Sequences - revision

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Arithmetic sequences, question 1

The fifth term of an arithmetic sequence is 20 and the twelfth term is 41.

(a) (i) Find the common difference.

(2)

(ii) Find the first term of the sequence.

(1)

(b) Calculate the eighty-fourth term.

(1)

(c) Calculate the sum of the first 200 terms.

(2)
(Total 6 marks)

Arithmetic sequences, question 1

Part (a) can be done in two ways.

We can form an a system of two equations with two unknowns:

$$\begin{cases} u_5 = u_1 + 4d \\ u_{12} = u_1 + 11d \end{cases}$$

and plugging in the values given we have:

$$\begin{cases} 20 = u_1 + 4d \\ 41 = u_1 + 11d \end{cases}$$

No we can solve this by hand or using the GDC and we get $d = 3$ and $u_1 = 8$.

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No we can solve this by hand or using the GDC and we get $d = 3$ and $u_1 = 8$.

Arithmetic sequences, question 1

We could have also formed one equation to find d first:

$$u_{12} = u_5 + 7d$$

So:

$$41 = 20 + 7d$$

which gives $d = 3$
Now we have:

$$u_5 = u_1 + 4d$$

So:

$$20 = u_1 + 12$$

which gives $u_1 = 8$.

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Arithmetic sequences, question 1

For part (b) we simpy use the general formula:

$$u_{84} = u_1 + 83d$$

so we have:

$$u_{84} = 8 + 83 \cdot 3 = 257$$

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Arithmetic sequences, question 1

For part (c) we can calculate u_{200} first:

$$u_{200} = u_1 + 199d$$

so:

$$u_{200} = 8 + 199 \cdot 3 = 605$$

and now we have:

$$S_{200} = \frac{200 \cdot (8 + 605)}{2} = 61300$$

We could have also used the second formula for the sum directly:

$$S_{200} = \frac{200 \cdot (2u_1 + 199d)}{2} = 100 \cdot 613 = 61300$$

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Arithmetic sequences, question 2

The sixth term of an arithmetic sequence is 24. The common difference is 8.

(a) Calculate the first term of the sequence.

The sum of the first n terms is 600.

(b) Calculate the value of n.

(Total 8 marks)

Arithmetic sequences, question 2

For part (a) we will use the general formula:

$$u_6 = u_1 + 5d$$

So we get:

$$24 = u_1 + 5 \cdot 8$$

which gives $u_1 = -16$.

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Arithmetic sequences, question 2

Now in part (b) we will use the formula for the sum of an arithmetic sequence (the second one):

$$S_n = \frac{n \cdot (2u_1 + (n-1)d)}{2}$$

This gives:

$$600 = \frac{n \cdot (-32 + (n-1)8)}{2}$$

No you can put this into solver or rearrange to get:

$$8n^2 - 40n - 1200 = 0$$

and use the polynomial solver. Either way remember that n has to be a natural number, you should get $n = 15$.

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Arithmetic sequences, question 3

Consider the arithmetic sequence 3, 9, 15, ..., 1353.

(a) Write down the common difference.

(1)

(b) Find the number of terms in the sequence.

(3)

(c) Find the sum of the sequence.

(2)
(Total 6 marks)

Arithmetic sequences, question 3

The common difference is of course 6. Remember the phrase "write down" means that either the answer has already been found in the previous parts or is very simple. For "write down" questions you do not need to show any work.

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Arithmetic sequences, question 3

For part (b) we use the general formula for an arithmetic sequence:

$$u_n = u_1 + (n-1)d$$

Now we plug in the values:

$$1353 = 3 + (n-1)6$$

Solving this gives $n = 226$

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Now we plug in the values:

$$1353 = 3 + (n-1)6$$

Solving this gives $n = 226$

Arithmetic sequences, question 3

For part (c) we use the formula for the sum of an arithmetic sequence:

$$S_n = \frac{n \cdot (u_1 + u_n)}{2}$$

So we have

$$S_{226} = \frac{226 \cdot (3 + 1353)}{2} = 153228$$

Arithmetic sequences, question 3

For part (c) we use the formula for the sum of an arithmetic sequence:

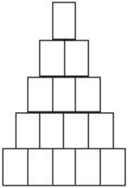
$$S_n = \frac{n \cdot (u_1 + u_n)}{2}$$

So we have

$$S_{226} = \frac{226 \cdot (3 + 1353)}{2} = 153228$$

Arithmetic sequences, question 4

Clara organizes cans in triangular piles, where each row has one less can than the row below. For example, the pile of 15 cans shown has 5 cans in the bottom row and 4 cans in the row above it.



(a) A pile has 20 cans in the bottom row. Show that the pile contains 210 cans.

(4)

(b) There are 3240 cans in a pile. How many cans are in the bottom row?

(4)

(c) (i) There are S cans and they are organized in a triangular pile with n cans in the bottom row. Show that $n^2 + n - 2S = 0$.

(ii) Clara has 2100 cans. Explain why she cannot organize them in a triangular pile.

(6)

(Total 14 marks)

Arithmetic sequences, question 4

In part (a) we notice that we have an arithmetic sequence with $u_1 = 1$ (the top row) $d = 1$ (each row has one more can than the one above), $n = 20$ and $u_{20} = 20$.

We need to calculate S_{20}. We have:

$$S_{20} = \frac{20 \cdot (1 + 20)}{2} = 210$$

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We need to calculate S_{20}. We have:

$$S_{20} = \frac{20 \cdot (1 + 20)}{2} = 210$$

Arithmetic sequences, question 4

Now in part (b) we still have $u_1 = 1$ and $d = 1$, we want to find u_n, but notice that $u_n = u_1 + (n-1)d = 1 + (n-1)1 = n$. We use the formula for the sum:

$$S_n = \frac{n \cdot (u_1 + u_n)}{2}$$

so:

$$3240 = \frac{n \cdot (1 + n)}{2}$$

Using GDC we get that $n = 80$

Arithmetic sequences, question 4

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Arithmetic sequences, question 4

In part (c) we use the same fomulae as in part (b) and get:

$$S = \frac{n \cdot (1+n)}{2}$$

Now we need to rearrange this to get:

$$2S = n^2 + n$$

So $n^2 + n - 2S = 0$

Now we have $S = 2100$, so we get the equation:

$$n^2 + n - 4200 = 0$$

We use GDC to solve this (polynomial solver) and get none of the solutions are natural numbers, so the cans cannot be organized in a triangular pile. Recall that n is the number of rows, it cannot be equal to 64.3 or -65.3.

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Geometric sequences, question 1

Portable telephones are first sold in the country *Cellmania* in 1990. During 1990, the number of units sold is 160. In 1991, the number of units sold is 240 and in 1992, the number of units sold is 360.

In 1993 it was noticed that the annual sales formed a geometric sequence with first term 160, the 2nd and 3rd terms being 240 and 360 respectively.

(a) What is the common ratio of this sequence?

(1)

Assume that this trend in sales continues.

(b) How many units will be sold during 2002?

(3)

(c) In what year does the number of units sold first exceed 5000?

(4)

Between 1990 and 1992, the total number of units sold is 760.

(d) What is the total number of units sold between 1990 and 2002?

(2)

During this period, the total population of *Cellmania* remains approximately 80 000.

(e) Use this information to suggest a reason why the geometric growth in sales would not continue.

(1)
(Total 11 marks)

Geometric sequences, question 1

Part (a): for the common ratio $r = \frac{240}{160} = \frac{3}{2}$.

Part (b): Here we need to count carefully. If u_1 is the number of units sold in 1990, u_2 the number of units sold in 1991, and so on, then we need to calculate u_{13}. We use the general formula:

$$u_{13} = u_1 \cdot r^{12} = 160 \cdot 1.5^{12} = 20759$$

Geometric sequences, question 1

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Geometric sequences, question 1

For part (c) we want to solve $u_n = 5000$, so we need to solve:

$$160 \cdot 1.5^{n-1} = 5000$$

We use GDC to get $n = 9.49...$, so $n = 10$ as we want the number of units sold to exceed 5000. Now u_{10} corresponds to the year 1999.

Geometric sequences, question 1

For part (c) we want to solve $u_n = 5000$, so we need to solve:

$$160 \cdot 1.5^{n-1} = 5000$$

We use GDC to get $n = 9.49..$, so $n = 10$ as we want the number of units sold to exceed 5000. Now u_{10} corresponds to the year 1999.

Geometric sequences, question 1

For part (d) we want to calculate S_{13}, we use the formula for the sum of a geometric sequence:

$$S_{13} = \frac{u_1(r^{13} - 1)}{r - 1} = \frac{160 \cdot (1.5^{13} - 1)}{1.5 - 1} = 61958$$

For part (e) we use common sense. If by 2002 61958 mobile phones were sold, then majority of the population already has one, so the sales should slow down.

Geometric sequences, question 1

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The test will include approx 2 basic questions on arithmetic sequences, 1 basic question on geometric sequences and some questions on applications of both of these sequences.