j Valid

Converse: If one integer is odd and one integer is even then the sum of the two integers is odd. Valid.

Inverse: If the sum of the two integers is not odd, then the integers are either both odd or both even. Valid.

Contrapositive: If two integers are either both even or both odd then the sum of the two integers is not odd. Valid.

k Valid

Converse: If two integers are both odd then the product of the two integers is odd. Valid.

Inverse: If the product of the two integers is not odd, then the two integers are not both odd. Valid.

Contrapositive: If two integers are not both odd then the product of the two integers is not odd. Valid.

l Valid

Converse: If $a^2 + b^2 = c^2$ then triangle ABC is right angled. Valid

Inverse: If triangle ABC is not right angled, then $a^2 + b^2 \neq c^2$. Valid.

Contrapositive: If $a^2 + b^2 \neq c^2$ then triangle ABC is not right angled. Valid

m **Direct argument:** If an integer is odd then its square is odd. Valid.

Converse: If the square of an integer is odd, then the integer is odd. Valid.

Inverse: If an integer is not odd then its square is not odd. Valid

Contrapositive: If the square of an integer is not odd then the integer is not odd. Valid.

n Valid

Converse: If triangle ABC has three equal sides then triangle ABC has three equal angles. Valid.

Inverse: If triangle ABC does not have three equal angles then triangle ABC does not have three equal sides. Valid.

Contrapositive: If triangle ABC does not have three equal sides then triangle ABC does not have three equal angles. Valid.

o Invalid eg: a rhombus

Converse: If quadrilateral ABCD has four equal angles then ABCD has four equal sides.

Invalid eg: a rectangle

Inverse: If quadrilateral ABCD does not have four equal sides, then ABCD does not have four equal angles. Invalid eq: a rectangle

Contrapositive: If quadrilateral ABCD does not have four equal angles then ABCD does not have four equal sides. Invalid eg: a rhombus

p Invalid. eq: $x = -5$

Converse: If $x = 5$, then $x^2 = 25$ valid

Inverse: If $x^2 \neq 25$, then $x \neq 5$ valid

Contrapositive: If $x \neq 5$, then $x^2 \neq 25$ Invalid eg: $x = -5$

q Valid

Converse: If $x = 3$, then $x^3 = 27$. Valid

Inverse: If $x^3 \neq 27$, then $x \neq 3$. Valid

Contrapositive: If $x \neq 3$, then $x^3 \neq 27$. Valid.

r Invalid. eq. $x < -5$

Converse: If $x > 5$, then $x^2 > 25$. Valid

Inverse: If $x^2 \leq 25$, then $x \leq 5$. Valid

Contrapositive: If $x \leq 5$, then $x^2 \leq 25$. Invalid eg: $x < -5$

s Valid

Converse: If $x < 3$, then $x^3 < 27$. Valid

Inverse: If $x^3 \geq 27$, then $x \geq 3$. Valid

Contrapositive: If $x \geq 3$, then $x^3 \geq 27$. Valid.

Review exercise

Paper 1 style questions

1 **a**

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$\neg(p \vee q) \Rightarrow \neg p \vee \neg q$
T	T	T	F	F	F	F	T
T	F	T	F	F	T	F	T
F	T	T	F	T	F	F	T
F	F	F	T	T	T	T	T

Since every entry in the root column is T $\neg(p \vee q) \Rightarrow \neg p \wedge \neg q$ is a valid argument.

b She does not dance well and she does not sing beautifully

2 **a** If the train leaves from gate 2, then it leaves today and not from gate 8.

b $\neg r \Leftrightarrow (p \vee q)$

3 **a**

p	q	$p \Rightarrow q$	$\neg p$	$\neg q$	$\neg q \vee p$	$\neg p \vee q$
T	T	T	F	F	T	T
T	F	F	F	T	T	F
F	T	T	T	F	F	T
F	F	T	T	T	T	T

b $(p \Rightarrow q) \Leftrightarrow (\neg p \vee q)$

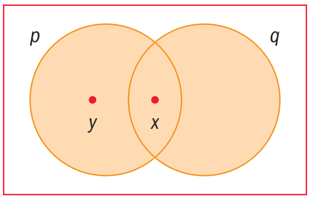
4 a

p	q	$\neg p$	$\neg p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

b i If $x > 3$ and $x^2 \not> 9$, p is T and q is F. From the table $\neg p \vee q$ is F.

ii If $x \not> 3$ and $x^2 > 9$, p is F and q is T. From the table $\neg p \vee q$ is T.

5 a, b



c i $\neg q \Rightarrow \neg p$ **ii** $\neg p \vee q$

iii $\neg q \Rightarrow p$ **iv** $p \wedge \neg q$

d Proposition **i** since it is the contrapositive of the given statement

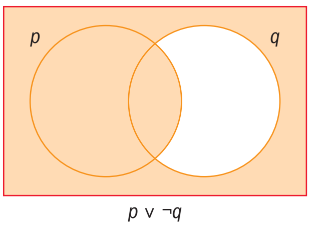
6 a i Picasso painted picture A or van Gogh did not paint picture A.

ii Picasso did not paint picture A and van Gogh painted picture A.

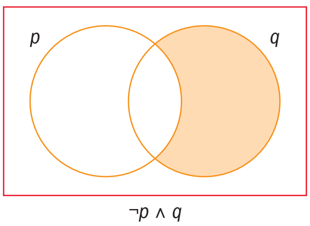
b

p	q	$\neg p$	$\neg q$	$p \vee \neg q$	$\neg p \wedge q$
T	T	F	F	T	F
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	F

c



$p \vee \neg q$



$\neg p \wedge q$

d i

$(p \vee \neg q)$	$(\neg p \wedge q)$	$(p \vee \neg q) \Leftrightarrow (\neg p \wedge q)$
T	F	F
T	F	F
F	T	F
T	F	F

ii Using the Venn diagrams the regions representing $p \vee \neg q$ and $\neg p \wedge q$ do not overlap hence the truth values of $(p \vee \neg q) \Leftrightarrow (\neg p \wedge q)$ are all false.

e A logical contradiction.

7 a x is a multiple of 3 or a factor of 90 and is not a multiple of 5

b $r \Rightarrow (p \vee \neg q)$

c

p	q	r	$q \vee r$	$\neg p$	$(q \vee r) \wedge \neg p$
T	T	T	T	F	F
T	T	F	T	F	F
T	F	T	T	F	F
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	F	T	F

p	q	r	$\neg q$	$p \vee \neg q$	$r \Rightarrow (p \vee \neg q)$
T	T	T	F	T	T
T	T	F	F	T	T
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	F	F
F	T	F	F	F	T
F	F	T	T	T	T
F	F	F	T	T	T

d

p	q	r	x
F	T	T	3
F	T	F	12
F	F	T	2

e

p	q	r	$(q \vee r) \wedge \neg p$	$r \Rightarrow (p \vee \neg q)$
T	T	T	F	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	F
F	T	F	T	T
F	F	T	T	T
F	F	F	F	T

The statements are equivalent only in the cases

p	q	r
F	T	F
F	F	T

i.e x is not a multiple of 5 and is either a multiple of 3 or a factor of 90 (but not both).