a If ABCD is a square, then ABCD is a quadrilateral.

b If ABCD is a rectangle, then ABCD is a parallelogram.

c If an integer is divisible by four then it is divisible by two.

d If an integer is divisible by three then it is an odd integer.

e If an integer is divisible by two then it is an even integer.

f If an integer is divisible by both four and three then it is divisible by twelve.

g If an integer is divisible by both four and two then it is divisible by eight.

h If the sum of two integers is even, then the two integers are both even.

i If the product of two integers is even, then the two integers are both even.

j If the sum of two integers is odd, then one of the integers is odd and the other is even.

k If the product of two integers is odd, then the two integers are both odd.

l If triangle ABC is right-angled, then $a^2 + b^2 = c^2$.

m The square of an odd integer is odd.

n If triangle ABC has three equal angles, then triangle ABC has three equal sides.

o If quadrilateral ABCD has four equal sides, then ABCD has four equal angles.

p If $x^2 = 25$, then $x = 5$.

q If $x^3 = 27$, then $x = 3$.

r If $x^2 > 25$, then $x > 5$.

s If $x^3 < 27$, then $x < 3$.

Extension material on CD:
Worksheet 9 - De Morgan's Laws

Review exercise

Paper 1 style questions

EXAM-STYLE QUESTION

1 **a** Copy and complete the truth table to show that $\neg(p \vee q) \Rightarrow \neg p \wedge \neg q$ is a valid argument.

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$\neg(p \vee q) \Rightarrow \neg p \wedge \neg q$
T	T			F	F		
T	F			F	T		
F	T			T	F		
F	F			T	T		

b Using the results of **a**, rewrite the following statement without using the phrase ‘It is not true . . .’

‘It is not true that she dances well, or sings beautifully.’

EXAM-STYLE QUESTIONS

2 The following propositions are given.

p: The train leaves from gate 2.
q: The train leaves from gate 8.
r: The train does not leave today.

a Write a sentence, in words, for the following logic statement:

$p \Rightarrow (\neg r \wedge \neg q)$.

b Write the following sentence as a logic statement using p, q, r and logic notation:

'The train leaves today if and only if it leaves from gate 2 or from gate 8.'

3 **a** Copy and complete the truth table.

p	q	$p \Rightarrow q$	$\neg p$	$\neg q$	$\neg q \vee p$	$\neg p \vee q$
T	T					
T	F					
F	T					
F	F					

b What identity is shown by the truth table?

4 **a** Copy and complete the following truth table for

p: $x > 3$
q: $x^2 > 9$

p	q	$\neg p$	$\neg p \vee q$
T	T		
T	F		
F	T		
F	F		

b Using the results of part **a**, and explaining your reasoning, is $\neg p \vee q$ true, or false, when

i $x > 3$ and $x^2 \ngtr 9$?
ii $x \ngtr 3$ and $x^2 > 9$?

[Note: the symbol $\ngtr$ denotes '**not** greater than'.]

5 p and q are two statements:

p: Ice creams are vanilla flavored.
q: Ice creams are full of raisins.

a Draw a Venn diagram to represent the statements above, carefully labeling all sets including the universal set.
Shade the region that represents $p \vee q$.

b On the Venn diagram, show

i a point x, representing a vanilla flavored ice cream full of raisins
ii a point y, representing a vanilla flavored ice cream not full of raisins.

c Write each of the following using logic symbols.

i If ice creams are not full of raisins, they are not vanilla flavored.

ii Ice creams are not vanilla flavored or they are full of raisins.

iii If ice creams are not full of raisins, they are vanilla flavored.

iv Ice creams are vanilla flavored and they are not full of raisins.

d State which one of the propositions in part **c** above is logically equivalent to:

'If ice creams are vanilla flavored, they are full of raisins.'
Give a reason.

6 The following propositions are given.

p: Picasso painted picture A.
q: Van Gogh painted picture A.

a Write a sentence in words to define the logic statements

i $p \vee \neg q$ **ii** $\neg p \wedge q$.

b Copy and complete the following truth table.

p	q	$\neg p$	$\neg q$	$p \vee \neg q$	$\neg p \wedge q$
T	T				
T	F				
F	T				
F	F				

c Draw two Venn diagrams and shade the area represented by $p \vee \neg q$ on the first diagram and $\neg p \wedge q$ on the second diagram.

d Deduce the truth values of the logic statement $(p \vee \neg q) \Leftrightarrow (\neg p \wedge q)$

i using the truth table

ii using the Venn diagrams.

Explain your answers clearly in words.

e Write down the name given to a logic statement such as $(p \vee \neg q) \Leftrightarrow (\neg p \wedge q)$.

7 The following propositions are given.

p: x is a multiple of 5.
q: x is a multiple of 3.
r: x is a factor of 90.

a Write a sentence, in words, for the statement: $(q \vee r) \wedge \neg p$.

b Write the following sentence as a logic statement using p, q, r and logic notation:

If x is a factor of 90 then x is either a multiple of 5 or x is not a multiple of 3.

c Use truth tables to determine the truth values of each of the following two statements:

$(q \vee r) \wedge \neg p$ and $r \Rightarrow (p \vee \neg q)$.

EXAM-STYLE QUESTION

d List the combinations of truth values of p, q and r that make the statement $(q \vee r) \wedge \neg p$ true.
Write down a possible value of x for each of these combinations of truth values.

e Construct a truth table to determine the conditions for equivalence between the two statements $(q \vee r) \wedge \neg p$ and $r \Rightarrow (p \vee \neg q)$
When the equivalence is true, describe in words the conditions on the value of x.

CHAPTER 9 SUMMARY

Introduction to logic

- A (simple) **statement** has a truth value of **true** or **false** (but not both).

Compound statements and symbols

- A **compound statement** is made up of simple statements joined together by **connectives.**
- The five connectives have these names and symbols:

NOT	**Negation**	$\neg$
AND	**Conjunction**	$\wedge$
OR	**Inclusive disjunction**	$\vee$
OR	**Exclusive disjunction**	$\underline{\vee}$
IF ... THEN	**Implication**	$\Rightarrow$

Truth tables: negation

- The **negation** of a statement p is written $\neg p$ (read as 'not-p'). The relation between any statement p and its negation $\neg p$ is shown in a **truth table**.

p	$\neg p$
T	F
F	T

Truth tables: conjunction (and)

- The **conjunction** of any two statements p and q is written $p \wedge q$. This **compound statement** is defined by this truth table.

p	q	$p \wedge q$
T	T	**T**
T	F	**F**
F	T	**F**
F	F	**F**

Continued on next page

Truth tables: resolving an ambiguity – the 'or' connective

- The **disjunction** of any two statements p and q is written $p \vee q$. This is '**inclusive or**' and it is defined by this truth table.

p	q	$p \vee q$
T	T	**T**
T	F	**T**
F	T	**T**
F	F	**F**

$p \vee q$ is true if either p or q or possibly both are true.

- **Exclusive disjunction** is written $p \veebar q$ and is defined by this truth table.

p	q	$p \veebar q$
T	T	**F**
T	F	**T**
F	T	**T**
F	F	**F**

Logical equivalence, tautologies and contradictions

- The statements $\neg p \wedge \neg q$ and $\neg(p \vee q)$ are said to be (**logically**) **equivalent**. Equivalence is shown by the symbol $\Leftrightarrow$ so we write

$$\neg p \wedge \neg q \Leftrightarrow \neg(p \vee q)$$

- $\neg p \vee \neg q$ is **not** equivalent to $\neg(p \vee q)$.
- A **tautology** is a compound statement which is **true whatever** the truth values of the simple statements it is made up from.
- A (logical) **contradiction** is a compound statement which is **false whatever** the truth values of its simple statements.

Compound statements made up from three simple statements

- $(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$

Arguments

- A compound statement that includes **implication** is called an **argument**.
- The truth table for implication is:

p	q	$p \Rightarrow q$
T	T	**T**
T	F	**F**
F	T	**T**
F	F	**T**

Continued on next page

- If the compound statement that represents an argument is a **tautology**, then the argument is **valid.**
- There are four important types of argument:
  - A **contradiction** is always false. (last column of truth table all Fs)
  - A **tautology** is always true. (last column of truth table all Ts)
  - A **valid** argument is always true. (last column of truth table all Ts)
  - An **invalid** argument is not always true. (last column of truth table has at least one F)

  These definitions mean that an invalid argument may (or may not) be a contradiction. A contradiction, however, is always an invalid argument.
- The truth table for **equivalence** ($p \Leftrightarrow q$) is:

p	q	$p \Leftrightarrow q$
T	T	**T**
T	F	**F**
F	T	**F**
F	F	**T**

- There are three commonly used arguments that are formed from the direct statement $p \Rightarrow q$:

$q \Rightarrow p$ the **converse** of the direct statement
$\neg p \Rightarrow \neg q$ the **inverse** of the direct statement
$\neg q \Rightarrow \neg p$ the **contrapositive** of the direct statement.

- This table summarizes the truth values for the direct argument, $p \Rightarrow q$, and the related conditionals.

p	q	Statement $p \Rightarrow q$	Converse $q \Rightarrow p$	Inverse $\neg p \Rightarrow \neg q$	Contrapositive $\neg q \Rightarrow \neg p$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T