Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematical studies
Standard level
Paper 2

Thursday 3 May 2018 (morning)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

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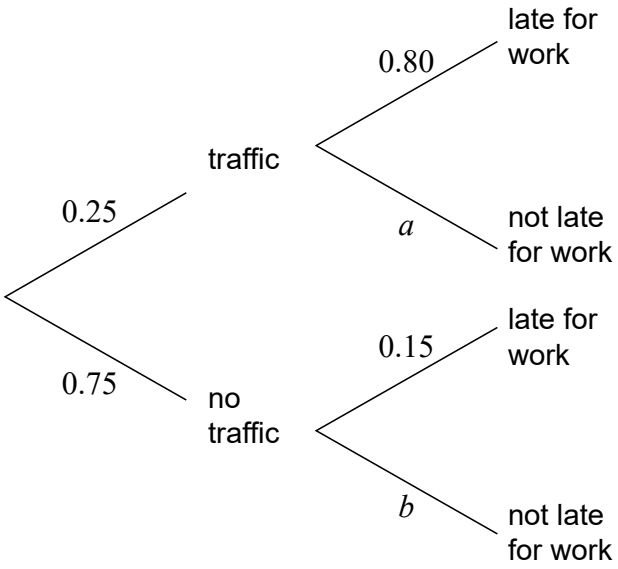
Answer **all** questions in the answer booklet provided. Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. [Maximum mark: 16]

In a company it is found that 25% of the employees encountered traffic on their way to work. From those who encountered traffic the probability of being late for work is 80%.

From those who did not encounter traffic, the probability of being late for work is 15%.

The tree diagram illustrates the information.



(a) Write down the value of

(i) a;

(ii) b. [2]

(b) Use the tree diagram to find the probability that an employee

(i) encountered traffic and was late for work;

(ii) was late for work;

(iii) encountered traffic given that they were late for work. [8]

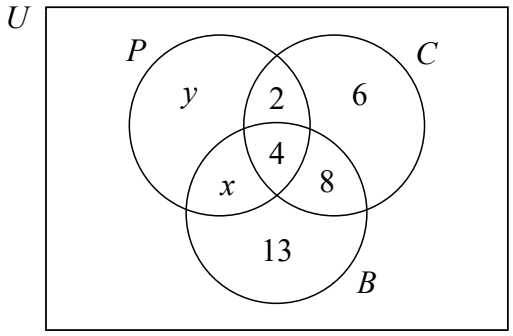
(This question continues on the following page)

(Question 1 continued)

The company investigates the different means of transport used by their employees in the past year to travel to work. It was found that the three most common means of transport used to travel to work were public transportation (P), car (C) and bicycle (B).

The company finds that 20 employees travelled by car, 28 travelled by bicycle and 19 travelled by public transportation in the last year.

Some of the information is shown in the Venn diagram.



(c) Find the value of

(i) x;

(ii) y. [2]

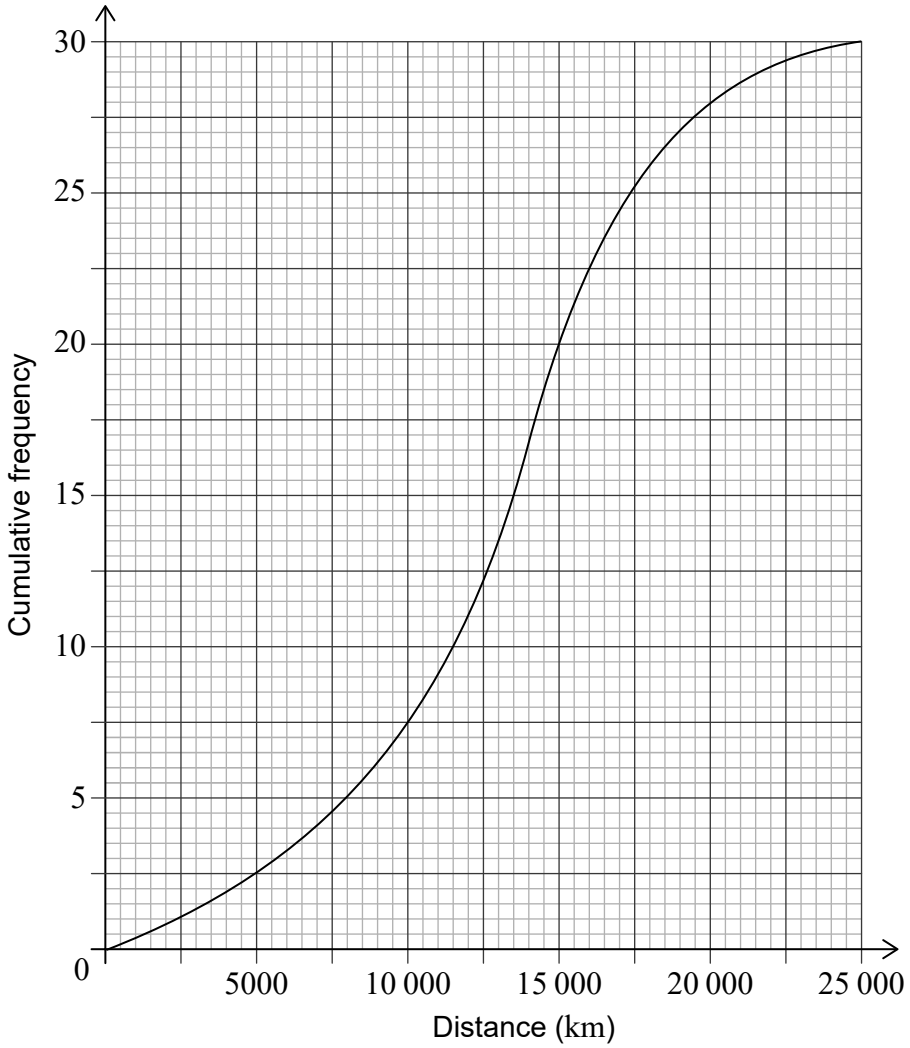
There are 54 employees in the company.

(d) Find the number of employees who, in the last year, did not travel to work by car, bicycle or public transportation. [2]

(e) Find $n\left((C \cup B) \cap P'\right)$. [2]

2. [Maximum mark: 15]

A transportation company owns 30 buses. The distance that each bus has travelled since being purchased by the company is recorded. The cumulative frequency curve for these data is shown.



(a) Find the number of buses that travelled a distance between $15\,000$ and $20\,000$ kilometres. [2]

(b) Use the cumulative frequency curve to find the

(i) median distance;

(ii) lower quartile;

(iii) upper quartile. [4]

(This question continues on the following page)

(Question 2 continued)

(c) Hence write down the interquartile range. [1]

(d) Write down the percentage of buses that travelled a distance greater than the upper quartile. [1]

(e) Find the number of buses that travelled a distance less than or equal to $12\,000\,\text{km}$. [1]

It is known that 8 buses travelled more than m kilometres.

(f) Find the value of m. [2]

The smallest distance travelled by one of the buses was $2500\,\text{km}$.
The longest distance travelled by one of the buses was $23\,000\,\text{km}$.

(g) **On graph paper**, draw a box-and-whisker diagram for these data. Use a scale of $2\,\text{cm}$ to represent $5000\,\text{km}$. [4]

3. [Maximum mark: 14]

The weight, W, of basketball players in a tournament is found to be normally distributed with a mean of $65\,\text{kg}$ and a standard deviation of $5\,\text{kg}$.

(a) (i) Find the probability that a basketball player has a weight that is less than $61\,\text{kg}$.

In a training session there are 40 basketball players.

(ii) Find the expected number of players with a weight less than $61\,\text{kg}$ in this training session. [4]

(b) The probability that a basketball player has a weight that is within 1.5 standard deviations of the mean is q.

(i) Sketch a normal curve to represent this probability.

(ii) Find the value of q. [3]

(c) Given that $\text{P}(W > k) = 0.225$, find the value of k. [2]

A basketball coach observed 60 of her players to determine whether their performance and their weight were independent of each other. Her observations were recorded as shown in the table.

		Performance	
		Satisfactory	**Excellent**
Weight	**Below average**	6	10
	Average	7	15
	Above average	12	10

She decided to conduct a χ^2 test for independence at the $5\,\%$ significance level.

(d) For this test,

(i) state the null hypothesis; [1]

(ii) find the p-value. [2]

(e) State a conclusion for this test. Justify your answer. [2]

4. [Maximum mark: 16]

A new café opened and during the first week their profit was $\$60$.

The café's profit increases by $\$10$ every week.

(a) Find the café's profit during the 11th week. [3]

(b) Calculate the café's **total** profit for the first 12 weeks. [3]

A new tea-shop opened at the same time as the café. During the first week their profit was also $\$60$.

The tea-shop's profit increases by $10\,\%$ every week.

(c) Find the tea-shop's profit during the 11th week. [3]

(d) Calculate the tea-shop's **total** profit for the first 12 weeks. [3]

In the $m\text{th}$ week the tea-shop's **total** profit exceeds the café's **total** profit, for the first time since they both opened.

(e) Find the value of m. [4]

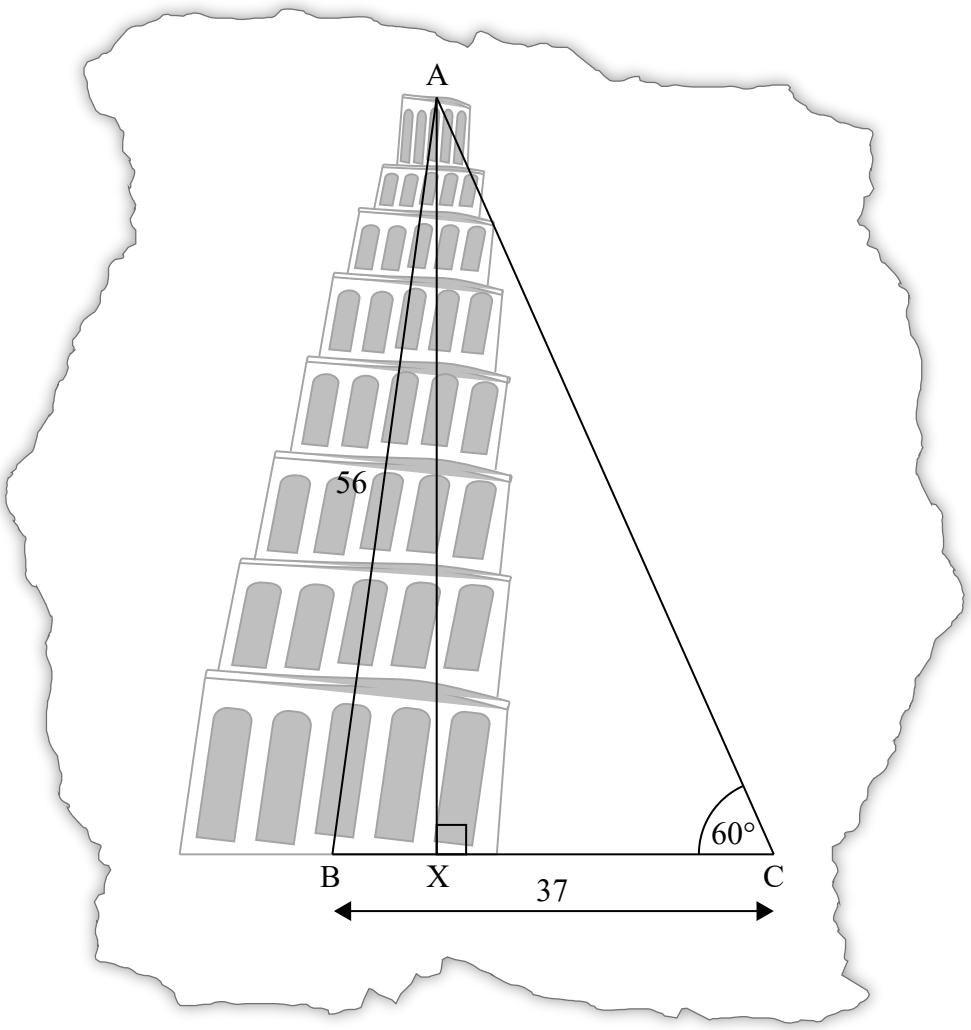
5. [Maximum mark: 14]

The Tower of Pisa is well known worldwide for how it leans.

Giovanni visits the Tower and wants to investigate how much it is leaning. He draws a diagram showing a non-right triangle, ABC.

On Giovanni's diagram the length of AB is $56\,\text{m}$, the length of BC is $37\,\text{m}$, and angle ACB is 60°. AX is the perpendicular height from A to BC.

diagram not to scale



(a) Use Giovanni's diagram to

(i) show that angle ABC, the angle at which the Tower is leaning relative to the horizontal, is 85° to the nearest degree.

(ii) calculate the length of AX.

(iii) find the length of BX, the horizontal displacement of the Tower. [9]

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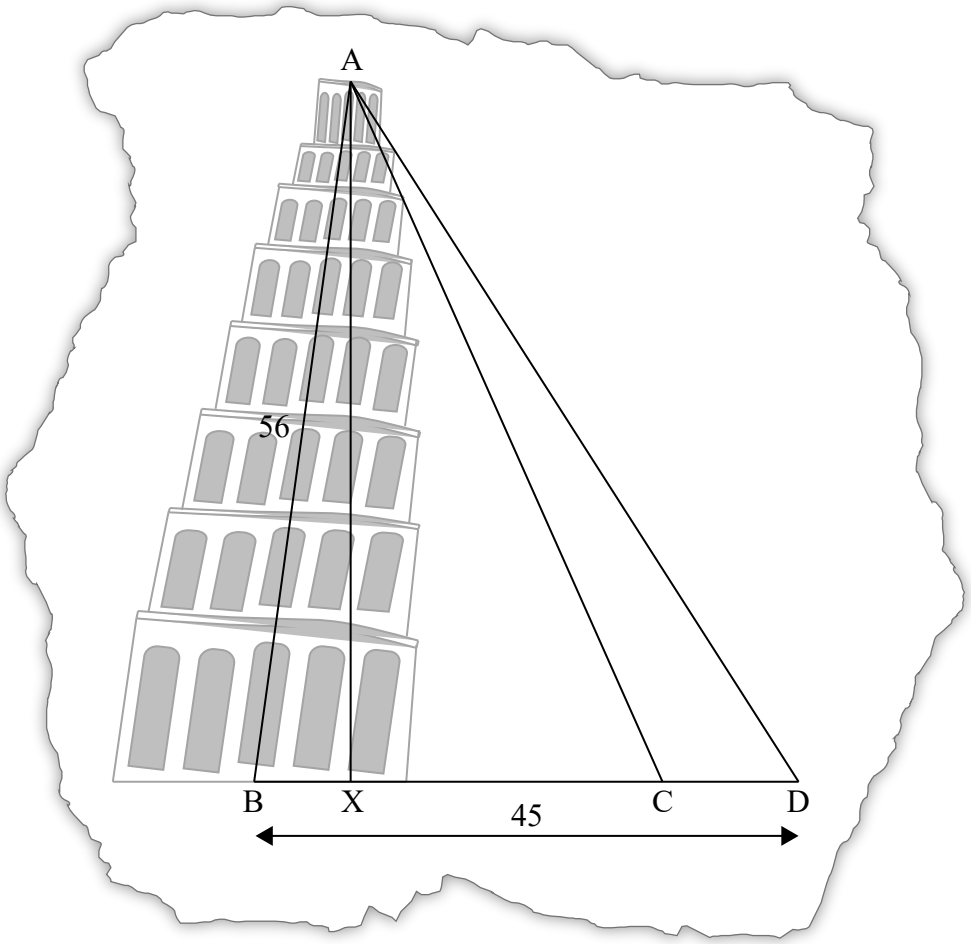
(Question 5 continued)

Giovanni's tourist guidebook says that the actual horizontal displacement of the Tower, BX, is 3.9 metres.

(b) Find the percentage error on Giovanni's diagram. [2]

Giovanni adds a point D to his diagram, such that $\mathrm{BD} = 45\,\mathrm{m}$, and another triangle is formed.

diagram not to scale



(c) Find the angle of elevation of A from D. [3]

6. [Maximum mark: 15]

Consider the curve $y = 2x^3 - 9x^2 + 12x + 2$, for $-1 < x < 3$.

(a) Sketch the curve for $-1 < x < 3$ and $-2 < y < 12$. [4]

(b) A teacher asks her students to make some observations about the curve.

Three students responded.
Nadia said "*The x-intercept of the curve is between -1 and zero*".
Rick said "*The curve is decreasing when $x < 1$*".
Paula said "*The gradient of the curve is less than zero between $x = 1$ and $x = 2$*".

State the name of the student who made an **incorrect** observation. [1]

(c) Find the value of y when $x = 1$. [2]

(d) Find $\frac{dy}{dx}$. [3]

(e) Show that the stationary points of the curve are at $x = 1$ and $x = 2$. [2]

(f) Given that $2x^3 - 9x^2 + 12x + 2 = k$ has **three** solutions, find the possible values of k. [3]