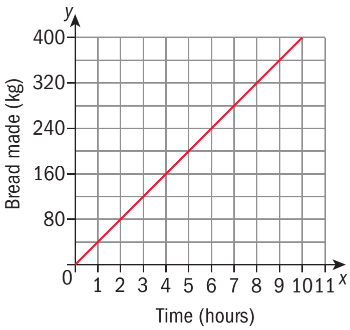
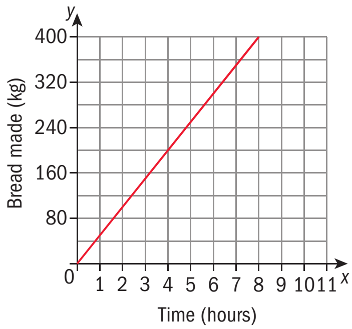
3.1 Gradient of a line

A bread factory has two bread-making machines, A and B.
Both machines make 400 kg of bread per day **at a constant rate**.
Machine A makes 400 kg in 10 hours.
Machine B makes the 400 kg in 8 hours.
For each machine, these graphs show the number of kilograms of bread made, y, in x hours. For example, in 2 hours machine A makes 80 kilograms of bread and machine B makes 100 kilograms of bread.

Machine A



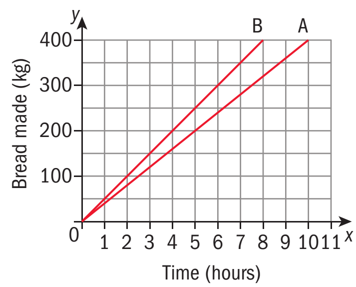
Machine B



This graph shows that machine A makes **40** kg of bread per hour.

This graph shows that machine B makes **50** kg of bread per hour.

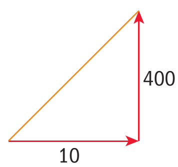
The next graph shows the number of kilograms of bread made by both machines.



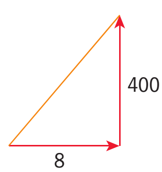
The line for machine B is steeper than the line for machine A. The **gradient** of a line tells you how steep it is. The gradient of line B is greater than the gradient of line A.

$$\text{The gradient of a line} = \frac{\text{vertical step}}{\text{horizontal step}}$$

$$\text{Gradient of line A} = \frac{\text{vertical step}}{\text{horizontal step}} = \frac{400}{10} = 40$$

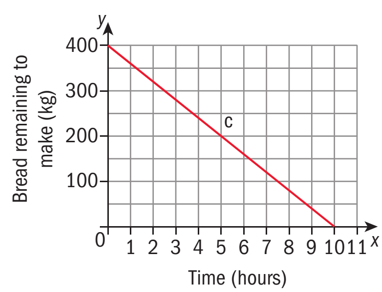


$$\text{Gradient of line B} = \frac{\text{vertical step}}{\text{horizontal step}} = \frac{400}{8} = 50$$



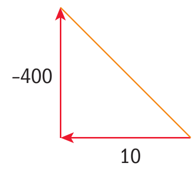
The gradient tells you the rate at which the machine is working:
A's rate = 40 kg per hour and
B's rate = 50 kg per hour

This graph shows the number of kilograms of bread still to be made by machine A. At the beginning of the day the machine has 400 kg to make, after 1 hour the machine has 360 kg to make, and so on.



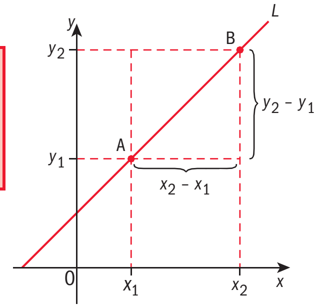
Line C has a negative gradient; it slopes downwards from left to right.

$$\text{Gradient of line C} = \frac{\text{vertical step}}{\text{horizontal step}}$$
$$= \frac{-400}{10}$$
$$= -40$$



Each hour there is 40 kg less bread to be made.

→ If A(x_1, y_1) and B(x_2, y_2) are two points that lie on line L, the gradient of L is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$



Note that the suffix order 2, then 1 in the gradient formula is the same in both the numerator and denominator.

Example 1

Find the gradient of the line L that passes through the points

a $A(1, 5)$ and $B(2, 8)$

b $A(0, 4)$ and $B(3, –2)$

c $A(2, 6)$ and $B(–1, 6)$

d $A(1, 5)$ and $B(1, –2)$

Answers

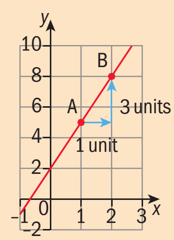
a $\left.\begin{array}{l} x_1 = 1 \\ y_1 = 5 \\ x_2 = 2 \\ y_2 = 8 \end{array}\right\} \Rightarrow m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{8-5}{2-1} = 3$

Substitute into the gradient formula.

Gradient = 3

For each 1 unit that x increases, y increases 3 units.



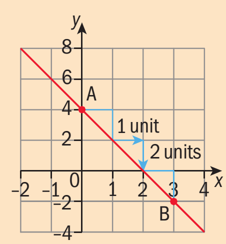
b $\left.\begin{array}{l} x_1 = 0 \\ y_2 = 4 \\ x_2 = 3 \\ y_2 = -2 \end{array}\right\} \Rightarrow m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{-2-4}{3-0} = -2$

Substitute into the gradient formula.

Gradient = −2

For each 1 unit that x ***increases****, y* ***decreases*** *by 2 units.*



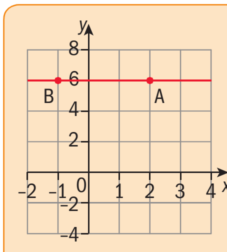
c $\left.\begin{array}{l} x_1 = 2 \\ y_1 = 6 \\ x_2 = -1 \\ y_2 = 6 \end{array}\right\} \Rightarrow m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{6-6}{-1-2} = 0$

Substitute into the gradient formula.

Gradient = 0

For each 1 unit that x ***increases****, y* ***remains constant****. The line is horizontal.*



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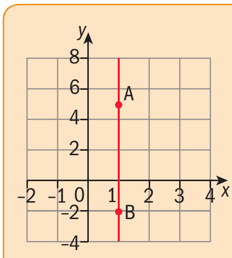
d
$$\left.\begin{aligned} x_1 &= 1 \\ y_1 &= 5 \\ x_2 &= 1 \\ y_2 &= -2 \end{aligned}\right\} \Rightarrow m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-2-5}{1-1} = \frac{-7}{0}$$

Substitute into the gradient formula.

Remember *that division by zero is not defined therefore* ***the gradient of this line is not defined****. The line is vertical.*



Exercise 3A

1 Plot the points A(2, 7), B(0, 9), C(0, −9) and D(2, −7) on a graph. Find the gradients of the lines

a AB **b** AC

c BD **d** CD

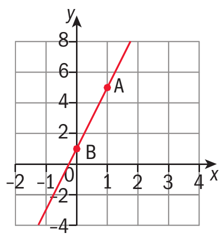
2 For each of these lines

i write down the coordinates of the points A and B

ii find the gradient.

The scales on the *x*-axis and *y*-axis are not always the same.

a



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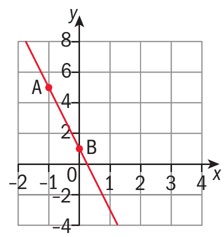

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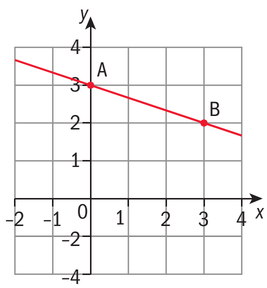

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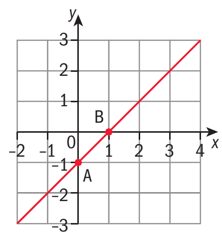

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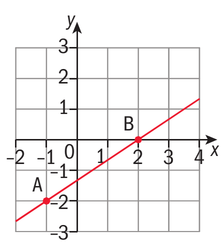

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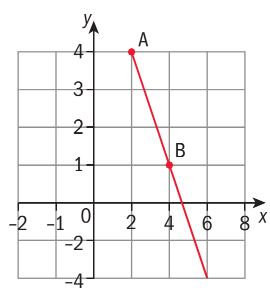


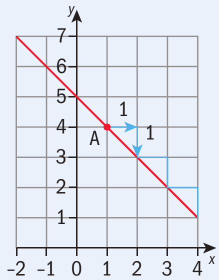
Example 2

a Draw a line that passes through the point A(1, 4) with gradient –1.

b Draw a line that passes through the point A(0, –2) with gradient $\frac{2}{3}$.

Answers

a



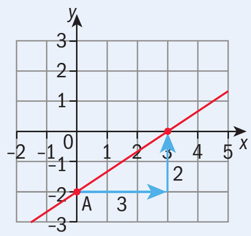
Plot the point A(1, 4).

The gradient is –1 so,

$$m = -1 = \frac{-1}{1} = \frac{y\text{-step}}{x\text{-step}}$$

so every time that x increases by 1 unit, y decreases by 1 unit.

b



Plot the point A(0, –2).

The gradient is $\frac{2}{3}$ *so,*

$$m = \frac{2}{3} = \frac{y\text{-step}}{x\text{-step}}$$

so every time that x increases by 3 units, y increases by 2 units.

Road gradients are often given as percentages or ratios. How do road signs show gradient in your country?

Exercise 3B

1 **a** Draw a line with gradient $\frac{1}{2}$ that passes through the point A(0, 3).

b Draw a line with gradient –3 that passes through the point B(1, 2).

c Draw a line with gradient 2 that passes through the point C(3, –1).

2 For each of these lines, points A, B and C lie on the same line.

i find the gradient of line AB.

ii find the second coordinate of point C:

a $A(2, 5)$, $B(3, 7)$ and $C(4, p)$

b $A(0, 2)$, $B(1, 6)$ and $C(2, t)$

c $A(0, 0)$, $B(1, -5)$ and $C(2, q)$

d $A(0, -1)$, $B(1, 0)$ and $C(4, s)$

e $A(-5, 1)$, $B(-6, 4)$ and $C(-4, r)$

You may use a graph or the gradient formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.

EXAM-STYLE QUESTIONS

3 The gradient of the line passing through points P(–1, 5) and Q(a, 10) is 4.

a Write down an expression in terms of a for the gradient of PQ.

b Find the value of a.

4 In line MN, every time that x increases by 1 unit, y increases by 0.5 units. Point M is (2, 6) and point N is (–3, t).

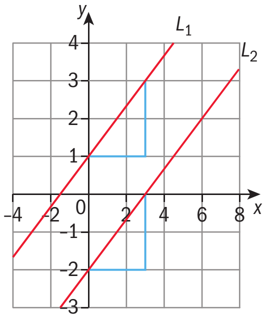
a Write down the gradient of MN.

b Write down an expression for the gradient of MN in terms of t.

c Find the value of t.

Parallel lines

→ **Parallel lines** have the **same gradient**. This means that
- if two lines are parallel then they have the same gradient
- if two lines have the same gradient then they are parallel.



The symbols, $L_1 \parallel L_2$ mean 'L_1 is parallel to L_2'.

Note that, although the gradient of a vertical line is not defined, two vertical lines are parallel.

Example 3

Line L_1 passes through the points A(0, –3) and B(–7, 4).

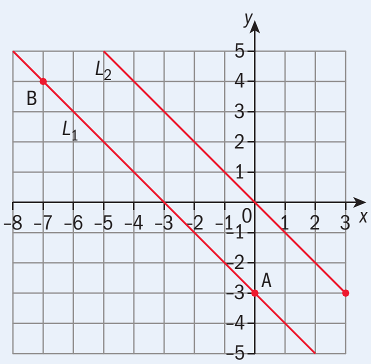
a Find the gradient of L_1. **b** Draw and label L_1.

c Draw and label a second line L_2 passing through the origin and parallel to L_1.

Answers

a $m = \dfrac{4-(-3)}{-7-0} = -1$ *Substitute into the gradient formula.*

b and **c** *For L_1, plot A and B and join them. For L_2, draw a line through the origin parallel to L_1.*



Remember that the ***origin*** is the point O(0, 0), the point where the *x*-axis and the *y*-axis meet.

Exercise 3C

1 Line L_1 passes through the points A(2, 5) and B(0, –4).

a Find the gradient of L_1. **b** Draw and label L_1.

c Draw and label a second line L_2 passing through the point C(0, 2) and parallel to L_1.

2 Decide whether each line is parallel to the *y*-axis, the *x*-axis or neither:

a the line passing through the points P(1, 7) and Q(12, 7)

b the line passing through the points P(1, 7) and T(1, –3)

c the line passing through the points P(1, 7) and M(2, 5).

3 Complete these statements to make them true.

a Any horizontal line is parallel to the ______-axis.

b Any vertical line is parallel to the ______-axis.

c Any horizontal line has gradient equal to ______.

4 PQ is parallel to the x-axis. The coordinates of P and Q are $(5, 3)$ and $(8, a)$ respectively. Write down the value of a.

5 MN is parallel to the y-axis. The coordinates of M and N are respectively $(m, 24)$ and $(-5, 2)$. Write down the value of m.

Perpendicular lines

→ Two lines are **perpendicular** if, and only if, they make an angle of 90°.
This means that
- if two lines are perpendicular then they make an angle of 90°
- if two lines make an angle of 90° then they are perpendicular.

The x-axis and the y-axis are perpendicular.

The next example shows you the **numerical relationship** between the gradients of two perpendicular lines that are not horizontal and vertical.

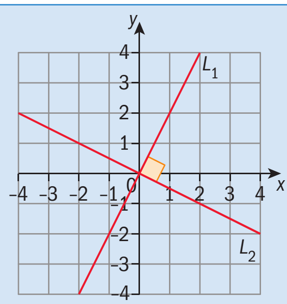
Any vertical line is perpendicular to any horizontal line.

Example 4

The diagram shows two perpendicular lines L_1 and L_2.

a Find the gradients of L_1 and L_2.

b Show that the product of their gradients is equal to -1.

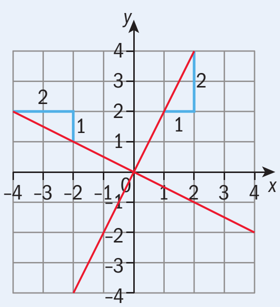


Answers

a Let m_1 be the gradient of L_1 and m_2 the gradient of L_2.

$m_1 = 2$ and $m_2 = -\frac{1}{2}$

Use the diagram to find m_1 and m_2.



b $2 \times -\frac{1}{2} = -1$

Note that the gradient of L_1 is positive and the gradient of L_2 is negative.

In general, if the gradient of a line is m, the gradient of a perpendicular line is $-\frac{1}{m}$

→ Two lines are **perpendicular** if the product of their gradients is -1.

a and b are reciprocal if $a \times b = 1$ or $a = \frac{1}{b}$
For example:
2 and $\frac{1}{2}$, $\frac{4}{3}$ and $\frac{3}{4}$

Exercise 3D

1 Which of these pairs of numbers are negative reciprocals?

a 2 and $-\frac{1}{2}$ **b** $-\frac{4}{3}$ and $\frac{3}{4}$ **c** 3 and $\frac{1}{3}$ **d** -1 and 1

2 Which of these pairs of gradients are of perpendicular lines?

a $\frac{2}{5}$ and $\frac{5}{2}$ **b** $\frac{4}{3}$ and $-\frac{3}{4}$ **c** -3 and $-\frac{1}{3}$ **d** 1 and -1

3 Find the gradient of lines that are perpendicular to a line with gradient

a -3 **b** $\frac{2}{3}$ **c** $-\frac{1}{4}$ **d** 1 **e** -1

4 Find the gradient of any line perpendicular to the line passing through the points

a A$(-2, 6)$ and B$(1, -1)$ **b** A$(5, 10)$ and B$(0, -2)$

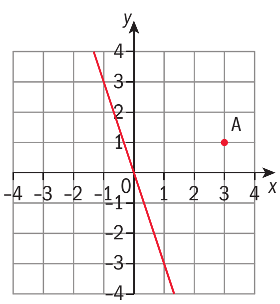
5 Each diagram shows a line and a point A.

i Write down the gradient of the line.

ii Write down the gradient of any line that is perpendicular to this line.

iii Copy the diagram and draw a line perpendicular to the red line passing through the point A.

a



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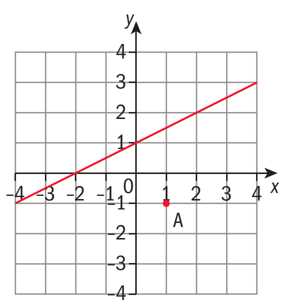

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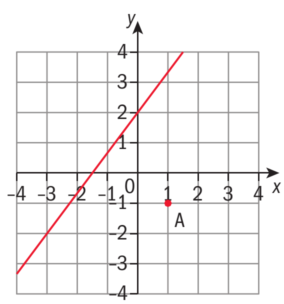


EXAM-STYLE QUESTIONS

6 Line L_1 passes through the points P$(0, 3)$ and Q$(-2, a)$.

a Find an expression for the gradient of L_1 in terms of a.

L_1 is perpendicular to line L_2. The gradient of L_2 is 2.

b Write down the gradient of L_1.

c Find the value of a.

7 The points A$(3, 5)$ and B$(5, -8)$ lie on the line L_1.

a Find the gradient of L_1.

A second line, L_2, is perpendicular to L_1.

b Write down the gradient of L_2.

L_2 passes through the points P$(5, 0)$ and Q$(t, 2)$.

c Find the value of t.

3.2 Equations of lines

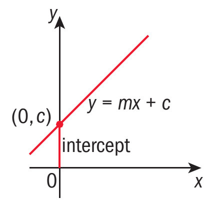
The coordinates x and y of **any** point on a line L are linked by an equation, called the **equation of the line**.

This means that:

- If a point Q lies on a line L then the coordinates of Q satisfy the equation of L.
- If the coordinates of any point Q satisfy the equation of a line L, then the point Q lies on L.

Values for variables x and y are said to **satisfy** an equation if, when the variables are replaced by the respective values, the two sides of the equation are equal.

→ The equation of a straight line can be written in the form $y = mx + c$, where
- m is the **gradient**
- c is the **y-intercept** (y-coordinate of the point where the line crosses the y-axis).



$y = mx + c$ is the **gradient-intercept** form of the straight line equation.

The equation $y = mx + c$ is in the Formula booklet. You will revisit this equation again in Chapter 4.

As well as $y = mx + c$, some people express the equation of a line as $y = ax + b$ or $y = mx + b$

Example 5

The line L passes through the point A(1, 7) and has gradient 5.
Find the equation of L.
Give your answer in the form $y = mx + c$.

Answer

Let P(x, y) be **any** point on L.	
The gradient of L is 5	
$\left.\begin{array}{l} x_1 = 1 \\ y_1 = 7 \\ x_2 = x \\ y_2 = y \end{array}\right\} \Rightarrow \frac{y-7}{x-1} = 5$	*Use the gradient formula with A and P, and equate to 5.*
$y - 7 = 5(x - 1)$	*Multiply both sides by $(x - 1)$*
$y - 7 = 5x - 5$	*Expand brackets.*
$y = 5x + 2$	*Add 7 to both sides.* *$y = mx + c$ where $m = 5$ and $c = 2$*
Use A(7, 1) to check: $7 = 5 \times 1 + 2$	**Check that** • *the coordinates of the point A(1, 7) satisfy the equation of the line.*

Note that in the equation $y = 5x + 2$
- 5 multiplies x, and the gradient of the line is $m = 5$
- Putting $x = 0$ in the equation of L, $y = 5 \times 0 + 2 = 2$

Therefore the point (0, 2) lies on L.

Example 6

The line L has gradient $\frac{1}{3}$ and passes through A $(2, -1)$.

a Find the equation of L. Give your answer in the form $y = mx + c$.

b Write down the point of intersection of L with the y-axis.

c Find the point of intersection of L with the x-axis.

d Draw the line L showing clearly the information found in **b** and **c**.

Answers

a $y = \frac{1}{3}x + c$

Substitute $m = \frac{1}{3}$ in the equation $y = mx + c$.

$-1 = \frac{1}{3} \times 2 + c$

Substitute the coordinates of point A(2,–1) in the equation of the line.

$-1 = \frac{2}{3} + c$

$c = -\frac{5}{3}$

Make c the subject of the equation.

$y = \frac{1}{3}x - \frac{5}{3}$

Substitute c in the equation of the line.

b $\left(0, -\frac{5}{3}\right)$

The line crosses the y-axis at the point (0, c).

c $0 = \frac{1}{3}x - \frac{5}{3}$

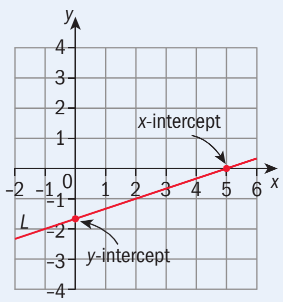
Any point on the x-axis has the form (k, 0).
Substitute y = 0 in the equation of L.

$\frac{1}{3}x = \frac{5}{3}$

$x = 5$

Therefore L intersects the x-axis at the point (5, 0).

d



Note that you could find the equation of L using the same method as in Example 5.

Exercise 3E

1 Find the equation of a line with

a gradient 3 that passes through the point A $(1, 4)$

b gradient $\frac{5}{3}$ that passes through the point A $(4, 8)$

c gradient –2 that passes through the point A $(-3, 0)$

Give your answers in the form $y = mx + c$.

2 For each of these lines write down

i the gradient

ii the point of intersection with the y-axis

iii the point of intersection with the x-axis.

a $y = 2x + 1$ b $y = -3x + 2$ c $y = -x + 3$ d $y = -\frac{2}{5}x - 1$

EXAM-STYLE QUESTIONS

3 A line has equation $y = \frac{3(x-6)}{2}$.

a Write the equation in the form $y = mx + c$.

b Write down the gradient of the line.

c Write down the y-intercept.

d Find the point of intersection of the line with the x-axis.

4 The line AB joins the points A(2, –4) and B(1, 1).

a Find the gradient of AB.

b Find the equation of AB in the form $y = mx + c$

5 The line PQ joins the points P(1, 3) and Q(2, 5).

a Find the gradient of PQ.

b Find the equation of PQ in the form $y = mx + c$

c Find the gradient of all lines perpendicular to PQ.

d Find the equation of a line perpendicular to PQ that passes through A(0, 2).

6 Line L_1 has gradient 3 and is perpendicular to line L_2.

a Write down the gradient of L_2.

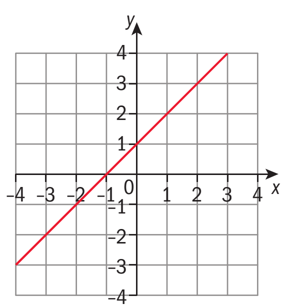
Line L_2 passes through the point P(5, 1).

b Find the equation of L_2. Give your answer in the form $y = mx + c$

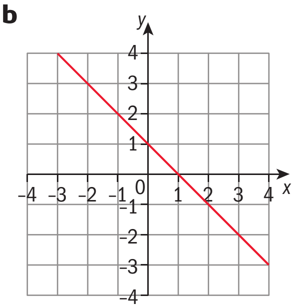
c Find the x-coordinate of the point where L_2 meets the x-axis.

7 Find the equations of these lines, in the form $y = mx + c$

a



b



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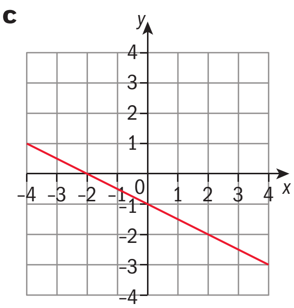

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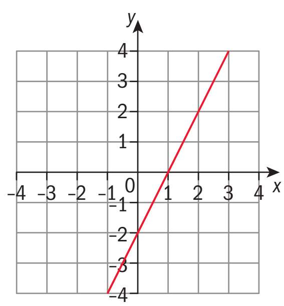

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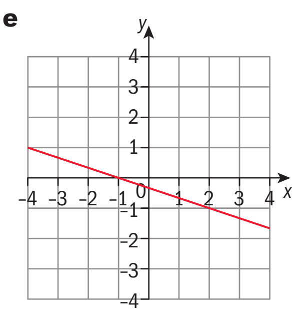

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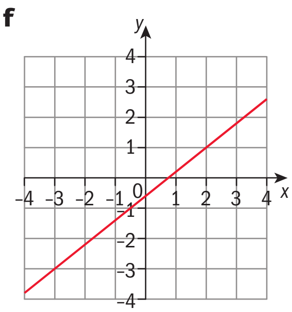


Example 7

a Line L joins the points A(–3, 5) and B(1, 2).
Find the equation of line L.
Give your answer in the form $ax + by + c = 0$ where $a, b, c \in \mathbb{Z}$

b The point $Q\left(\frac{5}{3}, t\right)$ lies on L. Find the value of t.

> The equation $ax + by + d = 0$ is called **the general form** and is also in the Formula booklet.

Answers

a The gradient of L is

$$m = \frac{2-5}{1-(-3)} = -\frac{3}{4}$$

Use the gradient formula with the coordinates of A and B.

Let P(x, y) be **any** point on L.
The gradient of L is also

Use the gradient formula with A and P (or B and P).

$$\left.\begin{array}{l} x_1 = -3 \\ y_1 = 5 \\ x_2 = x \\ y_2 = y \end{array}\right\} \Rightarrow m = \frac{y-5}{x-(-3)}$$

$$\frac{y-5}{x-(-3)} = -\frac{3}{4}$$

Equate gradients.

$$4(y-5) = -3(x+3)$$

Cross multiply.
$\frac{a}{b} = \frac{c}{d} \Leftrightarrow a \times d = b \times c$

$$4y - 20 = -3x - 9$$
$$3x + 4y - 11 = 0$$

Expand brackets.
Rearrange equation to form
$ax + by + d = 0$
$a = 3$, $b = 4$, $d = -11$
***Check that** both points A and B satisfy the equation of the line.*

> Note that any multiple of this equation would also be correct as long as $a, b, d \in \mathbb{Z}$, e.g. $-3x - 4y + 11 = 0$ or $6x + 8y - 22 = 0$

b The point $Q\left(\frac{5}{3}, t\right)$ lies on L
so its coordinates must satisfy the equation of L.

$$3x + 4y - 11 = 0$$
$$3 \times \frac{5}{3} + 4 \times t - 11 = 0$$

Substitute the coordinates of Q in the equation of L.

$$5 + 4t - 11 = 0$$
$$4t - 6 = 0$$
$$4t = 6$$
$$t = 1.5$$

Solve for t.

> **Discuss**: How many points do we need to determine a line?
> **Investigate:** the meaning of the word ‘collinear’. When do we say that three or more points are collinear?

→ The equation of a straight line can be written in the form
$$ax + by + c = 0$$
where a, b and $c \in \mathbb{Z}$.

Exercise 3F

1 Find the equations of these lines. Give your answers in the form $ax + by + c = 0$ where $a, b, c \in \mathbb{Z}$.

a A line with gradient -4 that passes through the point A(5, 0).

b A line with gradient $\frac{1}{2}$ that passes through the point A(2, 3).

c The line joining the points A(3, –2) and B(–1, 3).

d The line joining the points A(0, 5) and B(–5, 0).

2 Rewrite each of these equations in the form $y = mx + c$.

Make y the subject of the formula

a $3x + y = 0$ **b** $x + y + 1 = 0$ **c** $2x + y - 1 = 0$

d $2x - 4y = 0$ **e** $6x + 3y - 9 = 0$

3 The line L has equation $3x - 6y + 6 = 0$.

a Write down the equation of L in the form $y = mx + c$.

b Write down the x-intercept.

c Write down the y-intercept.

4 The equation of a line is $y = 2x - 6$

a Which of these points lie on this line?

A(3, 0), B(0, 3), C(1, –4), D(4, 2), E(10, 12), F(5, 4)

b The point (a, 7) lies on this line. Find the value of a.

c The point (7, t) lies on this line. Find the value of t.

5 The equation of a line is $-6x + 2y - 2 = 0$

a Which of these points lie on this line?

A(1, 4), B(0, 1), C(1, 0), D(2, 6), E($-\frac{1}{3}$, 0), F(–1, 2)

b The point (a, 3) lies on this line. Find the value of a.

c The point (10, t) lies on this line. Find the value of t.

6 The table has four equations and four pairs of conditions. Match each equation with the pair of conditions that satisfies that line.

Equation		Conditions	
A	$6x - 3y + 15 = 0$	E	The x-intercept is 2.5 and the y-intercept is 5
B	$y = 2x - 5$	F	The gradient is –2 and the line passes through the point (1, –7)
C	$10x + 5y + 25 = 0$	G	The line passes through the points (0, –5) and (2.5, 0)
D	$y = -2x + 5$	H	The y-intercept is (0, 5) and the gradient is 2

EXAM-STYLE QUESTION

7 The line L_1 has equation $2x - y + 6 = 0$

a Write down the gradient of L_1.

b Write down the y-intercept of L_1.

c The point A(c, 1.5) lies on L_1. Find the value of c.

d The point B(5, t) lies on L_1. Find the value of t.

Line L_2 is parallel to L_1.

e Write down the gradient of L_2.

f Find the equation of L_2 if it passes through C(0, 4).

EXAM-STYLE QUESTION

8 The line L_1 joins the points A(1, 2) and B(–1, 6).

a Find the equation of L_1.

C is the point (10, –16).

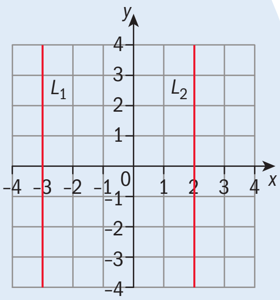
b Decide whether A, B and C are collinear, giving a reason for your answer.

Vertical and horizontal lines

Vertical lines are parallel to the y-axis.
Horizontal lines are parallel to the x-axis.

Investigation – vertical and horizontal lines

The diagram shows two **vertical** lines, L_1 and L_2.

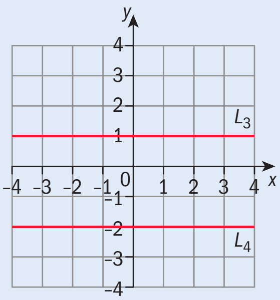


1 **a** Write down the coordinates of at least five points lying on L_1.
b What do you notice about the coordinates of the points from **a**? What do their coordinates have in common?
c What is the condition for a point to lie on L_1? Write down this condition in the form $x = k$ where k takes a particular value.

2 **a** Write down the coordinates of at least five points lying on L_2.
b What do you notice about the coordinates of the points from **a**? What do their coordinates have in common?
c What is the condition for a point to lie on L_2? Write down this condition in the form $x = k$ where k takes a particular value.

3 What is the equation of a vertical line passing through the point (1,–3)?

The diagram shows two **horizontal** lines, L_3 and L_4.



4 **a** Write down the coordinates of at least five points lying on L_3.
b What do you notice about the coordinates of the points from **a**? What do their coordinates have in common?
c What is the condition for a point to lie on L_3? Write down this condition in the form $y = k$ where k takes a particular value.

5 **a** Write down the coordinates of at least five points lying on L_4.
b What do you notice about the coordinates of the points from **a**? What do their coordinates have in common?
c What is the condition for a point to lie on L_4? Write down this condition in the form $y = k$ where k takes a particular value.

6 What is the equation of a horizontal line passing through the point (1, –3)?

→ • The equation of any vertical line is of the form $x = k$ where k is a constant.
• The equation of any horizontal line is of the form $y = k$ where k is a constant.

Intersection of lines in two dimensions

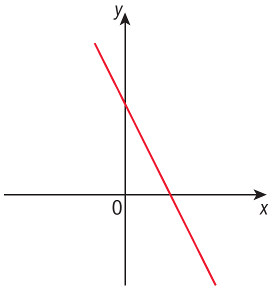
→ If two lines are parallel then they have the same gradient and do not intersect.

Parallel lines L_1 and L_2 can be:

- *Coincident lines (the same line)*

  e.g. $2x + y = 3$ and
  $6x + 3y = 9$

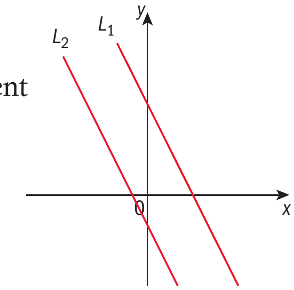
  $L_1 = L_2$ therefore they have the same gradient and the same y-intercept. There is an infinite number of points of intersection.



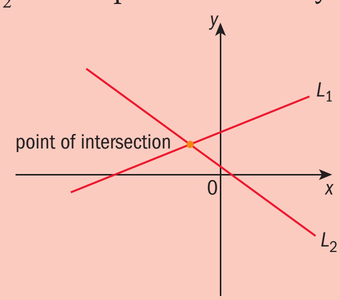
- *Different lines*

  e.g. $2x + y = 3$ and
  $2x + y = -1$

  L_1 and L_2 have the same gradient but different y-intercepts. There is no point of intersection.



→ If two lines L_1 and L_2 are not parallel then they intersect at just one point.



To find the point of intersection write $m_1x_1 + c_1 = m_2x_2 + c_2$ and solve for x.

Example 8

Find the point of intersection of the lines $y = 2x + 1$ and $-x - y + 4 = 0$.

Answer

Algebraically

$y = 2x + 1$ and $y = -x + 4$ — *Write both equations in the gradient–intercept form.*

$2x + 1 = -x + 4$ — *Equate expressions for y.*

$3x = 3$ — *Solve for x.*

$x = 1$

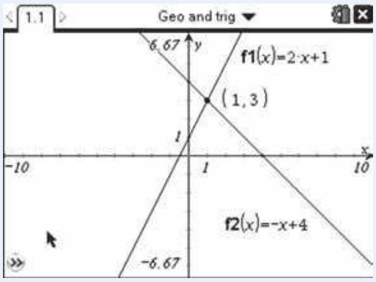
so $y = 2 \times 1 + 1$ — *Substitute for x in one of the equations to find y.*

$= 3$

The point of intersection is (1, 3).

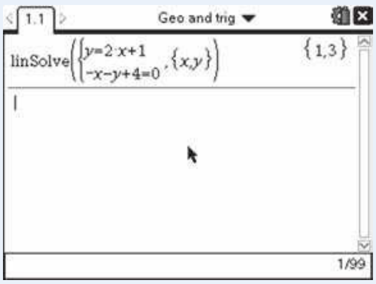
▶ Continued on next page

Using GDC Method 1



Rearrange both equations into the gradient–intercept form.

Using GDC Method 2



Solve the pair of simultaneous equations

$$\begin{cases} -2x + y = 1 \\ -x - y = -4 \end{cases}$$

For help with drawing graphs on your GDC, see Chapter 12, Section 3.4, Example 18.

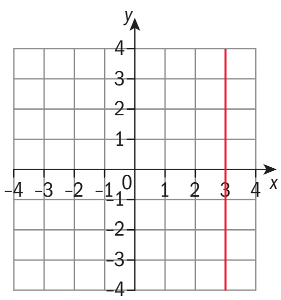
For help with solving simultaneous equations on your GDC, see Chapter 12, Section 1.1, Example 1.

GDC help on CD: *Alternative demonstrations for the TI-84 Plus and Casio FX-9860GII GDCs are on the CD.*

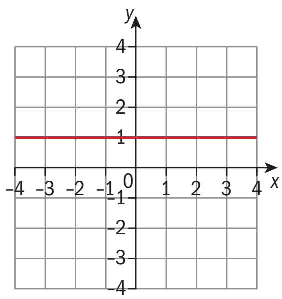
Exercise 3G

1 Write down the equations of these lines.

a



b



2 Find the point of intersection of each pair of lines.

a $y = 3x - 6$ and $y = -x + 2$

b $-x + 5y = 0$ and $\frac{1}{5}x + y - 2 = 0$

c $y = 3$ and $x = -7$

d $y = 1.5x + 4$ and $y = 1$

e $-x + 2y + 6 = 0$ and $x + y - 3 = 0$

f y-axis and $y = 4$

3 Show that the lines L_1 with equation $-5x + y + 1 = 0$ and L_2 with equation $10x - 2y + 4 = 0$ are parallel.

4 State, with reasons, whether each pair of lines meet at

i only one point **ii** an infinite number of points

iii no point.

a $y = 3(x - 5)$ and $x - \frac{1}{3}y + 6 = 0$

b $\frac{y+1}{x-2} = -1$ and $y = -x + 1$

c $y = 4x - 8$ and $4x - 2y = 0$

d $x - y + 3 = 0$ and $3x - 3y + 9 = 0$

EXAM-STYLE QUESTION

5 Line L_1 has gradient 5 and intersects line L_2 at the point A(1, 0).

a Find the equation of L_1.

Line L_2 is perpendicular to L_1.

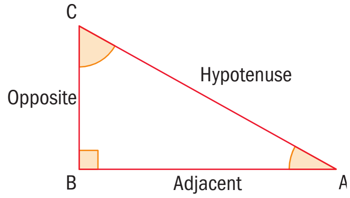
b Find the equation of L_2.

Point *A* lies on both lines.

3.3 The sine, cosine and tangent ratios

Some textbooks use 'right triangle' instead of right-angled triangle.

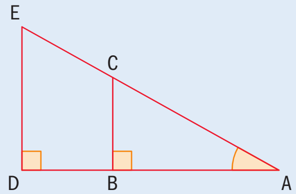
Trigonometry is the study of lengths and angles in triangles. This section looks at trigonometry in *right-angled* triangles. In a **right-angled triangle** the side opposite the right angle is the **hypotenuse**, which is the **longest side**.



- AC is the hypotenuse
- AB is adjacent to angle A (Â)
- BC is opposite Â

Investigation – right-angled triangles

Draw a diagram of two triangles like this.



1 Measure the angles at E and C. What do you notice?

2 Measure the lengths AB and AD. Calculate the ratio $\frac{AD}{AB}$

3 Measure the lengths AE and AC. Calculate the ratio $\frac{AE}{AC}$

4 Measure the lengths DE and BC. Calculate the ratio $\frac{DE}{BC}$

What do you notice about your answers to questions 2 to 4?

In the diagram the right-angled triangles ABC and ADE have the same angles, and corresponding sides are in the same ratio.

The ratios $\frac{AB}{AC}$, $\frac{BC}{AC}$ and $\frac{BC}{AB}$ in triangle ABC are respectively equal to the ratios $\frac{AD}{AE}$, $\frac{DE}{AE}$ and $\frac{DE}{AD}$ in triangle ADE.

Two triangles with the same angles and corresponding sides in the same ratio are called **similar triangles**.

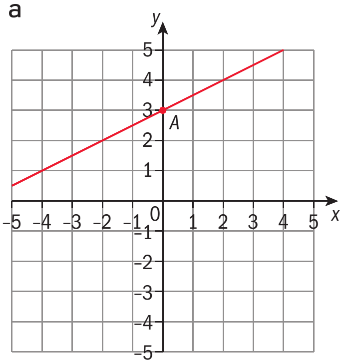
e i A(−1, −2), B(2, 0) ii $\frac{2}{3}$

f i A(2, 4), B(4, 1) ii $-\frac{3}{2}$

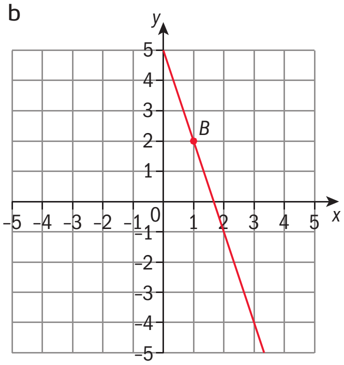
Exercise 3B

1

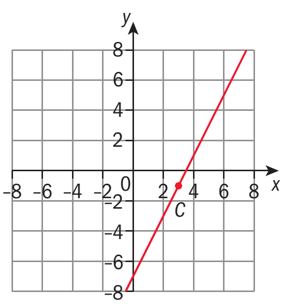
a



b



c



2 a i 2 ii $p = 9$

b i 4 ii $t = 10$

c i −5 ii $q = -10$

d i 1 ii $s = 3$

e i −3 ii $r = -2$

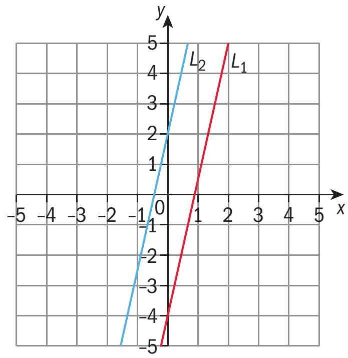
3 a $\frac{5}{a+1}$ b $a = \frac{1}{4}$

4 a 0.5 b $\frac{t-6}{-5}$ c $t = 3.5$

Exercise 3C

1 a 4.5

b & c



2 a Parallel to x-axis

b Parallel to y-axis c Neither

3 a x

b y c zero

4 $a = 3$ 5 $m = -5$

Exercise 3D

1 a, b, d

2 b, d

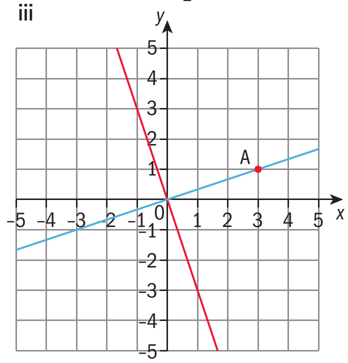
3 a $\frac{1}{3}$ b −1.5

c 4 d −1 e 1

4 a $\frac{3}{7}$ b $-\frac{5}{12}$

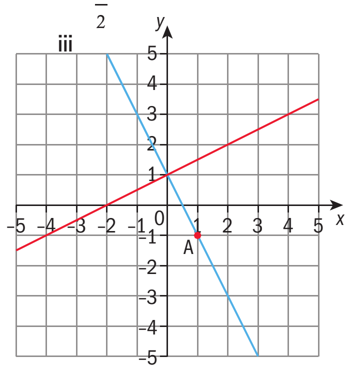
5 a i −3 ii $\frac{1}{2}$

iii



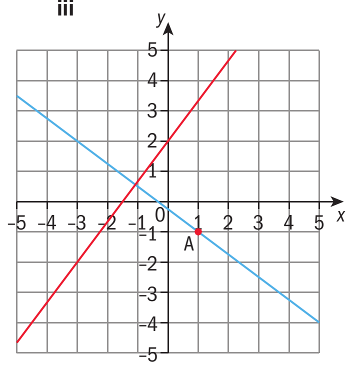
b i $\frac{1}{2}$ ii −2

iii



c i $\frac{4}{3}$ ii $-\frac{3}{4}$

iii



6 a $\frac{a-3}{-2}$ b $-\frac{1}{2}$ c $a = 4$

7 a $-\frac{13}{2}$ b $\frac{2}{13}$ c $t = 18$

Exercise 3E

1 a $y = 3x + 1$ b $y = \frac{5}{3}x + \frac{4}{3}$

c $y = -2x - 6$

2 a i 2 ii (0, 1)

iii $\left(-\frac{1}{2}, 0\right)$

b i −3 ii (0, 2)

iii $\left(\frac{2}{3}, 0\right)$

c i −1 ii (0, 3)

iii (3, 0)

d i $-\frac{2}{5}$ ii (0, −1)

iii (−2.5, 0)

3 a $y = 1.5x - 9$ b 1.5

c −9 d (6, 0)

4 a −5 b $y = -5x + 6$

5 a 2 b $y = 2x + 1$

c −0.5 d $y = -0.5x + 2$

6 a $-\frac{1}{3}$ b $y = -\frac{1}{3}x + \frac{8}{3}$

c 8

7 a $y = x + 1$ b $y = -x + 1$

c $y = -0.5x - 1$

d $y = 2x - 2$

e $y = \frac{1}{3}x - \frac{1}{3}$

f $y = \frac{4}{5}x - \frac{3}{5}$

Exercise 3F

1 a $4x + y - 20 = 0$

b $x - 2y + 4 = 0$

c $-5x - 4y + 7 = 0$

d $x - y + 5 = 0$

2 a $y = -3x$ b $y = -x - 1$
c $y = -2x + 1$ d $y = 0.5x$
e $y = -2x + 3$

3 a $y = 0.5x + 1$
b $x = -2$ c $y = 1$

4 a A, C, D, F
b $a = 6.5$ c $t = 8$

5 a A, B, E
b $a = \frac{2}{3}$ c $t = 31$

6

Line	Conditions
A	H
B	G
C	F
D	E

7 a 2 b 6 c $c = -2.25$
d $t = 16$ e 2
f $y = 2x + 4$ $(2x - y + 4 = 0)$

8 a $y = -2x + 4$
$(2x + y - 4 = 0)$
b Yes, A, B and C are collinear. The coordinates of A, B and C all satisfy the equation of L_1.

Investigation-vertical and horizontal lines

1 a $(-3, -1)$, $(-3, 0)$, $(-3, 1)$, $(-3, 2)$ and $(-3, 3)$
b All the coordinates have the x-coordinate as -3.
c To lie on L1, the x-coordinate must be -3. ie. $x = -3$

2 a $(2, -1)$, $(2, 0)$, $(2, 1)$, $(2, 2)$ and $(2, 3)$
b All the coordinates have the x-coordinate as 2
c To lie on L3, the x-coordinate must be 2. ie. $x = 2$

3 $x = 1$

4 a $(-1, 1)$, $(0, 1)$, $(1, 1)$, $(2, 1)$ and $(3, 1)$
b All the coordinates have the y-coordinate as 1
c To lie on L3, the y-coordinate must be 1. ie. $y = 1$

5 a $(-1, -2)$, $(0, -2)$, $(1, -2)$, $(2, -2)$ and $(3, -2)$
b All the coordinates have the y-coordinate as -2
c To lie on L4, the y-coordinate must be -2. ie. $y = -2$

6 $y = -3$

Exercise 3G

1 a $x = 3$ b $y = 1$

2 a $(2, 0)$ b $(5, 1)$ c $(-7, 3)$
d $(-2, 1)$ e $(4, -1)$ f $(0, 4)$

3 L_1: $y = 5x - 1$
L_2: $y = 5x + 2$
L_1 and L_2 have same gradient but different y-intercepts.

4 a no point
b an infinite number of points
c only one point
d infinite number of points

5 a $y = 5x - 5$ $(5x - y - 5 = 0)$
b $y = -\frac{1}{5}x + \frac{1}{5}$ $(x + 5y - 1 = 0)$

Investigation - right-angled triangles

1 Angles are identical

2 $\frac{AE}{AC} = 1.5$

3 $\frac{AD}{AB} = 1.5$

4 $\frac{DE}{BC} = 1.5$

All the ratios are identical.

Exercise 3H

1

H	Opp	Adj
XY	YZ	XZ
CB	AB	AC
RQ	PQ	PR

2 a $\cos\delta = \frac{AC}{AB}$
$\sin\delta = \frac{BC}{AB}$
$\tan\delta = \frac{BC}{AC}$

b $\cos\delta = \frac{QR}{PQ}$
$\sin\delta = \frac{PR}{PQ}$
$\tan\delta = \frac{PR}{QR}$

c $\cos\delta = \frac{EF}{DF}$
$\sin\delta = \frac{ED}{DF}$
$\tan\delta = \frac{ED}{EF}$

3 a i $\sin\alpha = \frac{4}{\sqrt{41}}$
ii $\cos\alpha = \frac{5}{\sqrt{41}}$
iii $\tan\alpha = \frac{4}{5}$

b i $\sin\alpha = \frac{\sqrt{28}}{8}$
ii $\cos\alpha = \frac{6}{8}$
iii $\tan\alpha = \frac{\sqrt{28}}{6}$

c i $\sin\alpha = \frac{10}{14}$
ii $\cos\alpha = \frac{\sqrt{96}}{14}$
iii $\tan\alpha = \frac{10}{\sqrt{96}}$

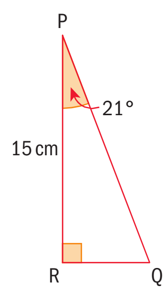
4 a $\sin\beta = \frac{x}{10}$
b $\cos\beta = \frac{x}{5}$
c $\tan\beta = \frac{x}{12}$
d $\tan\beta = \frac{7}{x}$
e $\sin\beta = \frac{14}{x}$
f $\cos\beta = \frac{3}{x}$

Exercise 3I

1 $h = 3.11$ cm 2 $x = 6.41$ cm
3 $m = 4.88$ cm 4 $y = 13.94$ cm
5 $t = 386.37$ m 6 $s = 86.60$ m

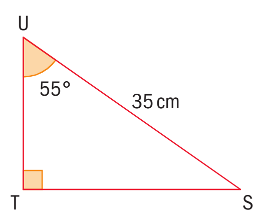
Exercise 3J

1 a



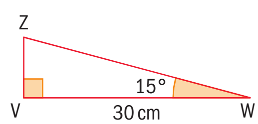
b $\hat{Q} = 69°$ c QR = 5.76 cm

2 a



b $\hat{S} = 35°$ c TU = 20.1 cm

3 a



b $\hat{Z} = 75°$ c VZ = 8.04 cm