Markscheme

May 2017

Mathematical studies

Standard level

Paper 2

Paper 2 Markscheme
Instructions to Examiners

Notes: **If in doubt about these instructions or any other marking issues, contact your team leader for clarification.**

1   Abbreviations

M   Marks awarded for **Method**

A   Marks awarded for an **Answer** or for **Accuracy**

R   Marks awarded for clear **Reasoning**

G   Marks awarded for correct solutions obtained from a **Graphic Display Calculator**, when no working shown.

AG   **Answer Given** in the question and consequently, marks not awarded.

ft   Marks that can be awarded as **follow through** from previous results in the question.

2   Method of Marking

(a)   All marking must be done in RM Assessor using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.

(b)   If a question part is completely correct use the number tick annotations to award full marks. If a part is completely wrong use the *A0* annotation, otherwise full annotations must be shown.

(c)   Working crossed out by the candidate should not be awarded any marks.

(d)   Where candidates have written two solutions to a question, only the first solution should be marked.

(e)   If correct working results in a correct answer but then further working is developed, indicating a lack of mathematical understanding full marks should **not** be awarded. In most such cases it will be a single final answer mark that is lost. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal.

Example:

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	Award the final *(A1)* *(ignore the further working)*
2.	$(x-6)(x+1)$	$x = 6$ and -1	Do **not** award the final *(A1)*

Example: Calculate the gradient of the line passing through the points $(5, 3)$ and $(0, 9)$.

Markscheme	Candidates' Scripts	Marking
$\dfrac{9-3}{0-5}$ *(M1)* *Award (M1) for correct substitution in gradient formula* $= -\dfrac{6}{5}$ *(A1)*	(i) $\dfrac{9-3}{0-5} = -\dfrac{6}{5}$ Gradient is $= -\dfrac{6}{5}$ *(There is clear understanding of the gradient.)* $y = -\dfrac{6}{5}x + 9$ (ii) $\dfrac{9-3}{0-5} = -\dfrac{6}{5}$ $y = -\dfrac{6}{5}x + 9$ *(There is confusion about what is required.)*	*(M1)* *(A1)* *(M1)* *(A0)*

3   Follow-through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with '**(ft)**'.

(a)   Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.

(b)   If an answer resulting from follow through is extremely unrealistic (*eg*, negative distances or incorrect by large order of magnitude) then the final **A** mark should not be awarded.

(c)   If a question is transformed by an error into a **different**, **much simpler question** then follow through may not apply.

(d)   To award follow through marks for a question part, **there must be working present for that part**. An isolated follow through answer, without working is regarded as incorrect and receives no marks **even if it is approximately correct**.

(e)   The exception to the above would be in a question which is testing the candidate's use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies.**

(f)   Inadvertent use of radians will be penalized the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

Example: Finding angles and lengths using trigonometry

Markscheme	Candidates' Scripts	Marking

(a) $\dfrac{\sin A}{3} = \dfrac{\sin 30}{4}$ **(M1)(A1)**
 	
*Award **(M1)** for substitution in sine rule formula, **(A1)** for correct substitutions.*
 	
  $A = 22.0° \ (22.0243\ldots)$ **(A1)(G2)**
	(a) $\dfrac{\sin A}{4} = \dfrac{\sin 30}{3}$
	
	      $A = 41.8°$
		(M1)(A0)
		
		(use of sine rule but with wrong values)
		
		(A0)
		(***Note:*** *the 2nd **(A1)** here was not marked* **(ft)** *and cannot be awarded because there was an earlier error in the **same** question part.)*

(b) $x = 7 \tan (22.0243\ldots°)$ **(M1)**
        $= 2.83 \ (2.83163\ldots)$ **(A1)(ft)**
	(b) case (i) $x = 7 \tan 41.8°$
	                    $= 6.26$
	but case (ii) 6.26
		(M1)
		(A1)(ft)
		(G0)
		since no working shown

4   Using the Markscheme

(a)   **A** marks are **dependent** on the preceding **M** mark being awarded, it is **not** possible to award **(M0)(A1)**. Once an **(M0)** has been awarded, all subsequent **A** marks are lost in that part of the question, even if calculations are performed correctly, until the next **M** mark.
The only exception to this will be for an answer where the accuracy is specified in the question – see section 5.

(b)   **A** marks are **dependent** on the **R** mark being awarded, it is **not** possible to award **(A1)(R0)**. Hence the **(A1)** cannot be awarded for an answer which is correct when no reason or the wrong reason is given.

(c)   In paper 2 candidates are expected to demonstrate their ability to communicate mathematics using appropriate working. Answers which are correct but not supported by adequate working will **not always receive full marks**, these unsupported answers are designated **G** in the mark scheme as an alternative to the full marks. Example **(M1)(A1)(A1)(G2)**.

Example: Using trigonometry to calculate an angle in a triangle.

Markscheme	Candidates' Scripts	Marking
(a) $\dfrac{\sin A}{3} = \dfrac{\sin 30}{4}$ **(M1)(A1)** *Award **(M1)** for substitution in sine rule formula, **(A1)** for correct substitutions.* $A = 22.0°\ (22.0243\ldots)$ **(A1)(G2)**	(i) $\dfrac{\sin A}{3} = \dfrac{\sin 30}{4}$ $A = 22.0°$ (ii) $A = 22.0°$ **Note: G** marks are used only if no working has been shown and the answer is correct.	**(M1)(A1)** **(A1)** **(G2)**

(d)   **Alternative methods** may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.
Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR**' *etc.*

(e)   Unless the question specifies otherwise, accept **equivalent forms**. For example: $\dfrac{\sin\theta}{\cos\theta}$ for $\tan\theta$.

On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.
Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:
the 3 significant figure answer worked through from full calculator display;
the exact value $\left(\text{for example } \dfrac{2}{3} \text{ if applicable}\right)$;
the full calculator display in the form $2.83163\ldots$ as in the example above.
Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.

(f)   As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; $1'7$; $1 \cdot 7$; $1,7$.

Decimal numbers less than 1 may be written with or without a leading zero: 0.49 or $.49$.

Different descriptions of an interval: $3 < x < 5$; $(3, 5)$; $]\,3, 5\,[$.

Different forms of notation for set properties (*eg,* complement): A'; $\overline{A}$; A^{c}; $U - A$; $(A$; $U \setminus A.$

Different forms of logic notation: $\neg p$; p' ; $\tilde{p}$; $\overline{p}$; $\sim p.$
$$p \Rightarrow q ;\ p \rightarrow q ;\ q \Leftarrow p .$$

Significance level may be written as α .

(g)   Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt and exception should be raised through RM Assessor to the team leader.

As with previous sessions there will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

5   Accuracy of Answers

Incorrect accuracy should be penalized once only in each question according to the rules below.

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1.   If the **candidate's answer** is seen to 4 sf or greater **and** would round to the required 3 sf answer, then award **(A1)** and ignore subsequent rounding.

2.   If the candidate's unrounded answer is **not** seen then award **(A1)** if the answer given is **correctly** rounded to 2 or more significant figures, otherwise **(A0)**.

Note: If the candidate's unrounded answer is **not** seen and the answer is given correct to 1 sf (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.

3.   If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarized in the table below and illustrated in the examples following.

	If candidates final answer is given …					
	Exact or to 4 or more sf (and would **round to the correct 3 sf**)	**Correct to 3 sf**	**Incorrect to 3 sf**	Correct to 2 sf [3]	Incorrect to 2 sf	Correct or incorrect to 1 sf
Unrounded answer seen[1]	Award the final **(A1)** irrespective of correct or incorrect rounding					
Unrounded answer not seen[2]	**(A1)**	**(A1)**	**(A0)**	**(A1)**	**(A0)**	**(A0)**
Treatment of subsequent parts	As per MS		Treat as follow through, only if working is seen.[3]			

Examples:

Markscheme	Candidates' Scripts		Marking
9.43 (9.43398…) *(A1)*	(i)	9.43398… is seen followed by 9; 9.4; 9.43; 9.434 *etc.* (correctly rounded)	*(A1)*
	(ii)	9.43398… is seen followed by 9.433; 9.44 *etc.* (incorrectly rounded)	*(A1)*
	(iii)	9.4	*(A1)*
	(iv)	9	*(A0)* *(correct to 1 sf)*
	(v)	9.3	*(A0)* *(incorrectly rounded to 2 sf)*
	(vi)	9.44	*(A0)* *(incorrectly rounded to 3 sf)*

Markscheme	Candidates' Scripts		Marking
7.44 (7.43798…) *(A1)*	(i)	7.43798… is seen followed by 7; 7.4; 7.44; 7.438 *etc.* (correctly rounded)	*(A1)*
	(ii)	7.43798… is seen followed by 7.437; 7.43 *etc.* (incorrectly rounded)	*(A1)*
	(iii)	7.4	*(A1)*
	(iv)	7	*(A0)* *(correct to 1 sf)*
	(v)	7.5	*(A0)* *(incorrectly rounded to 2 sf)*
	(vi)	7.43	*(A0)* *(incorrectly rounded to 3 sf)*

Example: ABC is a right angled triangle with angle $\text{ABC} = 90°$, $\text{AC} = 32 \text{ cm}$ and $\text{AB} = 30 \text{ cm}$. Find (a) the length of BC, (b) The area of triangle ABC.

Markscheme	Candidates' Scripts	Marking
(a) $\text{BC} = \sqrt{32^2 - 30^2}$ **(M1)** *Award **(M1)** for correct substitution in Pythagoras' formula* $= 11.1 \left(\sqrt{124}, 11.1355...\right)(\text{cm})$ **(A1)** (b) $\text{Area} = \dfrac{1}{2} \times 30 \times 11.1355...$ **(M1)** *Award **(M1)** for correct substitution in area of triangle formula* $= 167(167.032...)(\text{cm}^2)$ **(A1)(ft)**	(a) $\text{BC} = \sqrt{32^2 - 30^2}$ **(M1)** $11\,(\text{cm})$ **(A1)** *(2 sf answer only seen, but correct)* (b) case (i) $\text{Area} = \dfrac{1}{2} \times 30 \times 11$ **(M1)** *(working shown)* $= 165\,(\text{cm}^2)$ **(A1)(ft)** case (ii) $= 165\,(\text{cm}^2)$ **(M0)(A0)(ft)** *(No working shown, the answer 11 is treated as a ft, so no marks awarded here)*	

Certain answers obtained from the GDC are worth 2 marks and working will not be seen. In these cases only one mark should be lost for accuracy.

eg, Chi-squared, correlation coefficient, mean

Markscheme	Candidates' Scripts	Marking
Chi-squared $7.68\,(7.67543…)$ **(A2)**	(a) 7.7	**(G2)**
	(b) 7.67	**(G1)**
	(c) 7.6	**(G1)**
	(d) 8	**(G0)**
	(e) 7	**(G0)**
	(e) 7.66	**(G0)**

Regression line

Markscheme	Candidates' Scripts		Marking
$y = 0.888x + 13.5$ **(A2)** $(y = 0.887686\ldots x + 13.4895\ldots)$ If an answer is not in the form of an equation award at most **(A1)(A0)**.	(a)	$y = 0.89x + 13$	**(G2)** *(both accepted)*
	(b)	$y = 0.88x + 13$	**(G1)** *(one rounding error)*
	(c)	$y = 0.88x + 14$	**(G1)** *(rounding error repeated)*
	(d) (i)	$y = 0.9x + 13$	
	(ii)	$y = 0.8x + 13$	**(G1)** *(1 sf not accepted)*
	(e)	$0.88x + 13$	**(G0)** *(one rounding error and not an equation)*

Maximum/minimum/points of intersection

Markscheme	Candidates' Scripts		Marking
$(2.06, 4.49)$ **(A1)(A1)** $(2.06020\ldots, 4.49253\ldots)$	(a)	$(2.1, 4.5)$	**(A1)(A1)** *(both accepted)*
	(b)	$(2.0, 4.4)$	**(A1)** *(same rounding error twice)*
	(c)	$(2.06, 4.4)$	**(A1)** *(one rounding error)*
	(d)	$(2, 4.4)$	**(A0)** *(1sf not accepted, one rounding error)*

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**. Exact answers such as $\dfrac{1}{4}$ can be written as decimals to fewer than 3 significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential, however where an answer simplifies to an integer this is expected. Fractions that include a decimal in the numerator and/or the denominator are acceptable for showing correct substitution, but not as a final answer.

Ratios of π and answers taking the form of square roots of integers or any rational power of an integer (*eg,* $\sqrt{13}, 2^{\frac{2}{3}}, \sqrt[4]{5}$,) may be accepted as exact answers. All other powers (*eg,* of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. In **all** such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a **(ft)** mark regardless of an immediately preceding **(M0)**.

6   Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final **A** mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

Example: A financial question demands accuracy correct to 2 dp.

Markscheme	Candidates' Scripts		Marking
$231.62 (231.6189) **(A1)**	(i)	231.6	**(A0)**
	(ii)	232	**(A0)** *(Correct rounding to incorrect level)*
	(iii)	231.61	**(A0)**
	(iv)	232.00	**(A0)** *(Parts (iii) and (iv) are both incorrect rounding to correct level)*

7   Units in answers

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final **A** mark. The markscheme will give clear instructions to ensure that only one or two mark per paper can be lost for lack of units or incorrect units.
The units are considered only when the numerical answer is awarded **(A1)** under the accuracy rules given in Section 5.

Example:

Markscheme	Candidates' Scripts		Marking
(a)	$37000 \, \text{m}^2$ **(A1)**	(a)	$36000 \, \text{m}^2$	**(A0)** *(Incorrect answer so units not considered)*
(b)	$3200 \, \text{m}^3$ **(A1)**	(b)	$3200 \, \text{m}^2$	**(A0)** *(Incorrect units)*

If no method is shown and the answer is correct but with incorrect or missing units award G marks with a one mark penalty.

8   Graphic Display Calculators

Candidates will often be obtaining solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment 'I used my GDC' cannot receive a method mark.

1. (a) (i) $\dfrac{1}{5}$ $\left(\dfrac{18}{90}; 0.2; 20\%\right)$ **(A1)(A1)(G2)**

> **Note:** Award **(A1)** for correct numerator, **(A1)** for correct denominator.

(ii) $\dfrac{1}{9}$ $\left(\dfrac{10}{90}; 0.\overline{1}; 0.111111\ldots; 11.1\%\right)$ **(A1)(A1)(G2)**

> **Note:** Award **(A1)** for correct numerator, **(A1)** for correct denominator.

(iii) $\dfrac{5}{9}$ $\left(\dfrac{10}{18}; 0.\overline{5}; 0.555556\ldots; 55.6\%\right)$ **(A1)(A1)(G2)**

> **Note:** Award **(A1)** for correct numerator, **(A1)** for correct denominator.

[6 marks]

(b) $\dfrac{6}{20} \times \dfrac{5}{19}$ **(A1)(M1)**

> **Note:** Award **(A1)** for two correct fractions seen, **(M1)** for multiplying their two fractions.

$\dfrac{3}{38}$ $\left(\dfrac{30}{380}; 0.0789473\ldots; 7.89\%\right)$ **(A1)(G2)**

[3 marks]

(c) H_0: quality (of response) and topic (from the syllabus) are independent **(A1)**

> **Note:** Accept there is no association between quality (of response) and topic (from the syllabus). Do not accept "not related" or "not correlated" or "influenced".

[1 mark]

(d) $\dfrac{18}{90} \times \dfrac{60}{90} \times 90$ **OR** $\dfrac{18 \times 60}{90}$ **(M1)**

> **Note:** Award **(M1)** for correct substitution in expected value formula.

$(=) 12$ **(AG)**

> **Note:** The conclusion, $(=) 12$, must be seen for the **(A1)** to be awarded.

[1 mark]

(e) 3 **(A1)**

[1 mark]

(f) $\left(\chi^2_{calc} =\right)$ 1.46 $\left(1.46\overline{36}; 1.46363\ldots\right)$ **(G2)**

[2 marks]

continued…

Question 1 continued

(g) $1.46 < 7.815$ **OR** $0.690688… > 0.05$ **(R1)**

the null hypothesis is not rejected   **(A1)(ft)**

OR

the quality of the response and the topic are independent   **(A1)(ft)**

> **Note:** Award **(R1)** for a correct comparison of either their χ^2 statistic to the χ^2 critical value or the correct p-value $0.690688…$ to the test level, award **(A1)(ft)** for the correct result from that comparison. Accept "$\chi^2_{calc} < \chi^2_{crit}$" for the comparison, but only if their χ^2_{calc} value is explicitly seen in part (f). Follow through from their answers to part (f) and part (c). Do not award **(R0)(A1)**.

[2 marks]

Total [16 marks]

2.   (a) $p \Rightarrow (r \vee \neg q)$ *(A1)(A1)(A1)*

> **Note:**   Award *(A1)* for " $p \Rightarrow$ ".
> Award *(A1)* for " $r \vee \neg q$ " or " $r \vee q$ " (or " $\neg q \vee r$ "or " $q \vee r$ ")
> Award *(A1)* for " $\neg q$ ".
> Award at most *(A1)(A1)(A0)* if parentheses are missing for $r \vee \neg q$.
> Award *(A0)(A0)(A1)* for $(p \Rightarrow r) \vee \neg q$.

[3 marks]

(b) x is not a multiple of 5 and (x) is (either) a factor of 60 or (x) is a multiple of 4,
but not both   *(A1)(A1)(A1)*

> **Note:**   Award *(A1)* for " x is not a multiple of 5", *(A1)* for " (x) is a factor of 60 or (x) is a multiple
> of 4 but not both", *(A1)* for "and" in the correct position. Accept only "**but not both**" in
> the second *(A1).*
> Award at most *(A1)(A1)(A0)* for using extra statements such as "If ...then", "if and only
> if" etc.

[3 marks]

(c)

p	q	r	$\neg r$	$p \veebar q$	$\neg r \wedge (p \veebar q)$
T	T	T	**F**	**F**	**F**
T	T	F	**T**	**F**	**F**
T	F	T	**F**	**T**	**F**
T	F	F	**T**	**T**	**T**
F	T	T	**F**	**T**	**F**
F	T	F	**T**	**T**	**T**
F	F	T	**F**	**F**	**F**
F	F	F	**T**	**F**	**F**

(A1)(A1)(A1)(ft)

> **Note:** Award *(A1)* for each correct column. Last column follows through from previous two.

[3 marks]

(d)   because not all the entries in the $\neg r \wedge (p \veebar q)$ column are F   *(R1)*(ft)

> **Note:** If all entries in the last column of their truth table are T, award *(R1)*(ft) for an
> answer of "it is a tautology". Only award *(R1)*(ft) if the column is identified in the
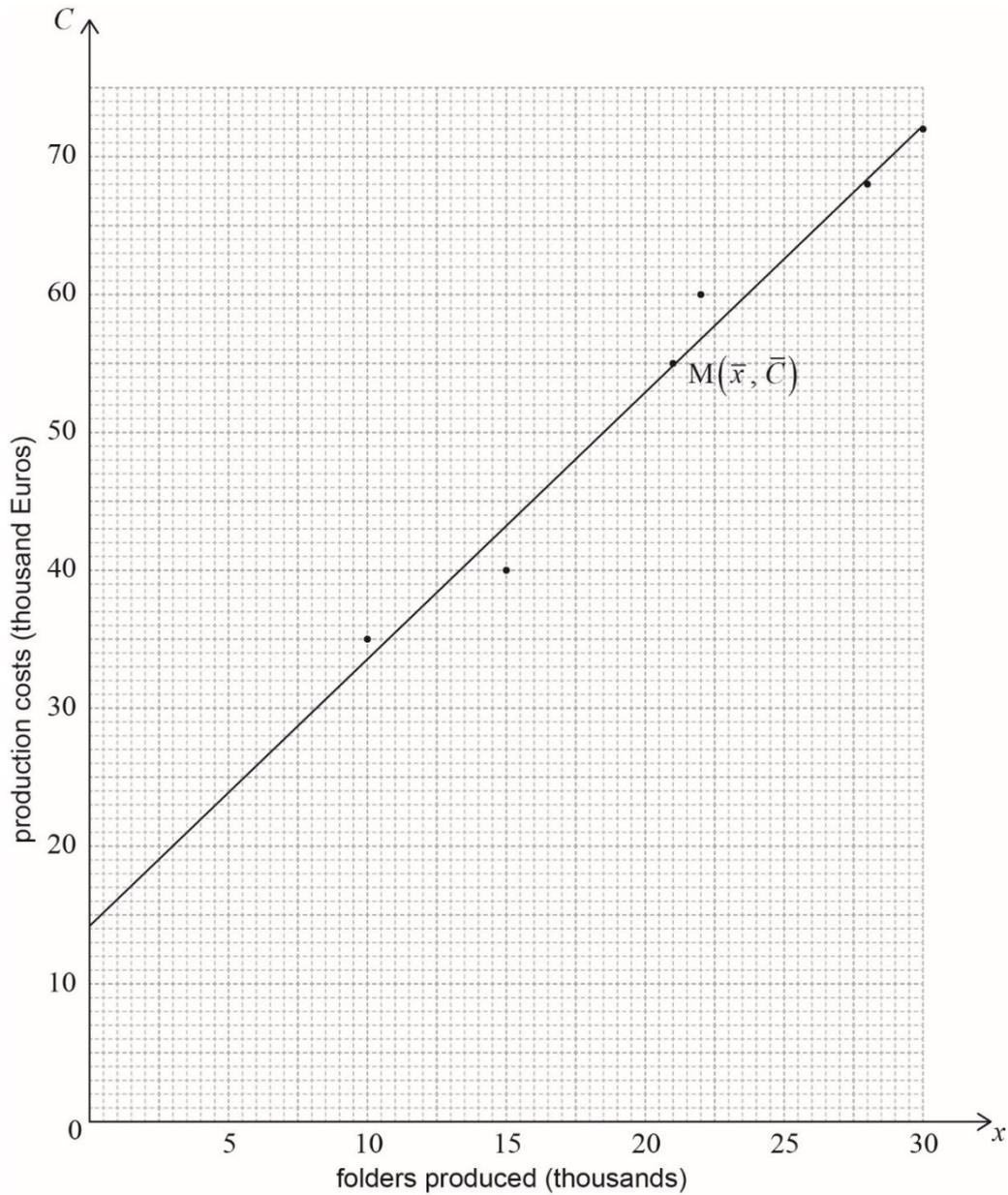> justification.

[1 mark]

(e)   accept one of: 1 **OR** 2 **OR** 3 **OR** 6   *(A1)*

> **Note:** Award *(A1)* for any **one** of the above answers.

[1 mark]
Total [11 marks]

3.   (a)

(A4)

Notes: Award **(A1)** for correct scales and labels. Award **(A0)** if axes are reversed and follow through for their points.
Award **(A3)** for all six points correctly plotted, **(A2)** for four or five points correctly plotted, **(A1)** for two or three points correctly plotted.
If graph paper has not been used, award at most **(A1)(A0)(A0)(A0)**. If accuracy cannot be determined award **(A0)(A0)(A0)(A0)**.

[4 marks]

Question 3 continued

(b)   (i)   ($\overline{x} =$) 21   **(A1)(G1)**

       (ii)   ($\overline{C} =$) 55   **(A1)(G1)**

Note: Accept (i) 21000 and (ii) 55000 seen.

[2 marks]

(c)   their mean point M labelled on diagram   **(A1)(ft)(G1)**

Note: Follow through from part (b).
Award **(A1)(ft)** if their part (b) is correct and their attempt at plotting $(21, 55)$ in part (a) is labelled M.
If graph paper not used, award **(A1)** if $(21, 55)$ is labelled. If their answer from part (b) is incorrect and accuracy cannot be determined, award **(A0)**.

[1 mark]

(d)   ($r =$) 0.990 (0.989568…)   **(G2)**

Note: Award **(G2)** for 0.99 seen. Award **(G1)** for 0.98 or 0.989. Do not accept 1.00.

[2 marks]

(e)   the correlation coefficient/r is (very) close to 1 **(R1)(ft)**

       OR

       the correlation is (very) strong   **(R1)(ft)**

Note: Follow through from their answer to part (d).

       OR

       the position of the data points on the scatter graphs suggests that the tendency is linear   **(R1)(ft)**

Note: Follow through from their scatter graph in part (a).

[1 mark]

(f) $C = 1.94x + 14.2$ ($C = 1.94097\ldots x + 14.2395\ldots$)   **(G2)**

Notes: Award **(G1)** for $1.94x$, **(G1)** for 14.2.
Award a maximum of **(G0)(G1)** if the answer is not an equation.
Award **(G0)(G1)(ft)** if gradient and C-intercept are swapped in the equation.

[2 marks]

continued…

Question 3 continued

(g)   straight line through their $M(21, 55)$ **(A1)(ft)**

   C-intercept of the line (or extension of line) passing through 14.2 (± 1)   **(A1)(ft)**

Notes: Follow through from part (f). In the event that the regression line is not straight
(ruler not used), award **(A0)(A1)(ft)** if line passes through both their $(21, 55)$
and $(0, 14.2)$, otherwise award **(A0)(A0)**. The line must pass *through* the
midpoint, not *near* this point. If it is not clear award **(A0)**.
If graph paper is not used, award at most **(A1)(ft)(A0)**.

[2 marks]

(h) $2.99x = 1.94097\ldots x + 14.2395\ldots$ **(M1)(M1)**

Note: Award **(M1)** for $2.99x$ seen and **(M1)** for equating to their equation of the
regression line. Accept an inequality sign.
Accept a correct graphical method involving their part (f) and $2.99x$.
Accept $C = 2.99x$ drawn on their scatter graph.

   $x = 13.5739\ldots$ (this step may be implied by their final answer)   **(A1)(ft)(G2)**
   $13\,600$ ($13\,574$)   **(A1)(ft)(G3)**

Note: Follow through from their answer to (f). Use of 3 sf gives an answer of $13\,524$.
Award **(G2)** for $13.5739...$ or 13.524 or a value which rounds to 13500 seen without
workings.
Award the last **(A1)(ft)** for correct multiplication by 1000 **and** an answer satisfying
revenue > **their** production cost.
Accept 13.6 thousand (folders).

[4 marks]

Total [18 marks]

4.　(a)　$(BD^2 =)\ \ 95^2 + 120^2 - 2 \times 95 \times 120 \times \cos 70°$　　　　　　　**(M1)(A1)**

> **Note:** Award **(M1)** for substituted cosine rule, **(A1)** for correct substitution.

　　　　$(BD =)\ \ 125\ (\text{m})\ \big(125.007\ldots\ (\text{m})\big)$　　　　　　　　　**(A1)(G2)**

　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　[3 marks]

(b)　$\dfrac{\sin DBC}{100} = \dfrac{\sin 110°}{125.007\ldots}$　　　　　　　　　　　　　**(M1)(A1)(ft)**

> **Note:** Award **(M1)** for substituted sine rule, **(A1)(ft)** for correct substitution.
> Follow through from their answer to part (a).

　　　　$(DBC =)\ \ 48.7384\ldots°$　　　　　　　　　　　　　　　　**(A1)(ft)**
　　　　$(DBC =)\ \ 48.7°$　　　　　　　　　　　　　　　　　　　　**(AG)**

> **Notes:** Award the final **(A1)(ft)** only if both their unrounded answer and $48.7°$ is
> seen. Follow through from their answer to part (a), only if their unrounded
> answer rounds to $48.7°$.

　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　[3 marks]

(c)　$\dfrac{1}{2} \times 125.007\ldots \times 100 \times \sin 21.3° + \dfrac{1}{2} \times 95 \times 120 \times \sin 70°$　　**(A1)(M1)(M1)**

> **Note:** Award **(A1)** for $21.3°\ (21.2615\ldots)$ seen, **(M1)** for substitution into (at least) one area
> of triangle formula in the form $\dfrac{1}{2} ab \sin c$, **(M1)** for **their** correct substitutions and
> adding the two areas.

　　　　$7630\,\text{m}^2\ \ (7626.70\ldots\,\text{m}^2)$　　　　　　　　　　　　**(A1)(ft)(G3)**

> **Notes:** Follow through from their answers to part (a). Accept $7620\,\text{m}^2\ (7622.79\ldots\,\text{m}^2)$ from
> use of $48.7384\ldots$

　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　[4 marks]

continued…

Question 4 continued

(d) $(CE =)\ 100 \times \sin 21.3°$ **(M1)**
        $(CE =)\ 36.3$ (m) $\left(36.3251\ldots \text{(m)}\right)$ **(A1)(ft)(G2)**

Note: Follow through from their angle $21.3°$ in part (c). Award **(M0)(A0)** for halving
            $110°$ and/or assuming E is the midpoint of BD in any method seen.

   OR

   area of $BCD = \dfrac{1}{2}\ BD \times CE$ **(M1)**
   $(CE =)\ 36.3$ (m) $\left(36.3251\ldots \text{(m)}\right)$ **(A1)(ft)(G2)**

Note: Follow through from parts (a) and (c). Award **(M0)(A0)** for halving $110°$ and/or
            assuming E is the midpoint of BD in any method seen.

[2 marks]

(e) $\sqrt{100^2 - 36.3251\ldots^2} + 100 + 36.3251\ldots$ **(M1)(M1)**

Note: Award **(M1)** for correct use of Pythagoras to find DE (or correct trigonometric equation,
          $100 \times \cos 21.3$, to find DE), **(M1)** for the sum of 100, their DE and their CE.

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        229 (m) $\left(229.494\ldots \text{(m)}\right)$ **(A1)(ft)(G2)**

Note: Follow through from part (d). Use of 3 sf values gives an answer of 230 (m) (229.5 (m)).

[3 marks]

Total [15 marks]

5. (a) $\dfrac{200-2x}{2}$ (or equivalent) **(M1)**

OR

$2x+2y=200$ (or equivalent) **(M1)**

Note: Award **(M1)** for a correct expression leading to $100-x$ (the $100-x$ does not need to be seen). The 200 must be seen for the **(M1)** to be awarded. Do not accept $100-x$ substituted in the perimeter of the rectangle formula.

$100-x$ **(AG)**

[1 mark]

(b) (area =) $x(100-x)$ **OR** $-x^2+100x$ (or equivalent) **(A1)**

[1 mark]

(c) $x=\dfrac{-100}{-2}$ **OR** $-2x+100=0$ **OR** graphical method **(M1)**

Note: Award **(M1)** for use of axis of symmetry formula or first derivative equated to zero or a sketch graph.

$x=50$ **(A1)(ft)(G2)**

Note: Follow through from part (b), provided x is positive and less than 100.

[2 marks]

(d) $50(100-50)\times2$ **(M1)(M1)**

Note: Award **(M1)** for substituting their x into their formula for area (accept "50×50" for the substituted formula), and **(M1)** for multiplying by 2. Award at most **(M0)(M1)** if their calculation does not lead to 5000 (BGN), although the 5000 (BGN) does not need to be seen explicitly.
Substitution of 50 into area formula may be seen in part (c).

5000 (BGN) **(AG)**

[2 marks]

continued…

Question 5 continued

(e) (i) $5000\left(1+\dfrac{4}{2\times100}\right)^{2\times6}$ **(M1)(A1)**

> **Note:** Award **(M1)** for substitution into compound interest formula,
> **(A1)** for correct substitution.

OR

$N = 6$
$I\% = 4$
$PV = -5000$
$P/Y = 1$
$C/Y = 2$ **(M1)(A1)**

> **Note:** Award **(A1)** for $C/Y = 2$ seen, **(M1)** for other correct entries.

OR

$N = 12$
$I\% = 4$
$PV = -5000$
$P/Y = 2$
$C/Y = 2$ **(M1)(A1)**

> **Note:** Award **(A1)** for $C/Y = 2$ seen, **(M1)** for other correct entries.

6341.21 (BGN)   **(A1)(G3)**

> **Note:** Award **(A1)** for correct answer, to two decimal places only.
> Award **(G2)** for 6341.20 or a correct, unrounded final answer if
> no working is seen (6341.2089…).

continued…

Question 5 continued

(ii) $5000\left(1+\dfrac{4}{2\times100}\right)^{2\times t}=7000$ ***(M1)(A1)*(ft)**

> **Note:** Award ***(M1)*** for using the compound interest formula with a variable for time,
> ***(A1)*(ft)** for substituting the correct values and equating to 7000.
> Follow through for their "2" from part (e)(i).

OR

$I\% = 4$
$PV = (\pm)\,5000$
$FV = \mp\,7000$
$P/Y = 1$
$C/Y = 2$ ***(M1)(A1)***

> **Note:** Award ***(A1)*** for 7000 seen, ***(M1)*** for the other correct entries.
> Award ***(M1)*** for their C/Y from part (e)(i).

OR

$I\% = 4$
$PV = (\pm)\,5000$
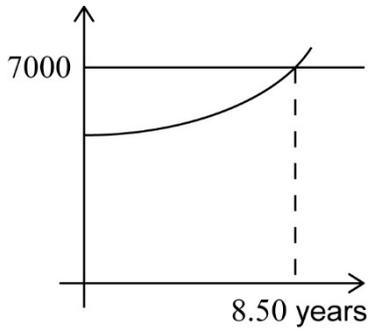$FV = \mp\,7000$
$P/Y = 2$
$C/Y = 2$ ***(M1)(A1)***

> **Note:** Award ***(A1)*** for 7000 seen, ***(M1)*** for the other correct entries.
> Award ***(M1)*** for their C/Y from part (e)(i).

continued…

Question 5 (e)(ii) continued

OR

8.50 years

(M1)(A1)(ft)

> **Note:** Award **(M1)** for a sketch with a straight line intercepted by appropriate curve,
> **(A1)(ft)** for numerical answer in the range of 8.4 and 8.5.
> Follow through from their part (e)(i).

$t = 8.50$ (years) $(8.49564\ldots)$ **(A1)(ft)(G3)**

> **Note:** Award only **(A1)** if $16.9912\ldots$ is seen without working. If working is seen, award at
> most **(M1)(A1)(A0)**.

[6 marks]

(f) $5000\left(1+\dfrac{r}{100}\right)^{12}=10000$ **(M1)**

> **Note:** Award **(M1)** for correct substitution into compound interest formula with $10\,000$ seen.

OR

$2=\left(1+\dfrac{r}{100}\right)^{12}$ **(M1)**

> **Note:** Award **(M1)** for correct substitution and simplification of compound interest formula,
> equating to 2.

$r = 5.95$ (%) $(5.94630\ldots)$ **(A1)(G2)**

[2 marks]

Total [14 marks]

6.   (a)   5   *(A1)*

> **Note:** Accept an answer of $(0, 5)$.

[1 mark]

(b) $\left(f'(x) = \right) - 4x^3 + 2ax$ *(A1)(A1)*

> **Note:** Award *(A1)* for $-4x^3$ and *(A1)* for $+2ax$. Award at most *(A1)(A0)* if extra terms are seen.

[2 marks]

(c)   (i) $-4 \times 2^3 + 2a \times 2 = 0$ *(M1)(M1)*

> **Note:** Award *(M1)* for substitution of $x = 2$ into their derivative, *(M1)* for equating their derivative, written in terms of a, to 0 leading to a correct answer (note, the 8 does not need to be seen).

  $a = 8$ *(AG)*

(ii) $\left(f(2) = \right) - 2^4 + 8 \times 2^2 + 5$ *(M1)*

> **Note:** Award *(M1)* for correct substitution of $x = 2$ and $a = 8$ into the formula of the function.

  21   *(A1)(G2)*

[4 marks]

(d)   (i) $(x =) - 2, \ (x =) 0$ *(A1)(A1)*

> **Note:** Award *(A1)* for each correct solution. Award at most *(A0)(A1)(ft)* if answers are given as $(-2, 21)$ and $(0, 5)$ or $(-2, 0)$ and $(0, 0)$.

(ii) $x < -2, \ 0 < x < 2$ *(A1)(ft)(A1)(ft)*

> **Note:** Award *(A1)(ft)* for $x < -2$, follow through from part (d)(i) provided their value is negative.
> Award *(A1)(ft)* for $0 < x < 2$, follow through only from their 0 from part (d)(i); 2 must be the upper limit.
> Accept interval notation.

[4 marks]

Question 6 continued

(e) $y \leq 21$ *(A1)(ft)(A1)*

> **Notes:** Award *(A1)(ft)* for 21 seen in an interval or an inequality, *(A1)* for " $y \leq$ ".
> Accept interval notation.
> Accept $-\infty < y \leq 21$ or $f(x) \leq 21$.
> Follow through from their answer to part (c)(ii). Award at most *(A1)(ft)(A0)* if x is seen instead of y. Do not award the second *(A1)* if a (finite) lower limit is seen.

[2 marks]

(f) 3 (solutions)   *(A1)*

[1 mark]

(g) $5 < m < 21$ or equivalent   *(A1)(ft)(A1)*

> **Note:** Award *(A1)(ft)* for 5 and 21 seen in an interval or an inequality, *(A1)* for correct strict inequalities. Follow through from their answers to parts (a) and (c)(ii).
> Accept interval notation.

[2 marks]

Total [16 marks]