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MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 1

Thursday 3 May 2012 (afternoon)

1 hour 30 minutes

Candidate session number		
0	0	

Examination code							
2	2	1	2	–	7	4	0	5

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **Mathematical Studies SL** *information booklet* is required for this paper.
- Answer all questions.
- Write your answers in the boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is *[90 marks]*.

Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Write your answers in the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1.   Consider $c = 5200$ and $d = 0.0000037$.

   (a)   Write down the value of $r = c \times d$.   *[1 mark]*

   (b)   Write down your value of r in the form $a \times 10^k$, where $1 \le a < 10$ and $k \in \mathbb{Z}$.   *[2 marks]*

   (c)   Consider the following statements about c, d and r. Only **three** of these statements are true.

   Circle the true statements.

$c \in \mathbb{N}$
$d \in \mathbb{Z}$
$d \in \mathbb{Q}$
$r < d$
$c + d \in \mathbb{R}$
$\dfrac{1}{r} > c$

[3 marks]

Working:

Answers:

(a)   . . . .

(b)   . . . .

2. Consider the propositions p and q.

> p: *I take swimming lessons*
> q: *I can swim 50 metres*

(a) Complete the truth table below.

p	q	$\neg q$	$p \vee \neg q$
T	T		
T	F		
F	T		
F	F		

[2 marks]

(b) Write the following compound proposition in symbolic form.

"I cannot swim 50 metres and I take swimming lessons."   *[2 marks]*

(c) Write the following compound proposition in words.

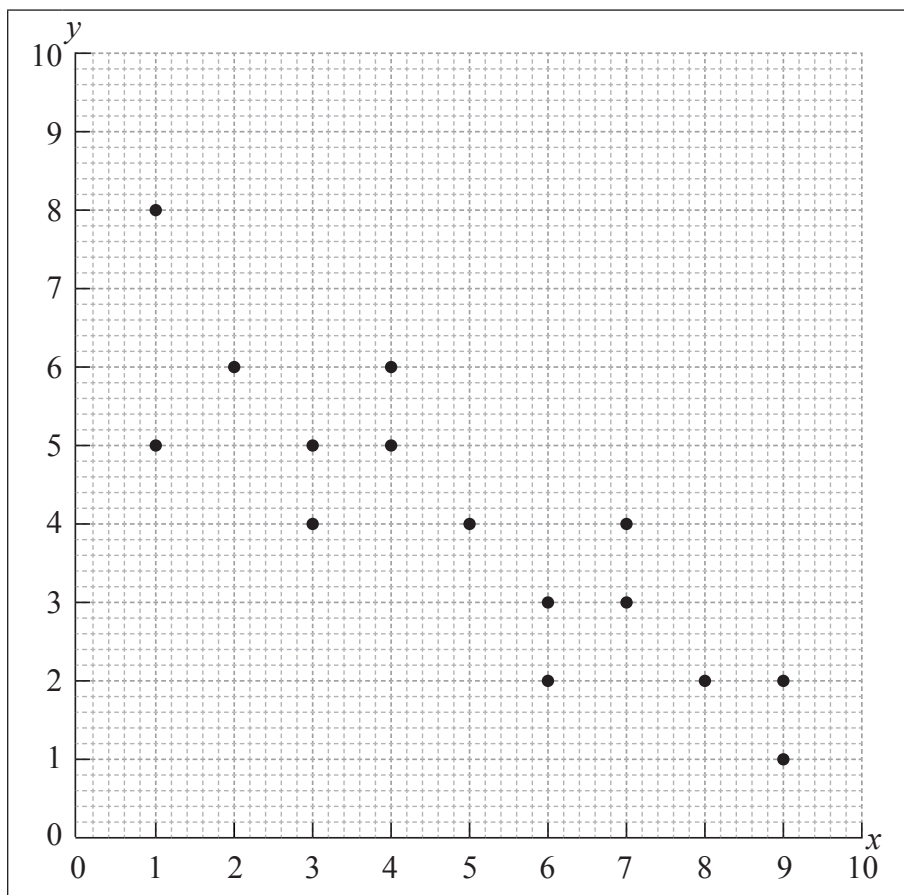$$q \Rightarrow \neg p$$

[2 marks]

Working:

Answers:

(b) ....

(c) ....

....

Turn over

3. Consider the following values of x and y and the scatter diagram which represents the information given in the table.

x	1	1	2	3	3	4	4	b	6	6	7	7	8	9	9
y	5	a	6	4	5	5	6	4	2	3	3	4	2	1	2

(a) Write down the value of

(i) a;

(ii) b. *[2 marks]*

(b) The mean of the x values is 5 and the mean of the y values is 4. Draw the line of best fit on the scatter diagram above. *[2 marks]*

(c) Use your line of best fit to estimate the value of y when $x = 6.5$. *[2 marks]*

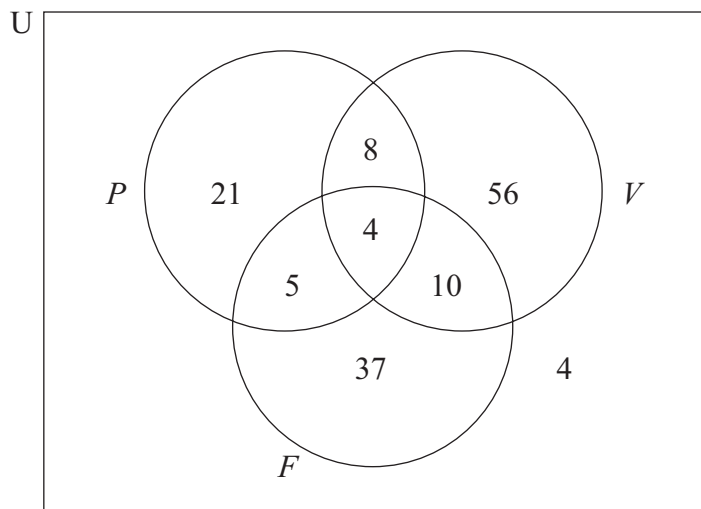*(This question continues on the following page)*

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(Question 3 continued)

Working:

Answers:

(a) (i) . . . .

    (ii) . . . .

(c) . . . .

Turn over

4. Music lessons in Piano (P), Violin (V) and Flute (F) are offered to students at a school.

The Venn diagram shows the number of students who learn each kind of instrument.

(a) Write down the total number of students in the school. *[1 mark]*

(b) Write down the number of students who

    (i) learn violin only;

    (ii) learn piano or flute or both;

    (iii) do not learn flute. *[3 marks]*

(c) Explain, in words, the meaning of the part of the diagram that represents the set $P \cap F'$. *[2 marks]*

Working:

Answers:

(a) . . . .

(b) (i) . . . .

    (ii) . . . .

    (iii) . . . .

(c) . . . .

. . . .

5. The tenth term of an arithmetic sequence is 32 and the common difference is –6.

(a) Find the first term of the sequence. *[2 marks]*

(b) Find the 21st term of the sequence. *[2 marks]*

(c) Find the sum of the first 30 terms of the sequence. *[2 marks]*

Working:

Answers:

(a) ....

(b) ....

(c) ....

Turn over

6. Water has a lower boiling point at higher altitudes. The relationship between the boiling point of water (T) and the height above sea level (h) can be described by the model $T = -0.0034h + 100$ where T is measured in degrees Celsius (°C) and h is measured in **metres** from sea level.

(a) Write down the boiling point of water at sea level. *[1 mark]*

(b) Use the model to calculate the boiling point of water at a height of 1.37 km above sea level. *[3 marks]*

Water boils at the top of Mt. Everest at $70\,°C$.

(c) Use the model to calculate the height above sea level of Mt. Everest. *[2 marks]*

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

7. In the diagram, triangle ABC is isosceles. AB = AC and angle ACB is 32°.
The length of side AC is x cm.

diagram not to scale

A

x

32°
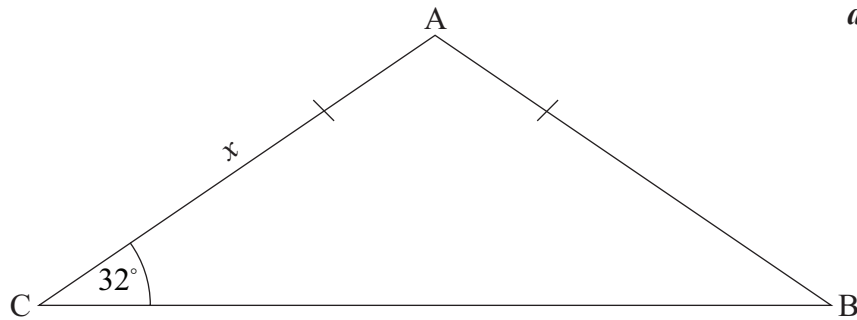
C

B

(a) Write down the size of angle CBA. *[1 mark]*

(b) Write down the size of angle CAB. *[1 mark]*

(c) The area of triangle ABC is 360 cm². Calculate the length of side AC.
Express your answer in **millimetres**. *[4 marks]*

Working:

Answers:

(a) ....

(b) ....

(c) ....

Turn over

8. A researcher consulted 500 men and women to see if the colour of the car they drove was independent of gender. A χ^2 test for independence was carried out.

(a) Write down the null hypothesis. *[1 mark]*

The colours of the cars were red, green, blue, black and silver.

(b) Find the number of degrees of freedom for this test. *[2 marks]*

At the 5 % significance level the χ^2_{calc} was found to be 8.73.

(c) Write down the critical value, χ^2_{crit}, for this test. *[1 mark]*

(d) State whether the null hypothesis was accepted. Give a reason for your answer. *[2 marks]*

Working:

Answers:

(a) ....

(b) ....

(c) ....

(d) ....

....

9.   ***In this question give all answers correct to two decimal places.***

Chiara is an Italian tourist visiting Sweden. The exchange rate for changing euros (€) into Swedish Krona (SEK) is $1€ = 10.275 \, \text{SEK}$.   She converts 350 euros into Swedish Krona at a bank which charges 2 % commission.

(a)   Calculate the amount of commission charged in **SEK**.   *[3 marks]*

(b)   Write down the amount of money she receives from the bank after commission.   *[1 mark]*

Chiara returns to Italy with 296 SEK. She changes this money back into euros at a bank and receives 32€. The bank does not charge commission.

(c)   Calculate the value in SEK of 1€.   *[2 marks]*

Working:

Answers:

(a)   . . . .

(b)   . . . .

(c)   . . . .

Turn over

10. The resting pulse rates of a group of 10 students who exercise regularly are given below.

$$65, 62, 75, 63, 69, 58, 65, 67, 55, 60$$

(a) Find the median resting pulse rate of the students. *[2 marks]*

(b) Find the mean resting pulse rate of the students. *[2 marks]*

A new student joins the class and the mean resting pulse rate of the group of 11 students becomes 65.

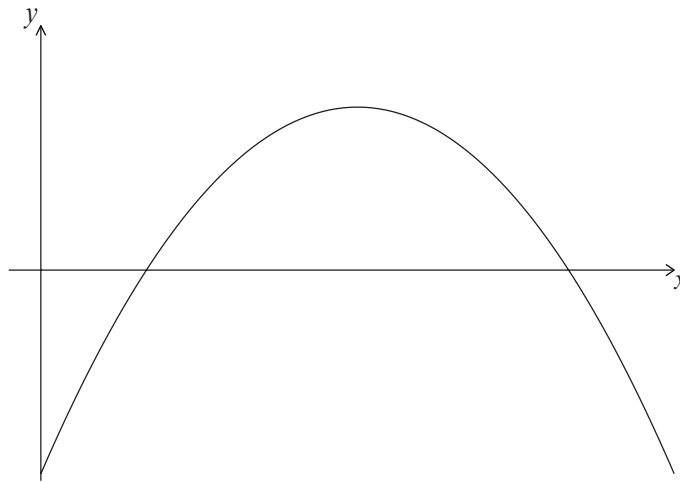(c) Find the resting pulse rate of the student who joined the group. *[2 marks]*

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

11. Part of the graph of the quadratic function f is given in the diagram below.

On this graph one of the x-intercepts is the point $(5, 0)$. The x-coordinate of the maximum point is 3.

The function f is given by $f(x) = -x^2 + bx + c$, where $b, c \in \mathbb{Z}$

(a) Find the value of

    (i) b;

    (ii) c.   *[3 marks]*

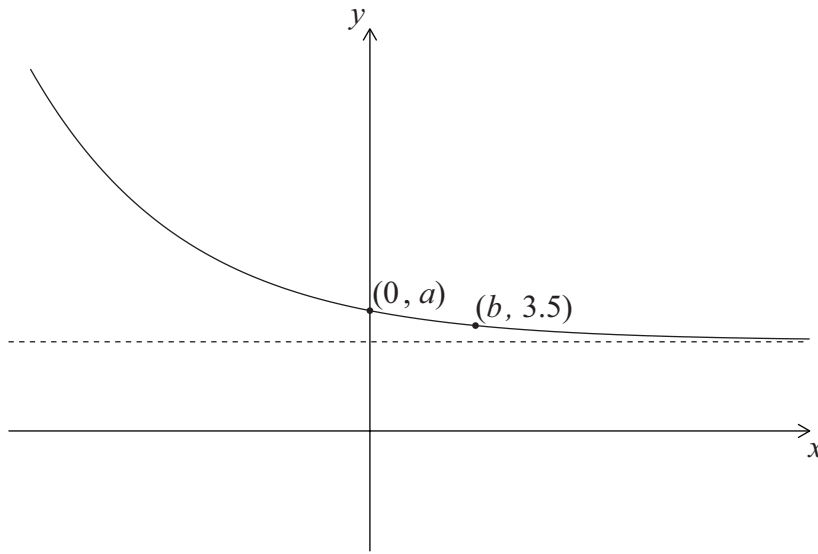The domain of f is $0 \le x \le 6$.

(b) Find the range of f.   *[3 marks]*

Working:

Answers:

(a) (i)   . . . .

     (ii)   . . . .

(b)   . . . .

Turn over

12. The diagram shows part of the graph of $y = 2^{-x} + 3$, and its horizontal asymptote. The graph passes through the points $(0, a)$ and $(b, 3.5)$.

(a) Find the value of

(i) a;

(ii) b.   *[4 marks]*

(b) Write down the equation of the horizontal asymptote to this graph.   *[2 marks]*

Working:

Answers:

(a) (i)   . . . .

(ii) . . . .

(b) . . . .

13. The equation of a curve is given as $y = 2x^2 - 5x + 4$.

(a)　Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.　　　　　　　　　　　　　　　　　　　　　　　*[2 marks]*

The equation of the line L is $6x + 2y = -1$.

(b)　Find the x-coordinate of the point on the curve $y = 2x^2 - 5x + 4$ where the tangent is parallel to L.　　　　　　　　　　　*[4 marks]*
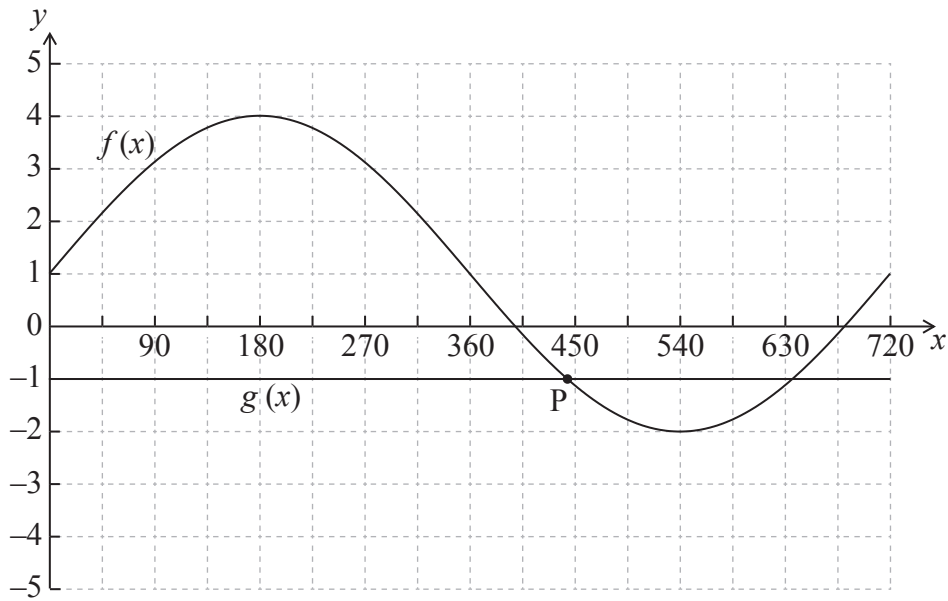
Working:

Answers:

(a) . . . .

(b) . . . .

Turn over

14. In the diagram below the graphs of $f(x)$ and $g(x)$ are shown for $0° \le x \le 720°$.
P is a point of intersection of the two graphs.

Let $f(x) = a\sin(bx°) + 1$.

(a) (i) Write down the value of a.

(ii) Find the value of b.   *[3 marks]*

(b) Use your graphic display calculator to find the *x*-coordinate of P. Give your
answer correct to two decimal places.   *[3 marks]*

Working:

Answers:

(a) (i) . . . .

(ii) . . . .

(b) . . . .

15. Javier starts training for a running race.

On the first day he runs 1.5 km. Every day he runs 10 % more than the day before.

(a) Write down the distance he runs on the second day of training. *[1 mark]*

(b) Calculate the **total** distance Javier runs in the first seven days of training. *[2 marks]*

Javier stops training on the day his total distance exceeds 100 km.

(c) Calculate the number of days Javier has trained for the running race. *[3 marks]*

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

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