Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematical studies
Standard level
Paper 2

Friday 5 May 2017 (morning)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

7 pages

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Answer **all** questions in the answer booklet provided. Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1.   [Maximum mark: 16]

In a school, all Mathematical Studies SL students were given a test. The test contained four questions, each one on a different topic from the syllabus. The quality of each response was classified as satisfactory or not satisfactory. Each student answered only three of the four questions, each on a separate answer sheet.

The table below shows the number of satisfactory and not satisfactory responses for each question.

		Topic from the syllabus				
		Calculus	Probability	Geometry	Logic	Total
Quality of response	**Satisfactory**	10	16	20	14	60
	Not satisfactory	8	6	10	6	30
	Total	18	22	30	20	90

(a)   If the teacher chooses a response at random, find the probability that

   (i)   it is a response to the Calculus question;

   (ii)   it is a satisfactory response to the Calculus question;

   (iii)   it is a satisfactory response, given that it is a response to the Calculus question.   [6]

(b)   The teacher groups the responses by topic, and chooses two responses to the Logic question. Find the probability that both are not satisfactory.   [3]

A χ^2 test is carried out at the 5% significance level for the data in the table.

(c)   State the null hypothesis for this test.   [1]

(d)   Show that the expected frequency of satisfactory Calculus responses is 12.   [1]

(e)   Write down the number of degrees of freedom for this test.   [1]

(f)   Use your graphic display calculator to find the χ^2 statistic for this data.   [2]

The critical value for this test is 7.815.

(g)   State the conclusion of this χ^2 test. Give a reason for your answer.   [2]

2.   [Maximum mark: 11]

Consider these three propositions, in which x is a natural number.

p: x is a factor of 60
q: x is a multiple of 4
r: x is a multiple of 5

(a)   Write down in symbolic form the compound proposition

"If x is a factor of 60 then x is a multiple of 5 or x is not a multiple of 4."   [3]

(b)   Write down in words the compound proposition $\neg r \wedge (p \veebar q)$.   [3]

(c)   **Copy** the following truth table and complete the last three columns.   [3]

p	q	r	$\neg r$	$p \veebar q$	$\neg r \wedge (p \veebar q)$
T	T	T			
T	T	F			
T	F	T			
T	F	F			T
F	T	T			
F	T	F			
F	F	T			
F	F	F			

(d)   State why the compound proposition $\neg r \wedge (p \veebar q)$ is not a logical contradiction.   [1]

(e)   A row from the truth table from part (c) is given below.

p	q	r	$\neg r$	$p \veebar q$	$\neg r \wedge (p \veebar q)$
T	F	F			T

Write down **one** value of x that satisfies these truth values.   [1]

Turn over

3.   [Maximum mark: 18]

The manager of a folder factory recorded the number of folders produced by the factory (in thousands) and the production costs (in thousand Euros), for six consecutive months.

	January	February	March	April	May	June
Folders produced, x (thousands)	10	15	22	30	28	21
Production costs, C (thousand Euros)	35	40	60	72	68	55

(a)   Draw a scatter diagram for this data. Use a scale of $2\,cm$ for 5000 folders on the horizontal axis and $2\,cm$ for $10\,000$ Euros on the vertical axis.   [4]

(b)   Write down, for this set of data

   (i)   the mean number of folders produced, $\overline{x}$;

   (ii)   the mean production cost, $\overline{C}$.   [2]

(c)   Label the point $\mathrm{M}\left(\overline{x},\,\overline{C}\right)$ on the scatter diagram.   [1]

(d)   Use your graphic display calculator to find the Pearson's product–moment correlation coefficient, r .   [2]

(e)   State a reason why the regression line C on x is appropriate to model the relationship between these variables.   [1]

(f)   Use your graphic display calculator to find the equation of the regression line C on x .   [2]

(g)   Draw the regression line C on x on the scatter diagram.   [2]

Every month the factory sells all the folders produced. Each folder is sold for 2.99 Euros.

(h)   Use the equation of the regression line to estimate the least number of folders that the factory needs to sell in a month to exceed its production cost for that month.   [4]

4. [Maximum mark: 15]

The quadrilateral ABCD represents a park, where $AB = 120\,m$, $AD = 95\,m$
and $DC = 100\,m$. Angle DAB is $70°$ and angle DCB is $110°$. This information is shown in
the following diagram.

diagram not to scale
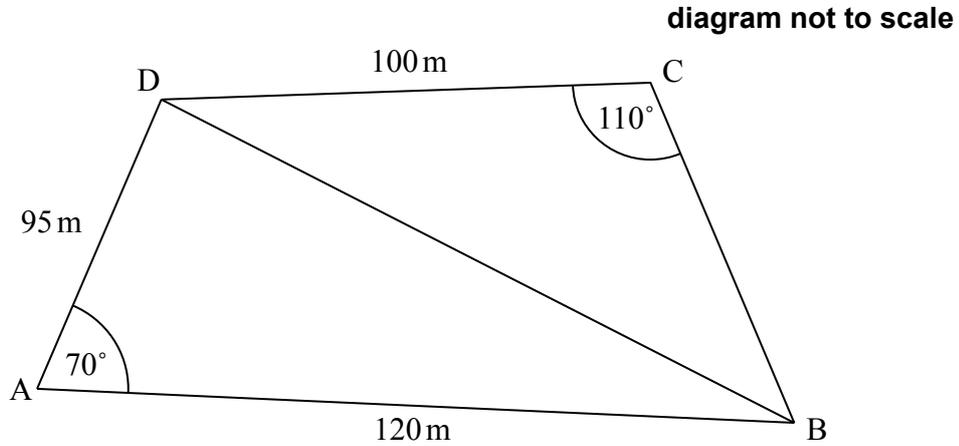
A straight path through the park joins the points B and D.

(a)   Find the length of the path BD.   [3]

(b)   Show that angle DBC is $48.7°$, correct to three significant figures.   [3]

(c)   Find the area of the park.   [4]

A new path, CE, is to be built such that E is the point on BD closest to C.

(d)   Find the length of the path CE.   [2]

The section of the park represented by triangle DCE will be used for a charity race. A track
will be marked along the sides of this section.

(e)   Calculate the total length of the track.   [3]

Turn over

5. [Maximum mark: 14]

Violeta plans to grow flowers in a rectangular plot. She places a fence to mark out the perimeter of the plot and uses 200 metres of fence. The length of the plot is x metres.
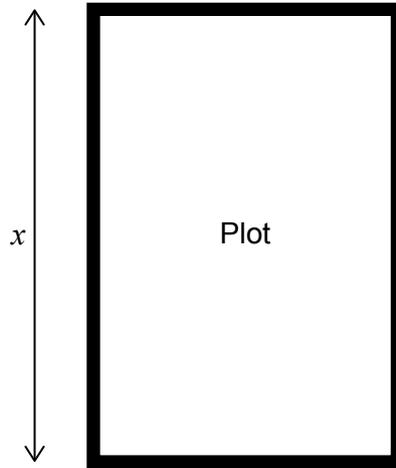
(a) Show that the width of the plot, in metres, is given by $100 - x$. [1]

(b) Write down the area of the plot in terms of x. [1]

Violeta places the fence so that the area of the plot is maximized.

(c) Find the value of x that maximizes the area of the plot. [2]

By selling her flowers, Violeta earns 2 Bulgarian Levs (BGN) per square metre of the plot.

(d) Show that Violeta earns 5000 BGN from selling the flowers grown on the plot. [2]

Violeta wants to invest her 5000 BGN.

(e) A bank offers a nominal annual interest rate of 4%, compounded **half-yearly**.

    (i) Find the amount of money that Violeta would have after 6 years. Give your answer correct to two decimal places.
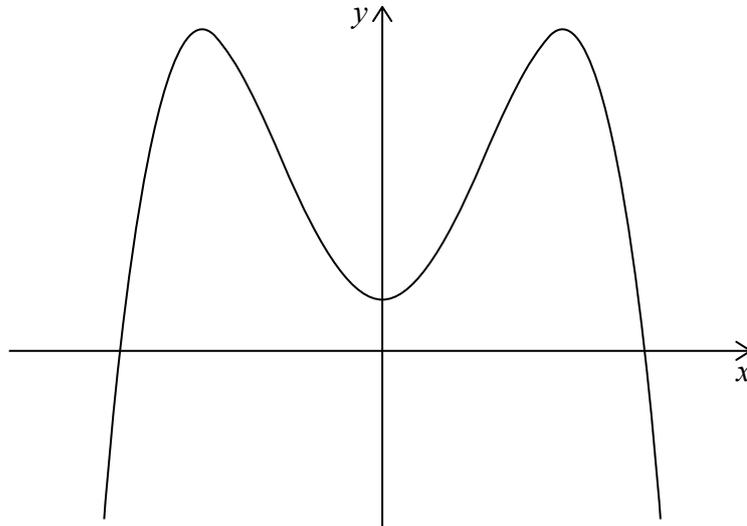
    (ii) Find how long it would take for the interest earned to be 2000 BGN. [6]

Another bank offers an interest rate of $r\%$ compounded **annually**, that would allow her to double her money in 12 years.

(f) Find the lowest possible value for r. [2]

6.   [Maximum mark: 16]

Consider the function $f(x) = -x^4 + ax^2 + 5$, where a is a constant. Part of the graph of $y = f(x)$ is shown below.

(a)   Write down the y-intercept of the graph.   [1]

(b)   Find $f'(x)$.   [2]

It is known that at the point where $x = 2$ the tangent to the graph of $y = f(x)$ is horizontal.

(c)   (i)   Show that $a = 8$.

        (ii)   Find $f(2)$.   [4]

There are two other points on the graph of $y = f(x)$ at which the tangent is horizontal.

(d)   Write down

        (i)   the x-coordinates of these two points;

        (ii)   the intervals where the gradient of the graph of $y = f(x)$ is positive.   [4]

(e)   Write down the range of $f(x)$.   [2]

(f)   Write down the number of possible solutions to the equation $f(x) = 5$.   [1]

(g)   The equation $f(x) = m$, where $m \in \mathbb{R}$, has four solutions. Find the possible values
        of m.   [2]