sun is shining I will wear my shorts. The sun is shining. Therefore, I will wear shorts.

b All teachers work hard. Marty is not a teacher. Therefore Marty does not work hard.

REVIEW SET 8C

1 Find the negation of:

a $x \leqslant 3$ for $x \in \mathbb{Z}$

b x is a comb, for $x \in \{\text{brush, comb, hairclip, bobby pin}\}$

c x is a tall woman for $x \in \{\text{women}\}$.

2 For $U = \{x \mid 1 \leqslant x \leqslant 20,\ x \in \mathbb{Z}\}$, consider the propositions

p: x is an even number and q: x is a square number.

a Illustrate the truth sets for p and q on a Venn diagram.

b Use your Venn diagram to find the truth set for:

i $p \wedge q$ **ii** $\neg p \vee q$ **iii** $\neg(p \veebar q)$

3 Write down, in words, the inverse, converse, and contrapositive for the implication:
"The diagonals of a rhombus are equal in length."

4 Consider the propositions p: cakes are sweet and q: cakes are full of sultanas.
Write each of the following using logic symbols:

a If cakes are not sweet then they are not full of sultanas.

b If cakes are not sweet then they are full of sultanas.

c Cakes are full of sultanas and they are not sweet.

d Cakes are not sweet or they are full of sultanas.

5 Consider the propositions:

p: The plane leaves from gate 5. q: The plane leaves from gate 2.

r: The plane does not leave this morning.

a Write the following logic statement in words: $p \Rightarrow (\neg r \wedge \neg q)$

b Write in symbols: The plane leaves this morning if and only if it leaves from gate 2 or from gate 5.

6 Construct truth tables for the following and state whether the statements are tautologies, logical contradictions, or neither:

a $(p \Rightarrow q) \wedge q \Rightarrow p$ **b** $(p \wedge q) \wedge \neg(p \vee q)$ **c** $\neg p \Leftrightarrow q$

d $(p \vee \neg q) \Rightarrow q$ **e** $(\neg p \vee q) \Rightarrow r$ **f** $p \wedge q \Rightarrow q$

7 Express the following in logical form. Determine whether or not the argument is valid.

a If Fred is a dog he has fur. If Fred has fur he has a cold nose.
Fred is a dog. Hence, Fred has a cold nose.

b If Viv is a judge, she wears a robe or a wig.
Viv does not wear a wig, nor is she a judge.
Therefore, Viv does not wear a robe.