Lines in 2d

Intro

We will revise all necessary information concerning lines in 2 dimensions.

Slope-intercept form

The slope(gradient)-intercept form is an equation of the form:

$$y = mx + c$$

where m is the slope (gradient) of the line and c is the y-intercept.

The slope represents the rate of change of the function - the ratio of the change in the y-coordinates (rise) to the change of the x-coordinates (run).

$$slope = \frac{rise}{run}$$

c represents the y-intercept, ie. the second coordinate of the point of the intersection of the line with the y-axis.

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Finding equation of a line through two points

We need to be able to find an equation of the line through two points. We will go through two methods of doing this.

Example 1, method 1

Find the equation of the line passing through $A(1,3)$ and $B(3,-1)$.

We will calculate the gradient first. We have:

$$m = \frac{rise}{run} = \frac{-1-3}{3-1} = -2$$

So we know that the equation will be of the form $y = -2x + c$. Now we need to substitute one of the points to calculate c. We will substitute $A(1,3)$, $x_A = 1$ and $y_A = 3$, so we have:

$$3 = -2 \cdot 1 + c$$

we get $c = 5$. The equation of the line is $y = -2x + 5$.

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Example 1, method 2

Find the equation of the line passing through $A(1,3)$ and $B(3,-1)$.

Since both points lie on the line we substitute the coordinates of the points to form two equations with two unkowns:

$$\begin{cases} 3 = m \cdot 1 + c \\ -1 = m \cdot 3 + c \end{cases}$$

We can solve this system algebraically or use GDC and we get $m = -2$ and $c = 5$, so the equation of the line is $y = -2x + 5$.

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Find the equation of the line passing through $C(3, -2)$ and $D(1, 1)$.

We will calculate the gradient first. We have:

$$m = \frac{rise}{run} = \frac{1-(-2)}{1-3} = -\frac{3}{2}$$

Now we know that the equation will be of the form $y = -\frac{3}{2}x + c$. We substitute one of the points to calculate c. We will substitute $C(3, -2)$, $x_C = 3$ and $y_C = -2$, so we have:

$$-2 = -\frac{3}{2} \cdot 3 + c$$

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We substitute the coordinates of the points to form two equations with two unknowns:

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Horizontal and vertical lines

Horizontal lines have a constant y-coordinates, the slope is 0 (the rise is 0, the line does not rise at all). Horizontal lines have equations of the form $y = c$.

Vertical lines have a constant x-coordinate, the slope is undefined (the run is 0). Vertical lines have equations of the form $x = d$.

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Horizontal and vertical lines

Find a horizontal and vertical lines through the point $A(1, -3)$.

The horizontal line has the equation $y = c$ and since the y-coordinate of the point is -3, the horizontal line will have the equation $y = -3$.

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Parallel and perpendicular lines

Parallel lines

Given lines

$$l_1: \quad y = m_1x + c_1 \qquad \text{and} \qquad l_2: \quad y = m_2x + c_2$$

l_1 is parallel to l_2 (denoted by $l_1 || l_2$) if both lines have the same gradient, that is $m_1 = m_2$.

Perpendicular lines

Given lines

$$l_1: \quad y = m_1x + c_1 \qquad \text{and} \qquad l_2: \quad y = m_2x + c_2$$

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Example 3

a) Find the equation of the line l_1 through $A(-2, 3)$ and $B(1, 2)$.

b) Find the equation of the line l_2 through $C(1, 5)$ and parallel to l_1.

c) Find the equation of the line l_3 through $D(-2, -3)$ and perpendicular to l_1.

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a) Find the equation of the line l_1 through $A(-2, 3)$ and $B(1, 2)$.

We calculate the gradient $m_1 = \frac{2-3}{1-(-2)} = -\frac{1}{3}$. So we have $y = -\frac{1}{3}x + c_1$. We substitute the point $A(-2, 3)$, to get:

$$3 = -\frac{1}{3} \cdot (-2) + c_1$$

and we solve for c_1 to get $c_1 = \frac{7}{3}$. So the equation l_1 is $y = -\frac{1}{3}x + \frac{7}{3}$.

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b) Find the equation of the line l_2 through $C(1, 5)$ and parallel to l_1.

Since l_2 is parallel to l_1 we have $m_2 = m_1 = -\frac{1}{3}$. So the equation of l_2 will be of the form $y = -\frac{1}{3}x + c_2$. Now we use the point C, to get the equation:

$$5 = -\frac{1}{3} \cdot 1 + c_2$$

this gives $c_2 = \frac{16}{3}$. So the equation of the line l_2 is $y = -\frac{1}{3}x + \frac{16}{3}$

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c) Find the equation of the line l_3 through $D(-2, -3)$ and perpendicular to l_1.

Since l_3 is perpendicular to l_1 we have $m_3 \cdot m_1 = -1$, so $m_3 = 3$ So the equation of l_3 will be of the form $y = 3x + c_3$. Now we use the point D, to get the equation:

$$-3 = 3 \cdot (-2) + c_3$$

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this gives $c_3 = 3$. So the equation of the line l_2 is $y = 3x + 3$

Lines in standard form

The standard form of a line is an equation:

$$Ax + By + D = 0$$

if possible the coefficient A, B, D should be integers and $A > 0$.

In order to turn an equation in the slope-intercept form into a standard form, we simply move all terms to one side of the equation and multiply both sides by a common denominator of all coefficients.

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Example 4

Find an equation of a line perpendicular to $y = -2x + 3$ and passing through $(1, 5)$. Write the equation in the standard form.

We start by finding the equation in the slope-intercept form. Since the line has to be perpendicular to $y = -2x + 3$, we must have $m \cdot (-2) = -1$ and we get $m = \frac{1}{2}$. The equation has the form $y = \frac{1}{2}x + c$. We will use the point $(1, 5)$ to find c. We get the equation:

$$5 = \frac{1}{2} \cdot 1 + c$$

We get $c = \frac{9}{2}$. So the equation in the slope-intercept form is $y = \frac{1}{2}x + \frac{9}{2}$.

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Now we need to turn this form into the standard form. We move all terms to one side to get:

$$\frac{1}{2}x - y + \frac{9}{2} = 0$$

Now we multiply both sides by 2 to cancel the denominators and we get:

$$x - 2y + 9 = 0$$

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Mid-point

Mid-point of a line segment has coordinates that are averages of the corresponding coordinates of the endpoints:

Mid-point

Let $A(x_A, y_A)$ and $B(x_B, y_B)$, the midpoint M of the line segment AB has coordinates $M(\frac{x_A+x_B}{2}, \frac{y_A+y_B}{2})$.

Example 5

Find the equation of the perpendicular bisector of the line segment AB, where $A(2,5)$ and $B(-1,3)$. Write the equation in the standard form.

The perpendicular bisector is the line that bisects (divides into two equal parts) a line segment and is perpendicular to that line segment.

We start by finding the gradient of the line through A and B.

$$m_{AB} = \frac{3-5}{-1-2} = \frac{2}{3}$$

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Now our line has to be perpendicular to the line segment AB, so the gradient has to satisfy $m \cdot \frac{2}{3} = -1$, so $m = -\frac{3}{2}$.

So the line will be of the form $y = -\frac{3}{2}x + c$. Now the line will pass through the mid-point of the line segment AB. The midpoint M_{AB} will have coordinates $M(\frac{2-1}{2}, \frac{5+3}{2})$, so we have $M(\frac{1}{2}, 4)$.

We can now use the point M and substitute into the equation of the line, to find c:

$$4 = -\frac{3}{2} \cdot \frac{1}{2} + c$$

so $c = \frac{19}{4}$. This means that the perpendicular bisector will have the equation $y = -\frac{3}{2}x + \frac{19}{4}$.

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Distance between two points

Given two points we can find the distance between them using Pythagorean Theorem

Distance

Given points $A(x_A, y_A)$ and $B(x_B, y_B)$, the distance between these points is given by the formula

$$|AB| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$

We simply calculate the horizontal distance $(x_B - x_A)$ and the vertical distance $(y_B - y_A)$ and apply the Pythagorean Theorem.

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Example 6

Consider the triangle with vertices $A(1,3), B(3,4)$ and $C(-1,7)$. Show that this is a right triangle and calculate its area.

We will calculate the gradients of all line segments:

$$m_{AB} = \frac{4-3}{3-1} = \frac{1}{2}$$
$$m_{BC} = \frac{7-4}{-1-3} = -\frac{3}{4}$$
$$m_{AC} = \frac{7-3}{-1-1} = -2$$

Now we note that $m_{AB} \cdot m_{AC} = \frac{1}{2} \cdot (-2) = -1$, so the lines AB and AC are perpendicular and the triangle is right-angled.

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$$|AB| = \sqrt{(3-1)^2 + (4-3)^2} = \sqrt{5}$$

$$|AC| = \sqrt{(-1-1)^2 + (7-3)^2} = \sqrt{20}$$

So the area of the triangle is equal to:

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