1. (7 points)   Consider the curve given by the equation:

$$(x^2 + y^2 - 1)^3 - x^2 y^3 = 0$$

(a) Find $\frac{dy}{dx}$.

(b) Calculate the equation of tangent line to the curve at the point where $y = 1$ and $x > 0$.

2. (10 points) Consider the function

$$f(x) = xe^{kx}$$

where $k \in \mathbb{R}^+$

(a) Find the first four derivatives of f.

(b) Conjecture the formula for the n-th derivative of f.

(c) Prove your conjecture using mathematical induction.

3. (6 points) Find the equations of the tangent and the normal to the curve

$$\ln \left(x^2 - y + 1\right) = 8x - y^2$$

at the point $(2, 4)$.

4. (6 points) The table below shows some values of two functions, f and g, and of their derivatives f' and g'.

x	1	2	3	4
$f(x)$	2	4	1	3
$g(x)$	1	4	2	3
$f'(x)$	2	2	1	1
$g'(x)$	4	3	1	2

eg. $f(1) = 2$ and $g'(4) = 2$.

Calculate:

(a) $(f(x) + g(x))'$ when $x = 2$,

(b) $(f(x) \cdot g(x))'$ when $x = 1$,

(c) $(f(g(x)))'$ when $x = 3$.

5. (7 points) Given that
$$x^n + y^n = 1$$

show that
$$\frac{d^2y}{dx^2} = -\frac{(n-1)x^{n-2}}{y^{2n-1}}$$

6. (6 points) A champagne glass is in the shape of an inverted cone of depth 9cm and radius 3cm. Champagne is poured into the glass at the rate of $2\pi \ cm^3 s^{-1}$. Find the rate at which the depth of the champagne in the glass is increasing 4 seconds after pouring has commenced.

7. (8 points) The diagram below shows a rope running through a pulley at P and bearing a weight W at one end. The other end is held $5ft$ above the ground in the hand M of a worker. Suppose the pulley is $25ft$ above ground, the rope is $45ft$ long, and the worker is walking rapidly away from the vertical line PW at the rate of $6ft/sec$. How fast is the weight being raised when the worker's hand is $21ft$ away from PW?
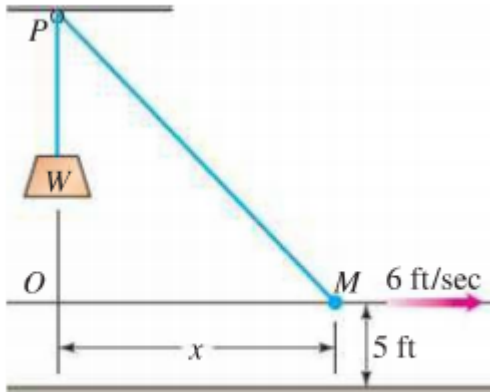
8. (7 points) Consider the function given by the formula $f(x) = x^3 - 4x$

   (a) Sketch the graph of $y = f(x)$, clearly indicating the axes intercepts and local maximum and minimum points.

   Now consider the curve given by the equation $y^2 = x^3 - 4x$.

   (b) State the value of x for which there is a value of y, for which the point (x, y) lies on that curve.

   (c) Find the $\frac{dy}{dx}$

   (d) Find the coordinates of the points where the tangent to the curve is horizontal.

   (e) [**bonus question**] Sketch the graph of the curve.

9. (7 points) Consider the sequence a_n given by the formula $a_n = 4^{b_n}$, where b_n is another sequence. Given that a_n is a geometric sequence with the common ratio equal to 16, and given that $b_1 + b_2 + b_3 + ... + b_{20} = 480$, find the value of b_1.