

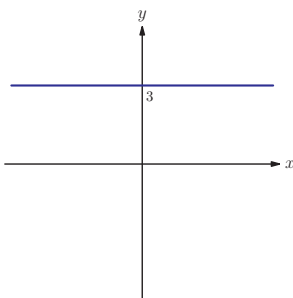
Long questions

1. (a) $z_1 = \sqrt{2} \left(\cos\left(-\frac{\pi}{6}\right) + i \sin\left(-\frac{\pi}{6}\right) \right)$
 $z_2 = \sqrt{2} \left(\cos\left(-\frac{\pi}{4}\right) + i \sin\left(-\frac{\pi}{4}\right) \right)$
- (c) $\cos\frac{\pi}{12} = \frac{\sqrt{6} + \sqrt{2}}{4}$, $\sin\frac{\pi}{12} = \frac{\sqrt{6} - \sqrt{2}}{4}$
2. (a) $\cos\left(-\frac{\pi}{6}\right) + i \sin\left(-\frac{\pi}{6}\right)$
 (b) $c = 1$
 (c) $m = -6$, $n = 4$
3. (a) (i) $Z^3 \cos^3 \theta - 3 \cos \theta \sin^2 \theta +$
 $(3 \cos^2 \theta \sin \theta - \sin^3 \theta) i$
 (c) $\frac{23\sqrt{2}}{20}$
5. (a) (i) $x_1 + x_2 + x_3 = -\frac{b}{a}$, $x_1 x_2 + x_2 x_3 + x_3 x_1 = -\frac{d}{a}$
6. (c) (ii) $\frac{2}{3} \pm \frac{\sqrt{5}}{3} i$, $-\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$
7. (a) $\omega^2 = e^{\frac{4\pi i}{5}}$, $\omega^3 = e^{-\frac{4\pi i}{5}}$, $\omega^4 = e^{-\frac{2\pi i}{5}}$
 (c) $4 \cos^2\left(\frac{2\pi}{5}\right) + 2 \cos\left(\frac{2\pi}{5}\right) - 1 = 0$
8. (a) $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$
 $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$
 (f) $2 - \sqrt{3}$
9. (b) (i) a (ii) $b(\cos \theta + i \sin \theta)$
 (iii) $AB = \sqrt{b^2 \sin^2 \theta + (a - b \cos \theta)^2}$

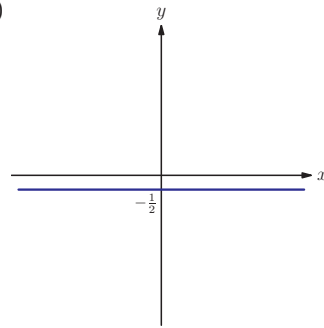
Chapter 16

Exercise 16A

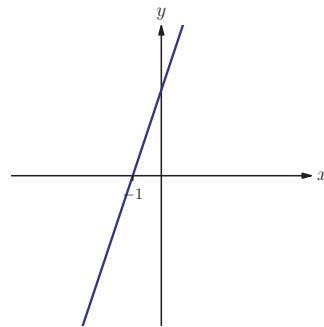
1. (a) (i)



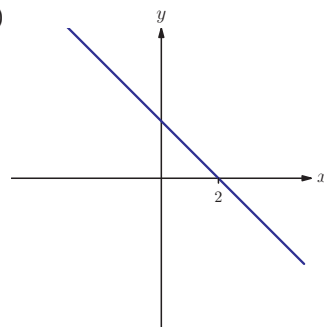
- (ii)



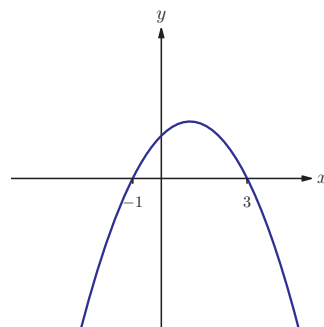
- (b) (i)



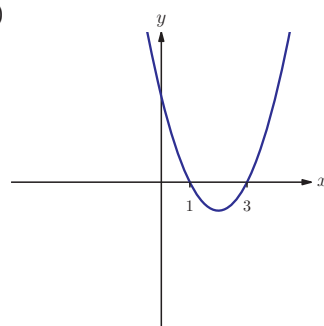
- (ii)



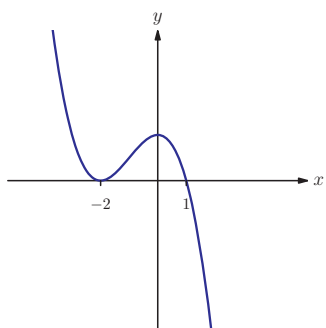
- (c) (i)



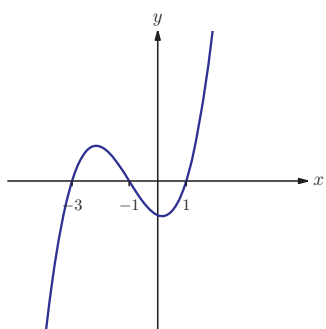
- (ii)



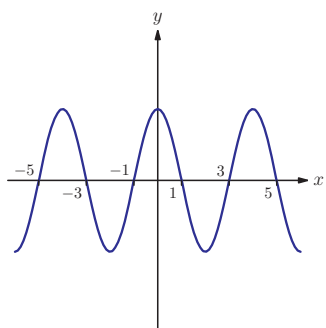
(d) (i)



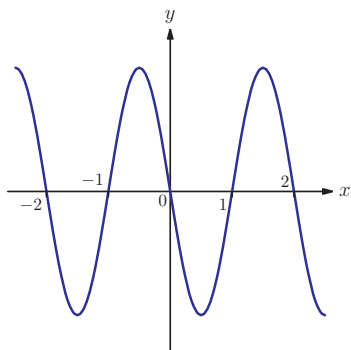
(ii)



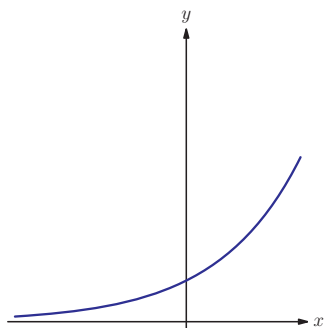
(e) (i)



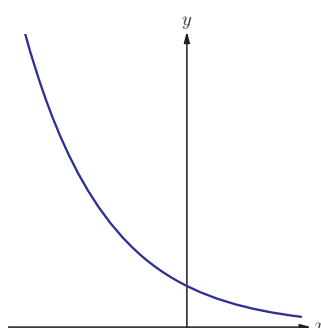
(ii)



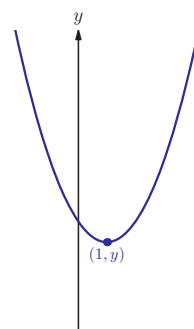
(f) (i)



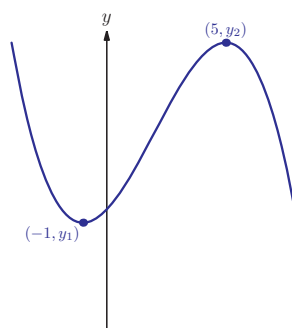
(ii)



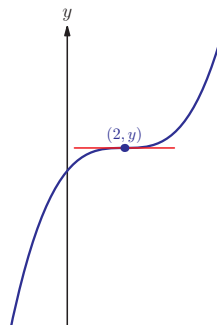
2. (a)



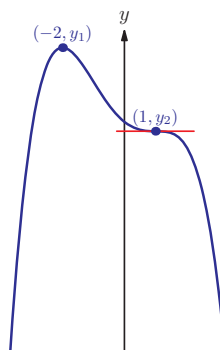
(b)



(c)



(d)



3. (a) Sometimes true
 (b) Sometimes true
 (c) Always true
 (d) Sometimes true
 (e) Sometimes true
 (f) Sometimes true

Exercise 16B

1. (a) (i) $f'(x) = 3x^2$ (ii) $f'(x) = 4x^3$
 (b) (i) $f'(x) = -4$ (ii) $f'(x) = 6x$
 (c) (i) $f'(x) = 2x$ (ii) $f'(x) = 2x - 3$

Exercise 16C

1. (a) (i) $y' = 4x^3$ (ii) $y' = 1$
 (b) (i) $y' = 21x^6$ (ii) $y' = -20x^4$
 (c) (i) $y' = 0$ (ii) $y' = 0$
 (d) (i) $y' = 12x^2 - 10x + 2$
 (ii) $y' = 8x^3 + 9x^2 - 1$
 (e) (i) $y' = 2x^5$ (ii) $y' = -\frac{3}{2}x$
 (f) (i) $y' = 7 - \frac{3}{2}x^2$ (ii) $y' = -20x^3 + x^4$
 (g) (i) $y' = \frac{3}{2}x^{\frac{1}{2}}$ (ii) $y' = \frac{2}{3}x^{-\frac{1}{3}}$
 (h) (i) $y' = 8x^{\frac{1}{3}}$ (ii) $y' = \frac{1}{2}x^{-\frac{1}{6}}$
 (i) (i) $y' = 12x^3 - 2x + 6x^{-\frac{3}{5}}$
 (ii) $y' = 3x^2 - x^{\frac{2}{3}} + \frac{2}{3}x^{-\frac{1}{2}}$
 (j) (i) $y' = -x^{-2}$ (ii) $y' = 3x^{-4}$
 (k) (i) $y' = -\frac{1}{2}x^{-\frac{3}{2}}$ (ii) $y' = 6x^{-\frac{7}{4}}$
 (l) (i) $y' = 5 \times \frac{4}{3}x^{-\frac{7}{2}}$ (ii) $y' = x^{-\frac{10}{7}} - 8x^{-7}$
2. (a) (i) $y' = \frac{1}{3}x^{-\frac{2}{3}}$ (ii) $y' = \frac{4}{5}x^{-\frac{1}{5}}$
 (b) (i) $y' = -6x^{-3}$ (ii) $y' = 4x^{-11}$
 (c) (i) $y' = -\frac{1}{2}x^{-\frac{3}{2}}$ (ii) $y' = -2x^{-\frac{7}{4}}$
 (d) (i) $y' = 9x^2 - 8x$
 (ii) $y' = \frac{7}{2}x^{\frac{5}{2}} - 3x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}$
 (e) (i) $y' = \frac{4}{3}x^{\frac{1}{3}} + \frac{2}{3}x^{-\frac{2}{3}} - 1$
 (ii) $y' = 2x - 8x^{-3}$

- (f) (i) $y' = 9x^2 + 2x^{-2}$
 (ii) $y' = \frac{15}{2}x^{\frac{2}{3}} - \frac{1}{2}x^{-\frac{4}{3}}$

3. (a) (i) -1 (ii) $\frac{3}{2}$
 (b) (i) $-2x - 1$ (ii) $4x^3 + 2$

Exercise 16D

1. (a) $\frac{dz}{dt}$ (b) $\frac{dQ}{dP}$
 (c) $\frac{dR}{dm}$ (d) $\frac{dV}{dt}$
 (e) $\frac{dy}{dx}$ (f) $\frac{d^2z}{dy^2}$
 (g) $\frac{d^2H}{dm^2}$
2. (a) (i) $\frac{5}{3}x^{-\frac{2}{3}}$ (ii) $15q^4$
 (b) (i) $3 - 7t^{-2}$ (ii) $1 - c^{-2}$
 (c) (i) $18 + 6x$ (ii) $6t^{-3}$
3. (a) (i) 30 (ii) $\frac{227}{36}$
 Did you think about doing this on the calculator?
 (b) (i) 7 (ii) -29999.8
 (c) (i) 12 (ii) -10
 (d) (i) 24 (ii) 32
 (e) (i) 6 (ii) $\frac{7}{2\sqrt{6}}$
4. (a) (i) $2ax + 1 - a$ (ii) $3x^2$
 (b) (i) $\frac{1}{2}\sqrt{\frac{b}{a}}$ (ii) $6a^2v$
5. (a) (i) 54 (ii) 384
 (b) (i) 8 (ii) $\frac{1}{108}$
 (c) (i) 0 (ii) 42
6. (a) (i) ± 2 (ii) 1
 (b) (i) ± 17 (ii) 6
7. (a) (i) $x \in \left[-\infty, -\frac{2\sqrt{3}}{3}\right] \cup \left[\frac{2\sqrt{3}}{3}, \infty\right]$
 (ii) $x \in [0, 2]$
 (b) (i) $x \in \left[-\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}\right]$ (ii) $x > \frac{1}{4}$
 (c) (i) $x > 0$ (ii) $x > -2$
 (d) (i) $1 < x < 2$ (ii) $-3 < x < 3$

8. $(-0.199, 0.913)$,
 $(1.29, -0.181), (3.91, 30.3)$

9. $x > \frac{-1 + \sqrt{22}}{3}$ and $x < \frac{-1 - \sqrt{22}}{3}$

10. $-3 < x < 1$ and $x > 1$

11. $n!$

Exercise 16E

1. (a) (i) $3\cos x$ (ii) $-2\sin x$
 (b) (i) $2 + 5\sin x$ (ii) $\sec^2 x$
 (c) (i) $\frac{\cos x - 2\sin x}{5}$ (ii) $\frac{1}{2}\sec^2 x - \frac{1}{3}\cos x$

2. π

3. $\frac{22 - \pi^2}{12}$

4. $x = \frac{\pi}{4}, \frac{5\pi}{4}$

5. $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

Exercise 16F

1. (a) (i) $3e^x$ (ii) $\frac{2e^x}{5}$
 (b) (i) $\frac{-2}{x}$ (ii) $\frac{1}{3x}$
 (c) (i) $\frac{1}{5x} - 3 + 4e^x$ (ii) $-\frac{e^x}{2} + \frac{3}{x}$

2. (i) $2 - \frac{7}{\ln 4}$
 (ii) $3 - \frac{1}{2\ln 3}$

3. $x = \ln 3$

4. $x = 3$

5. (a) (i) $\frac{3}{x}$ (ii) $\frac{1}{x}$
 (b) (i) e^{3+x} (ii) e^{x-3}
 (c) (i) $2x$ (ii) $3e^2x^2$
 (d) (i) $\frac{1}{x\ln 3}$ (ii) $\frac{4}{x\ln 6}$

Exercise 16G

1. (a) Tangent: $11x - 4y = 4$
 Normal: $4x + 11y = 126$

(b) Tangent: $4x - y + 1 - \pi = 0$
 Normal: $4x + 16y - \pi - 16 = 0$

(c) Tangent: $5x + y + 2 - 10\ln 5 = 0$
 Normal: $x - 5y - 10 - \ln 25 = 0$

2. $\left(\frac{1}{16}, \frac{7}{16}\right)$

3. $y = 3x - \ln 4 + 2$

4. -0.0541 or 2.05

5. $(-1, 4)$

6. $(0.410, 0.348)$

7. (a) $\sqrt{3}$ (b) 2.3°

Exercise 16H

1. (a) (i) $(0, 0)$ Local Maximum
 $\left(\frac{10}{3}, -\frac{500}{27}\right)$ Local Minimum

In the exam you should give the value of $\frac{d^2y}{dx^2}$ to justify your choice.

(ii) $(0, 0)$ Local Maximum
 $(-2, 16)$ Local Minimum

(b) (i) $(-2, -16)$
 Local Minimum
 $\left(\frac{2\pi}{3}, \frac{3\sqrt{3} + 2\pi}{6}\right)$
 Local Maximum

(ii) $\left(-\frac{2\pi}{3}, -\frac{3\sqrt{3} + 2\pi}{6}\right)$
 Local Minimum
 $(0, 3)$ Local Maximum

(c) (i) $(\pi, -1)$ Local Minimum
 $(2\pi, 3)$ Local Maximum
 (ii) $(4, \ln 4 - 2)$ Local Maximum
 $(\ln(2.5), 5 - 5\ln(2.5))$
 Local Minimum

2. $(x_1, f(x_1))$ could be a point of inflexion.

3. $(-4, 92)$ Local Maximum
 $(2, -16)$ Local Minimum

4. $\left(\frac{1}{4}, -\frac{1}{4}\right)$ Local Minimum

5. (0.245, 4.12) Local Maximum

(3.39, -4.12) Local Minimum

7. $y \geq -21$

8. $y \geq 6 - 4\ln 4$

9. (0, 0) Local Minimum

$\left(-\frac{4}{k}, -\frac{32}{k^2}\right)$ Local Maximum

Exercise 16I

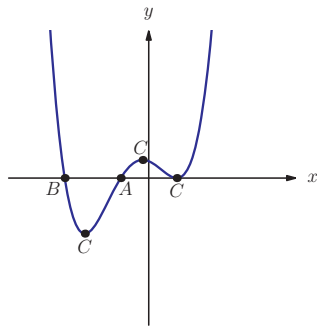
1. $(\ln 2, 2 - (\ln 2)^2)$

2. (1, 4), (-1, -10)

4. $\left(\frac{\pi}{2}, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, \frac{3\pi}{2}\right)$

As the question doesn't state how many points of inflexion there are, you need to show that both of these are by considering the gradient either side.

6.



Exercise 16J

1. Min 1, max e

2. (a) 225 m (b) 60 m

3. Min $-6\sqrt{3}$, max 80

4. Min $3 - 3\ln 3$, max $e^2 - 6$

5. Min 0, max 4π

6. 2

7. 2.25

8. Min 2, max 4

9. (a) 40 million litres (b) $\frac{\pi}{2}, \frac{3\pi}{2}$

10. (a) 2 g/tray (b) 0 g/tray (c) 1.94 g/tray

11. (a) 4 litres (b) 41.5 litres
(c) 20 seconds

12. (a) $2x - \frac{x^2}{10}$ (b) 10m^2 (c) $0 < x < 10$

Mixed examination practice 16

Short questions

1. $y = e^{\frac{\pi}{2}x} - \frac{\pi}{2}e^{\frac{\pi}{2}} + e^{\frac{\pi}{2}} + 2$

2. $x = 2$

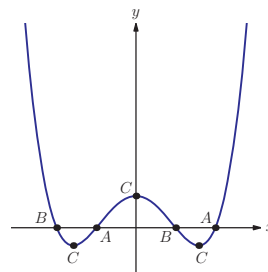
3. $b = 8, c = -7$

4. $\left(2, -\frac{2}{3}\right)$

5. $\frac{\pi}{6} \pm k\pi$ local minima $-\frac{\pi}{6} \pm k\pi$ local maxima $k \in \mathbb{Z}$

6. $-\frac{1}{5}x^3 + \frac{12}{5}x^2 - 3x + 2$

7.



8. $a = \frac{1}{\sqrt[4]{3}}$

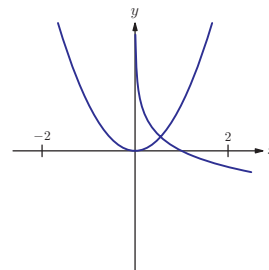
Long questions

1. (b) $3a + b = 6$ (c) $a = 1, b = 3$ (d) $(-7, -192)$

2. (a) (i) $\left(-\frac{1}{3}, \frac{86}{27}\right)$ (ii) $\left(\frac{1}{3}, \frac{70}{27}\right)$

(b) (i) $y = -\frac{8}{9}x + \frac{78}{27}$ (ii) $\frac{3 \pm 2\sqrt{3}}{9}$

3. (a)



(b) 0.548 (c) 0.302

4. (a) (i) 11 000 (ii) 2.42 h

(b) (i) $\frac{dP}{dt} = e^t - 3$ (ii) 8.7 h

(c) (i) $\frac{d^2P}{dt^2} = e^t$, the rate of change of the growth rate of the bacteria

(ii) 9704, $\frac{d^2P}{dt^2} = 3 > 0$, hence local minimum

Chapter 17

Exercise 17A

1. (a) $\frac{3}{4}x^4, \frac{3}{4}x^4 + 1; \frac{3}{4}x^4 - 1$ (b) 1, -1, 2
2. (a) (i) $x^3(+c)$ (ii) $x^5(+c)$
 (b) (i) $\frac{1}{x}(+c)$ (ii) $\frac{1}{x^4}(+c)$
 (c) (i) $\sqrt{x}(+c)$ (ii) $\sqrt[3]{x}(+c)$
 (d) (i) $2x^5(+c)$ (ii) $4x^3(+c)$

Exercise 17B

1. (a) $\frac{3}{4}x^4, \frac{3}{4}x^4 + 1, \frac{3}{4}x^4 - 1$ (b) 1, -1, 2
2. (a) (i) $\frac{7}{5}x^5 + c$ (ii) $\frac{1}{9}x^3 + c$
 (b) (i) $-\frac{1}{2t} + c$ (ii) $-\frac{4}{y^2} + c$

Exercise 17C

1. (a) (i) $x^9(+c)$ (ii) $x^{12}(+c)$
 (b) (i) $\frac{x^2}{2}(+c)$ (ii) $\frac{x^4}{4}(+c)$
 (c) (i) $9x(+c)$ (ii) $\frac{x}{2}(+c)$
 (d) (i) $\frac{x^6}{2}(+c)$ (ii) $\frac{9}{5}x^5(+c)$
 (e) (i) $2x^{\frac{3}{2}}(+c)$ (ii) $\frac{9}{4}x^{\frac{4}{3}}(+c)$
 (f) (i) $-\frac{5}{x}(+c)$ (ii) $-\frac{1}{x^2}(+c)$
2. (a) (i) $3t(+c)$ (ii) $7z(+c)$
 (b) (i) $\frac{q^6}{6}(+c)$ (ii) $\frac{r^{11}}{11}(+c)$
 (c) (i) $\frac{15}{2}g^{\frac{8}{5}}(+c)$ (ii) $\frac{10}{9}y^{\frac{9}{2}}(+c)$
 (d) (i) $-\frac{4}{h}(+c)$ (ii) $-\frac{1}{3p^3}(+c)$
3. (a) (i) $\frac{x^3}{3} + \frac{x^4}{4} + c$ (ii) $\frac{x^5}{5} - x^2 + 5x + c$
 (b) (i) $-\frac{1}{6t^2} - \frac{1}{12t^3} + c$ (ii) $-\frac{5}{v} + \frac{1}{v^4} + c$
 (c) (i) $\frac{2}{5}x^{\frac{5}{2}} + c$ (ii) $\frac{18}{7}x^{\frac{7}{6}} + c$
 (d) (i) $\frac{x^4}{4} + x^3 + \frac{3x^2}{2} + x + c$
 (ii) $\frac{x^4}{4} + \frac{4x^3}{3} + 2x^2 + c$

4. $2\sqrt{x} + \frac{2}{3}x^{\frac{3}{2}} + c$

Exercise 17D

1. (a) (i) $2\ln x + c$ (ii) $3\ln x + c$
 (b) (i) $\frac{1}{2}\ln x + c$ (ii) $\frac{1}{3}\ln x + c$
 (c) (i) $\frac{x^2}{2} - \ln x + c$ (ii) $\frac{x^3}{3} + 5\ln x + c$
 (d) (i) $3\ln x - \frac{2}{x} + c$ (ii) $\ln x + \frac{2}{\sqrt{x}} + c$
2. (a) (i) $5e^x + c$ (ii) $9e^x + c$
 (b) (i) $\frac{2e^x}{5} + c$ (ii) $\frac{7e^x}{11} + c$
 (c) (i) $\frac{e^x}{2} + \frac{3x^2}{4} + c$ (ii) $\frac{e^x}{5} + \frac{x^4}{20} + c$

Exercise 17E

1. (a) (i) $-\cos x - \sin x + c$ (ii) $3\sin x - 4\cos x + c$
 (b) (i) $x + \ln|\sec x| + c$ (ii) $-\frac{1}{2}\cos x + \frac{1}{3}\ln|\sec x| + c$
 (c) (i) $\frac{x^2}{14} - \frac{\cos x}{7} + c$ (ii) $\frac{1}{9}x^{\frac{3}{2}} + \frac{\sin x}{6} + c$
 (d) (i) $x - \sin x + \cos x + c$ (ii) $-2\cos x - \sin x + c$
2. $\frac{1}{2}\ln|\sec x| + \frac{x}{2} + c$
3. $\sin x - \cos x + c$

ANSWER HINT(3)

Use an identity

Exercise 17F

1. (a) (i) $y = \frac{x^2}{2} + 5$ (ii) $y = 2x^3 + 5$
 (b) (i) $y = 2\sqrt{x} + 4$ (ii) $y = -\frac{1}{x} + 4$
 (c) (i) $y = 2e^x + 2x - 1 - 2e$ (ii) $y = e^x - 5$
 (d) (i) $y = x + \ln x - 1$ (ii) $y = \frac{1}{2}\ln x + 4$
 (e) (i) $y = \sin x - \cos x$ (ii) $y = 3\ln|\sec x| + 4$

2. (a) $f(x) = \frac{1}{2} \ln x + c$
 (b) $y = \frac{1}{2} \ln x - \frac{1}{2} \ln 2 + 7$
3. (a) $x = -2, y'' = 2x = 4 < 0$, hence local maximum
4. $y = \ln \left| \frac{e^5}{x} \right|$

Exercise 17G

1. (a) (i) 320 (ii) 28.5
 (b) (i) 1 (ii) -2
 (c) (i) $e - 1$ (ii) $3e - \frac{3}{e}$
2. (a) (i) 0.995 (3SF) (ii) 0.0997 (3SF)
 (b) (i) 1.46 (3SF) (ii) 1
3. $e^\pi + \pi + 1$
5. 20
6. $a = 16$

Exercise 17H

1. (a) (i) $\frac{7}{3}$ (ii) $\frac{1}{4}$
 (b) (i) $\frac{2}{3}$ (ii) $\frac{22}{3}$
 (c) (i) $\frac{11}{4}$ (ii) $13\frac{1}{6}$
2. 9
3. (a) 6
 (b) $\frac{22}{3}$
4. 2

ANSWER HINT(4)

There is an easier way than splitting this integral into two parts

5. $\frac{9}{2}$

Exercise 17I

1. (a) (i) 9.13 (ii) 2.50
 (b) (i) 0.828 (ii) 41.3
 (c) (i) 2.35 (ii) 5.38
2. 6

3. $e^2 - 3$
4. 25
5. $a = \sqrt{b}; a = \sqrt[3]{2b^{\frac{3}{2}} + 2 - 3b};$
 $a = 1 + \sqrt{3}; \text{Area} = 3 + 2\sqrt{3}$

Exercise 17J

1. (a) (i) $\frac{32}{3}$ (ii) $\frac{1}{6}$
 (b) (i) 9 (ii) $\frac{1}{3}$
 (c) (i) $\frac{9}{8}$ (ii) $\frac{1}{3}$
2. $\frac{32}{3}$
3. $e^2 - \frac{11}{3}$
4. 0.462
6. $2 - \sqrt{2}$
7. 8
8. $m = 4$

Mixed examination practice 17

Short questions

1. $f(x) = \frac{1}{2} - \cos x$
2. 0.201
3. $\frac{4k^3}{3}$
4. 3

ANSWER HINT(4)

You did not need to do anything with the red area to do this question

5. $\ln x + \frac{2}{5} x^{\frac{5}{2}} + c$
6. (a) $a = \sqrt{2}$ (b) $\frac{1}{2}$
7. $2\sqrt{3} - \frac{2}{3}\pi$
8. (a) (Local) Minimum
 (b) $x^3 + 3x^2 - 45x + 100$

Long questions

1. (a) $(-a, 0)$ and $(3a, 8a^2)$
 (b) $\frac{64}{3}a^3$ (c) $\frac{15}{16}$
 2. (b) $\arcsin a$ (c) $1 - \sqrt{1 - a^2}$
 (d) $a \arcsin a + \sqrt{1 - a^2} - 1$

Chapter 18

Exercise 18A

1. (a) (i) $7(2x-3)(x^2-3x+1)^6$
 (ii) $15x^2(x^3+1)^4$
 (b) (i) $(2x-2)e^{x^2-2x}$
 (ii) $-3x^2e^{4-x^3}$
 (c) (i) $-6e^x(2e^x+1)^{-4}$
 (ii) $20e^x(2-5e^x)^{-5}$
 (d) (i) $6x \cos(3x^2+1)$
 (ii) $-(2x+2)\sin(x^2+2x)$
 (e) (i) $-3\sin x \cos^2 x$
 (ii) $4\cos x \sin^3 x$
 (f) (i) $\frac{2-15x^2}{2x-5x^3}$ (ii) $\frac{8x}{4x^2-1}$
 (g) (i) $\frac{16}{x}(4\ln x-1)^3$ (ii) $-\frac{5}{x}(\ln x+3)^{-6}$
 2. (a) (i) $10(2x+3)^4$ (ii) $32(4x-1)^7$
 (b) (i) $4(5-x)^{-5}$ (ii) $7(1-x)^{-8}$
 (c) (i) $4\sin(1-4x)$ (ii) $\sin(2-x)$
 (d) (i) $\frac{5}{5x+2}$ (ii) $\frac{1}{x-4}$
 (e) (i) $-3\csc^2(3x)$ (ii) $-5\csc(5x)\cot(5x)$
 (f) (i) $2\sec(2x+1)\tan(2x+1)$
 (ii) $-\sec^2(1-x)$
 3. (a) (i) $6\sec^2(3x)\tan(3x)$
 (ii) $4\tan(2x)\sec^2(2x)$
 (b) (i) $6\sin(3x)\cos(3x)e^{\sin^2(3x)}$
 (ii) $\frac{2\ln(2x)}{x}e^{(\ln 2x)^2}$

- (c) (i) $-16\sin(2x)\cos(2x)(1-2\sin^2(2x))$
 (ii) $-24\sin 3x(4\cos 3x+1)$

- (d) (i) $\frac{6\sin 2x}{1-3\cos 2x}$ (ii) $\frac{5\sin 5x}{2-\cos 5x}$

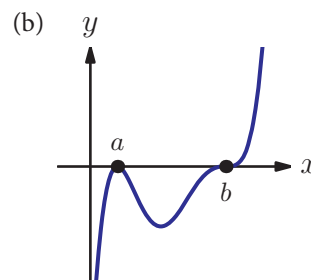
4. $y = \frac{27\sqrt{2}}{8}x - \frac{77}{12}$

5. $\left(\frac{\pi}{2}, e\right), \left(\frac{3\pi}{2}, e^{-1}\right)$

6. (a) $-2\csc^2 x \cot x$ (b) $x = -\frac{\pi}{4}, \frac{3\pi}{4}$

7. 7

8. (a) $\frac{qa+pb}{p+q}$

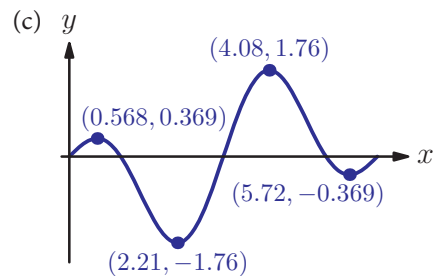


(c) q is even.

9. (a) Left post $e^{-1} > e^{-4}$ (c) $\sqrt[3]{2} + \frac{1}{\sqrt[3]{4}}$

10. (a) $x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$

- (b) $x = 0.568, 2.21, 4.08, 5.72$



Exercise 18B

1. (a) (i) $2x \cos x - x^2 \sin x$
 (ii) $-x^{-2} \sin x + x^{-1} \cos x$
 (b) (i) $-2x^{-3} \ln x + x^{-3}$ (ii) $\ln x + 1$
 (c) (i) $3x^2\sqrt{2x+1} + x^3(2x+1)^{-\frac{1}{2}}$
 (ii) $-x^{-2}\sqrt{4x} + 2x^{-1}(4x)^{-\frac{1}{2}}$
 (d) (i) $2e^{2x} \tan x + e^{2x} \sec^2 x$
 (ii) $e^{x+1} \sec 3x + 3e^{x+1} \sec 3x \tan 3x$

2. (a) (i) $3(x+1)^3(x-2)^4(3x-1)$
(ii) $(x-3)^6(x+5)^3(11x+23)$
(b) (i) $(2x-1)^3(1-3x)^2$
 $(-42x+17)$
(ii) $(1-x)^4(4x+1)(-28x+3)$
3. $(6x^2+4x+3)e^{2x}$
4. $(9x^2+12x+2)e^{3x}$
5. $x = -\frac{1}{2}, 2$
6. $x = 3, -\frac{1}{3}, \frac{7}{4}$
7. $(0.538, 0.473), (1.82, -0.87), (3.29, 0.962), (4.81, -0.933)$
8. $e^x(1+x)\cos(xe^x)$
9. (a) $\ln x + 1$
(b) $x \ln x - x$
10. $\left(\frac{3\pi}{4}, -\frac{\sqrt{2}}{2}e^{-\frac{3\pi}{4}}\right)$
11. $a = 4, b = 5$
12. (a) $y = e^{\ln x^x}$ (b) $(\ln x + 1)x^x$
(c) $(e^{-1}, e^{-e^{-1}})$

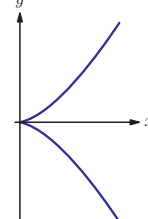
Exercise 18C

1. (a) (i) $\frac{2}{(x+1)^2}$ (ii) $\frac{-5}{(x-3)^2}$
(b) (i) $\frac{x(2x+1)^{\frac{1}{2}} - (2x+1)^{\frac{1}{2}}}{x^2}$
(ii) $\frac{2x(x-1)^{\frac{1}{2}} - \frac{1}{2}x^2(x-1)^{-\frac{1}{2}}}{x-1}$
(c) (i) $\frac{2(x^2-x-2)}{(x^2+2)^2}$ (ii) $-\frac{x^2+2x+4}{(1+x)^2}$
(d) (i) $\frac{1-\ln 3x}{x^2}$ (ii) $\frac{x-2x \ln 2x}{x^4}$
2. $y = \frac{\pi^2}{4}x + \frac{16-\pi^4}{8\pi}$
3. $(0,0), (1,1)$
4. $a = -1$
5. $\left(e, \frac{1}{e}\right)$ Local maximum

6. $x \in (0,2), x \neq 1$

7. $a = 3, b = 4, p = \frac{3}{2}$

Exercise 18D

1. (a) (i) $\frac{2}{3}$ (ii) $\frac{1}{2}$
(b) (i) 0 (ii) -1
(c) (i) -1 (ii) $\frac{5}{3}$
(d) (i) -1 (ii) $-\frac{1}{2}$
2. (a) (i) $\frac{2x}{y^2}$ (ii) $\frac{-2x^3}{3y}$
(b) (i) $\frac{y(8x-y)}{2x(y-2x)}$ (ii) $\frac{y}{2y-x}$
(c) (i) $\frac{1-2y}{2x-4y-1}$ (ii) $\frac{y}{2y-x}$
(d) (i) $\frac{y(2x-e^y)}{xye^y-4}$ (ii) $\frac{\cos x - 3\sin y}{3x \cos y - 2\sin y}$
3. (i) $(3,2), (-3,-2)$
(ii) $(\sqrt{2}, 4\sqrt{2}), (-\sqrt{2}, -4\sqrt{2})$
4. (a) (i) $3 \ln 3$ (ii) $25 \ln 5$
(b) (i) $-4 \ln 2$ (ii) $-3 \ln 3$
(c) (i) $\frac{3 \ln 2}{8}$ (ii) $4 \ln 4$
(d) (i) $-3 \ln 3$ (ii) $-\frac{\ln 5}{5}$
5. (c) $\frac{d}{dx}(\ln kx) = \frac{1}{x}, \frac{d}{dx}(\ln x^n) = \frac{n}{x}$
6. 4
7. $5x - 2y = 16$
8. $\frac{dy}{dx} = \frac{y2^y}{1-x2^y-\ln 2}$
9. $(2, e^4)$
10. (a) 
- (b) $y = 3x - 4$ (c) $(1-4)$

Exercise 18E

1. (a) (i) $\frac{-3}{\sqrt{1-9x^2}}$ (ii) $\frac{-2}{\sqrt{1-4x^2}}$

(b) (i) $\frac{2}{4+x^2}$ (ii) $\frac{10}{25+4x^2}$

(c) (i) $\arcsin x + \frac{x}{\sqrt{1-x^2}}$

(ii) $2x \arccos x - \frac{x^2}{\sqrt{1-x^2}}$

(d) (i) $\frac{2x}{1+(x^2+1)^2}$

(ii) $\frac{-2x}{\sqrt{1-(1-x^2)^2}}$

2. $-\frac{3}{\sqrt{35}}$

4. $\frac{dy}{dx} = -\frac{1+\tan^2\left(\frac{1}{x}\right)}{x^2}$

5. (a) $\arcsin x + \frac{x}{\sqrt{1-x^2}}$

(b) $x \arcsin x + \sqrt{1-x^2} + c$

Mixed examination practice 18

Short questions

1. (a) $2x \arcsin x + \frac{x^2}{\sqrt{1-x^2}}$ (b) $\frac{e^y}{8y - xe^y}$

2. $\frac{2x}{\sqrt{1-(1-x^2)^2}}$

3. $\frac{16}{225}$

4. $y = \frac{14}{9}x + \frac{88}{9}$

5. $\frac{2(1-3x^4)}{(1+x^4)^2}$

6. $-\frac{1}{7}$

7. $\frac{5}{2}$

8. (b) $-\ln \frac{b}{c}, \frac{a}{2b}$

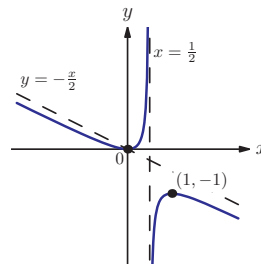
9. $\left(1, \frac{2\sqrt{e}-2}{\sqrt{e}}\right), \left(1, \frac{2\sqrt{e}+2}{\sqrt{e}}\right)$

Long questions

1. (a) $x = \frac{1}{2}$ (b) $(0,0), (1,-1)$

(c) $(0,0)$ local min $(1,-1)$ local max

(d)



2. (a) (ii) $\frac{(\ln 2)^2 x^2 - 4x \ln 2 + 2}{2^x}$

(b) (i) $\frac{2}{\ln 2}$ (c) $\frac{2 \pm \sqrt{2}}{\ln 2}$

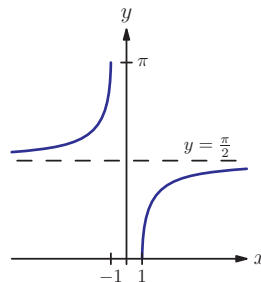
3. (c) $k = 3, p = 1$

4. (a) $(2,4), (-2,-4)$

(c) $(2,4)$ local max; $(-2,-4)$ local min

5. (a) $x \geq 1, x \leq -1$

(b)



(d) $\frac{1}{x\sqrt{x^2-1}}$

Chapter 19

Exercise 19A

1. (a) (i) $(x+3)^5 + c$ (ii) $\frac{1}{6}(x-2)^6 + c$

(b) (i) $\frac{1}{32}(4x-5)^8 + c$ (ii) $2\left(\frac{1}{8}x+1\right)^4 + c$

(c) (i) $-\frac{8}{7}\left(3-\frac{1}{2}x\right)^7 + c$ (ii) $-\frac{1}{9}(4-x)^9 + c$

(d) (i) $\frac{1}{3}(2x-1)^{\frac{3}{2}} + c$ (ii) $-\frac{4}{5}(2-5x)^{\frac{7}{4}} + c$

(e) (i) $4\left(2+\frac{x}{3}\right)^{\frac{3}{4}} + c$ (ii) $2(4-3x)^{-1} + c$

2. (a) (i) $e^{3x} + c$ (ii) $\frac{1}{2}e^{2x+5} + c$
 (b) (i) $6e^{\frac{2x-1}{3}} + c$ (ii) $2e^{\frac{1}{3}x} + c$
 (c) (i) $2e^{-3x} + c$ (ii) $-\frac{1}{4}e^{-4x} + c$
 (d) (i) $8e^{\frac{x}{4}} + c$ (ii) $-\frac{3}{2}e^{\frac{2}{3}x} + c$
3. (a) (i) $\ln|x+4| + c$ (ii) $\ln|5x-2| + c$
 (b) (i) $\frac{2}{3}\ln|3x+4| + c$ (ii) $-4\ln|2x-5| + c$
 (c) (i) $\frac{3}{4}\ln|1-4x| + c$ (ii) $-\frac{1}{2}\ln|7-2x| + c$
 (d) (i) $x+3\ln|5-x| + c$ (ii) $3x - \ln|3-x| + c$
4. (a) $\csc x + c$ (b) $\tan 3x + c$
 (c) $\frac{1}{3}\cos(2-3x) + c$ (d) $-4\cot\left(\frac{1}{4}x\right) + c$
 (e) $\frac{1}{2}\sin 4x + c$ (f) $2\sec\frac{x}{2} + c$
5. Both; $\frac{1}{3}\ln(3x) + c = \frac{1}{3}\ln x + \frac{1}{3}\ln 3 + c = \frac{1}{3}\ln x + c'$
6. 0.492

Exercise 19B

1. (a) (i) $\frac{1}{8}(x^2+3)^4 + c$ (ii) $\frac{1}{4}(x^2-1)^6 + c$
 (b) (i) $\frac{1}{15}(3x^2-15x+4)^5 + c$
 (ii) $\frac{1}{12}(x^3+3x^2-5)^4 + c$
 (c) (i) $\ln|x^2+3| + c$
 (ii) $2\ln|x^3-6x+1| + c$
 (d) (i) $-\frac{2}{9}\cos^6 3x + c$
 (ii) $\frac{1}{8}\sin^4 2x + c$
 (e) (i) $\frac{1}{2}e^{3x^2-1} + c$
 (ii) $\frac{3}{2}e^{x^2} + c$
 (f) (i) $\ln\sqrt{e^{2x+3}+4} + c$ (ii) $\ln^4\sqrt{3+4\sin x} + c$
 (g) (i) $4\tan^4 2x + c$ (ii) $4\sec^6\left(\frac{x}{4}\right) + c$

- (h) (i) $-\frac{1}{4}\csc x^4 + c$
 (ii) $-\ln(\sqrt{3+\cot 2x}) + c$
- (i) (i) $-\sqrt{3-x^2} + c$
 (ii) $-\frac{1}{3}(e^{-4x+1})^{\frac{3}{2}} + c$

3. (a) (i) $\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + c$
 (ii) $\frac{2}{7}(x-2)^{7/2} + \frac{8}{5}(x-2)^{5/2} + \frac{8}{3}(x-2)^{3/2} + c$
 (b) (i) $\frac{2}{9}(x-5)^9 + \frac{5}{4}(x-5)^8 + c$
 (ii) $\frac{1}{7}(x+3)^7 - \frac{1}{2}(x+3)^6 + c$

4. (a) (i) $\frac{1}{24}(2x-1)^6 + \frac{1}{20}(2x-1)^5 + c$
 (ii) $\frac{1}{7}(3x+2)^7 - \frac{1}{3}(3x+2)^6 + c$
 (b) (i) $\frac{2}{5}(x-3)^{\frac{5}{2}} + 2(x-3)^{\frac{3}{2}} + c$
 (ii) $\frac{2}{75}(5x-6)^{5/2} + \frac{22}{75}(5x-6)^{3/2} + c$
 (c) (i) $\frac{2}{5}(x-5)^{5/2} + \frac{20}{3}(x-5)^{3/2} + 50(x-5)^{1/2} + c$
 (ii) $-\frac{1}{(2x-3)} - \frac{13}{2(2x-3)^2} + c$

5. (a) (i) $9-8\ln 2$
 (ii) $12\ln\frac{5}{3} - 5\frac{1}{15}$
 (b) (i) $\ln\left(\frac{3}{2}\right)$ (ii) $\ln\left(\frac{3}{2}\right)$
 (c) (i) $\frac{\pi}{18}$ (ii) $\frac{\pi}{12}$

6. $e^5 - e^{-1}$
 7. $\ln 8$
 8. $\frac{2}{3}(x-2)^{3/2} + 4(x-2)^{1/2} + c$

9. (b) $\ln|x^2 + x + 1| + c$

10. $\frac{1}{4}\tan(\ln(x^2)) + c$

11. $-\frac{1}{4\sin^4 x} + c$

12. $2\sqrt{3} - 2$

Exercise 19C

1. (a) $\frac{1}{3}\sec 3x + c$ (b) $-\cot x + c$

(c) $-\frac{1}{4}\cos 4x + c$

(d) $\frac{1}{2}(-3\cot 2x + \csc 2x) + c$

(e) $\sin x + \cos x + c$

2. (a) $\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + c$ (b) $-\frac{1}{\sin x} - \sin x + c$

(c) $-\frac{1}{4}e^{\cos 2x} + c$ (d) $\frac{1}{15}\tan^5 3x + c$

(e) $-\frac{1}{4}\sqrt{1 + \cos 4x} + c$

3. (a) (i) $x + \frac{1}{2}\sin 2x + c$

(ii) $\frac{1}{2}\left(\frac{1}{6}\sin 6x + x\right) + c$

(b) (i) $4\tan\left(\frac{x}{2}\right) - 2x + c$

(ii) $\frac{1}{3}\tan 3x - x + c$

4. (a) (i) $\frac{\pi}{2}$ (ii) $6\sqrt{3} - 2\pi$

(b) (i) $1 - \ln 2$ (ii) $\frac{9\pi}{8} - 1$

5. They are all right, with different $+c$:

$$\frac{1}{2}\sin^2 x = -\frac{1}{2}\cos^2 x + \frac{1}{2} = -\frac{1}{4}\cos 2x + \frac{1}{4}$$

6. $\frac{1}{2}x - \frac{3}{4}\sin\left(\frac{2x}{3}\right) + c$

7. (b) $\frac{1}{2}\tan^2 x - \ln|\sec x| + c$

8. 3

9. (b) $-\frac{2}{3}\cos^3 x + \cos x + c$

10. 4

11. (b) (i) $z^5 - 5z^3 + 10z - \frac{10}{z} + \frac{5}{z^3} - \frac{1}{z^5}$

(ii) $a = 1, b = -5, c = 10$

(c) $\frac{1}{16}\left(-\frac{1}{10}\cos^5 2x + \frac{1}{3}\cos^3 2x - \frac{1}{2}\cos 2x\right) + c$

Exercise 19D

1. (a) (i) $\frac{1}{\sqrt{2}}\arctan(\sqrt{2}x)$

(ii) $\frac{1}{\sqrt{5}}\arctan(\sqrt{5}x)$

(b) (i) $\frac{1}{\sqrt{3}}\arcsin\sqrt{3}x + c$

(ii) $\frac{1}{2}\arcsin 2x + c$

(c) (i) $3\arctan\left(\frac{x}{3}\right) + c$

(ii) $\sqrt{10}\arctan\left(\frac{x}{\sqrt{10}}\right) + c$

(d) (i) $2\arcsin\left(\frac{x}{5}\right) + c$

(ii) $5\arcsin\left(\frac{x}{2}\right) + c$

2. (a) (i) $\arctan(x+2) + c$

(ii) $\arctan(x-3) + c$

(b) (i) $\arcsin(x-4) + c$

(ii) $\arcsin(x-1) + c$

(c) (i) $6\sqrt{2}\arctan\left(\frac{x+5}{\sqrt{2}}\right) + c$

(ii) $\frac{5}{2}\arcsin\left(\frac{2x+3}{3}\right) + c$

3. $\frac{\pi}{2}$

4. (a) $2(x+1)^2 + 9$

(b) $\frac{1}{\sqrt{2}}\arctan\left(\frac{\sqrt{2}(x+1)}{3}\right) + c$

5. (a) $2^2 - 3(x-1)^2$
 (b) $\frac{\sqrt{3}\pi}{9}$

7. $\frac{\pi}{12}$

Exercise 19E

1. (a) $\frac{1}{6}(2x-3)^3 + c$ (b) $-\frac{1}{5}\ln|2-5x| + c$

2. (a) $\frac{5}{2}\ln(x^2+6) + \frac{1}{\sqrt{6}}\arctan\left(\frac{x}{\sqrt{6}}\right) + c$

(b) $-\sqrt{4-x^2} - 3\arcsin\left(\frac{x}{2}\right) + c$

(c) $4\ln|x^2+8x+25| - 3\arctan\left(\frac{x+4}{3}\right) + c$

(d) $-\sqrt{-x^2+6x-7} - 2\arcsin\left(\frac{x-3}{\sqrt{2}}\right) + c$

3. (a) (i) $x - \ln|x+2| + c$ (ii) $2x + 5\ln|x-1| + c$

(b) (i) $\frac{1}{2}x^2 + 3x + 11\ln|x-3| + c$ (ii) $\frac{1}{2}x^2 - 3x + 14\ln|x+5| + c$

(c) $x + \frac{5}{2}\ln|x^2+3| - \frac{2}{\sqrt{3}}\arctan\left(\frac{x}{\sqrt{3}}\right) + c$

4. (a) $\frac{1}{x-2} - \frac{1}{x+3}$ (b) $\ln\left|\frac{x-2}{x+3}\right| + c$

5. $\frac{\pi}{4}$

6. (a) $\frac{2}{x+1} + \frac{2}{1-x}$ (b) 8

7. $-4\sqrt{1-x^2} + 5\arcsin x + c$

8. (a) $2(x-2)^2 + 9$

(b) $\frac{1}{2}\ln|(x-2)^2 + \frac{9}{2}| + 2\sqrt{2}\arctan\left(\frac{\sqrt{2}(x-2)}{3}\right) + c$

Exercise 19F

1. (a) (i) $\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$
 (ii) $-2x\cos\left(\frac{x}{2}\right) + 4\sin\left(\frac{x}{2}\right) + c$

(b) (i) $-2xe^{-2x} + e^{-2x} + c$

(ii) $\frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x} + c$

(c) (i) $x^2\ln 5x - \frac{1}{2}x^2 + c$

(ii) $\frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + c$

(d) (i) $\frac{1}{3}x^2\sin 3x +$

$\frac{2}{9}x\cos 3x -$

$\frac{2}{27}\sin 3x + c$

(ii) $-x^2\cos x + 2x\sin x + 2\cos x + c$

(e) $x^2e^{\frac{x}{4}} - 8xe^{\frac{x}{4}} - 32e^{\frac{x}{4}} + c$

(f) $-\frac{\ln x}{2x^2} - \frac{1}{4x^2} + c$

(g) $x(\ln x)^2 - 2x\ln x + 2x + c$

2. (a) $x\arctan x - \frac{1}{2}\ln(1+x^2) + c$

(b) $x\ln(2x+1) - x + \frac{1}{2}\ln(2x+1) + c$

3. (a) $\frac{\pi}{2} - 1$ (b) $\frac{1}{2}(1 - \ln 2)$

(c) $\frac{1}{2}(1 - \ln 2)$

4. No answers required

5. $-\frac{2}{3}xe^{-3x} - \frac{2}{9}e^{-3x} + c$

6. $\frac{5e^6 + 1}{36}$

7. (a) Proof

(b) $x\tan x - \ln|\sec x| + c$

8. 0.360 [7]

9. $e^2 + 1$

Mixed examination practice 19

Short questions

- $\frac{\pi}{2}$
- $\frac{1}{2}x \sin 2x + \frac{1}{4} \cos 2x + c$
- 6.36
- $\frac{\sqrt{3}}{4}$
- (a) $-\frac{1}{3} \ln |1 - 3x| + c$
(b) $-\frac{1}{2} (2x + 3)^{-1} + c$
- $x \ln x - x + c$
- (a) $\frac{e^{-2x}}{e^{-2x} + 3}$
(b) $\frac{1}{2} \ln \left| \frac{1}{e^{-2x} + 3} \right| + c$
- $3 \ln |x^2 + 4| + 2 \arctan \left(\frac{x}{2} \right) + c$
- (b) $\ln \frac{7}{4}$
- $\ln (\ln |x|) + c$
- $\frac{8}{3} \left(\frac{1}{2}x - 1 \right)^{3/2} + 8 \left(\frac{1}{2}x - 1 \right)^{1/2} + c$
- $3 - 3 \ln \left(\frac{7}{4} \right)$
- $x \arctan x - \frac{1}{2} \ln |1 + x^2| + c$
- $\frac{e-1}{e+1}$

Long questions

- (a) $A = 2$ $B = 2$
(b) $2 \ln |x + 2| - \ln |x^2 + 1| + \arctan x + c$ (c) $\frac{4\pi}{3} - \frac{3}{2}$

- (a) $x + c$ (b) $\ln |\sin x + \cos x| + c$
(c) $\frac{1}{2} (x - \ln |\sin x + \cos x|) + c$
- (a) $\frac{1}{2} (1 + t^2)$ (c) 1
- (c) (i) $\frac{1}{32} \sin 4a + \frac{1}{4} \sin 2a + \frac{3}{8} a$
(ii) 2.96

Chapter 20

Exercise 20A

- (a) (i) $144x^3$ (ii) $6x^2(x^3 + 1)$
(b) (i) $-6x \sin(3x^2)$ (ii) $2x \sec^2(x^2 + 1)$
- (a) (i) 50 (ii) -12
(b) (i) -6 (ii) 1
(c) (i) $\pm \frac{1}{3}$ (ii) -2
- (a) (i) 45 (ii) 176
(b) (i) 0.24 (ii) 0.00667
- $113 \text{ cm}^2 \text{ s}^{-1}$
- 2 cms^{-1}
- 2 cms^{-1}
- 8.92 cm
- 19.1 kmh^{-1}

Exercise 20B

- (a) (i) $v = -8e^{-2t}$, $a = 16e^{-2t}$
(ii) $v = -6e^{3t}$, $a = -18e^{3t}$
(b) (i) $v = \frac{5}{2} \cos \left(\frac{t}{2} \right)$, $a = -\frac{5}{4} \sin \left(\frac{t}{2} \right)$
(ii) $v = 6 \sin(2t)$, $a = 12 \cos(2t)$
- (a) (i) $t^3 - t$ (ii) $\frac{1}{2}t - \frac{1}{8}t^4$
(b) (i) $2 - 2e^{-t}$ (ii) $t + \frac{1}{2}e^{2t} - \frac{1}{2}$
(c) (i) $3 \ln \left(\frac{t+2}{2} \right)$ (ii) $3t - \ln(t+1)$
- (a) (i) 1.73 (ii) 3.16
(b) (i) 2.22 (ii) 0.746
(c) (i) 3.23 (ii) 7.06

4. (a) (i) $-\frac{2}{81}$ (ii) $\frac{9}{2}$
 (b) (i) $\frac{6}{125}$ (ii) $60e^{-\frac{10}{3}}$
6. (a) $-\frac{t^2}{(t^2+1)^2}$ (b) $\frac{1}{2}\ln(26) = 1.63$ (3SF)
7. 10.3 m
8. (a) 13.6 m (b) 16.4 m
9. 6.25
10. 0.0733
11. (a) 1.04 ms^{-1} (b) 0.319 ms^{-1}

Exercise 20C

1. (a) (i) 304.8π (ii) $\frac{18}{7}\pi$
 (b) (i) $\left(\frac{e^4}{4} + e^2 - \frac{1}{4}\right)\pi$ (ii) $\left(\frac{5}{2} - 4e^{-2} - \frac{e^{-4}}{2}\right)\pi$
 (c) (i) 2π (ii) 2π
2. (a) (i) 101 (ii) 134
 (b) (i) 12.6 (ii) 45.7
 (c) (i) 3.59 (ii) 0.771
3. 19.0
4. π
5. $\pi(e^4 - 1)$
6. (a) (i) $y = -\frac{h}{r}x + h$
7. 7.17
8. $\frac{243\pi}{10}$
9. $\sqrt[3]{\frac{4}{3}}$
10. 1.02
11. (a) (1,4), (9,12) (b) 154
12. (a) (0,3), (4,19) (b) 630
13. $\frac{5e^2 - 3}{6}\pi$

Exercise 20D

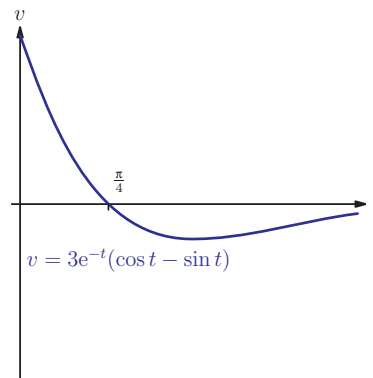
1. (a) (i) 2 (ii) $\frac{49}{12}$
 (b) (i) $2\sqrt{3}$ (ii) $4\sqrt{2}$
 (c) (i) $4\sqrt{2}$ (ii) $\frac{8}{3}$

2. (a) $210x - 2x^2$ (b) $x = 52.5, y = 105$
3. (a) $x(12 - 2x)^2$ (b) $x = 2$
4. 48
5. $\left(\frac{2\sqrt{3}}{3}, \frac{8}{3}\right)$
6. (a) (i) $(\pi - x, \sin x)$ (ii) $(\pi - 2x)\sin x$
 (c) 1.12
7. 889.5 cm^3
8. 788.3 cm^3
9. (a) 3 and 3 (b) 0 and 6
10. (b) $r = 5.56, h = 7.86$
13. $\left(\sqrt{7/2}, \frac{7}{2}\right)$

Mixed examination practice 20

Short questions

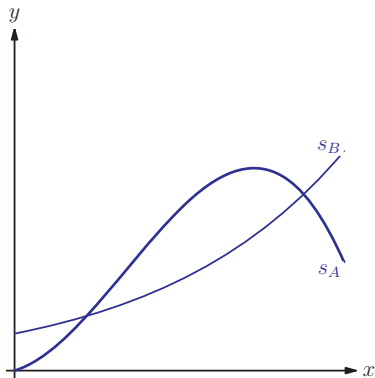
1. $\frac{\pi a^5}{30}$
2. (a) 6.25 m (b) 14.25 m
3. $x = y = 4$
4. 20.9 cm
5. (a) -0.169 (b)



6. (a) $100h - \frac{1}{2}h^2$ (c) $k = 6$
7. 240 km/h
8. (b) $\frac{2}{\sqrt{3}}$

Long questions

- (i) $4x$ (ii) $2\pi y$
 - $2 - \frac{\pi y}{2}$ (d) 44.0%
- (0,12) and (4.5, -3.75) (b) 10.125
 - (i) $\pi \int_0^{4.5} (9x - 2x^2)^2 dx$ (ii) 773
- $\frac{\sqrt{5}}{2}$
- (i) 0.5 ms^{-2} (ii) $3(t+1)\ln(t+1) - 3t$
(iii) 17.3 m
 - (i) 3.49 (ii) 8 ms^{-1}
(iii) 7.47 s
- (b) $h = 30 - 2t$ (c) $25 - 6t$
(d) 13 cms^{-1}
- $\frac{e+1}{2}$
- (a) $6 \text{ ms}^{-1} \frac{d^2v}{dt^2} = -1$, hence local maximum
(b) 0.445 ms^{-2}
(c) $s_A = -\frac{1}{6}t^3 + \frac{3}{2}t^2 + \frac{3}{2}t$,
 $s_B = 5e^{0.2t}$
 - (i)



- (ii) 1.95, 7.81
- (ii) 7.69 km
- (i) $AX = \frac{9}{\sin \theta}$, $BX = \frac{\sqrt{3}}{\cos \theta}$
(ii) $8\sqrt{3}$
 - $8\sqrt{3}$

Chapter 21

Exercise 21A

- (a) Discrete (b) Discrete
 - (c) Continuous (d) Continuous
 - (e) Discrete (f) Discrete
- (a) Only shoppers' opinions found.
 - (b) Truants would not be there.
 - (c) People who partake in internet surveys may not be representative.
 - (d) If you live in a big household your chances of being sampled are lower than if you live by yourself.

Exercise 21B

- (i) 7.23 (ii) 6.57
 - (i) 11.1 (ii) 35.3
- (i) $\frac{\sqrt{56}}{5}$ (ii) $\frac{\sqrt{68}}{3}$
- (a) 12 (b) 2.92
- 18
- 4.96
- 11.7
- 118888.2

Exercise 21C

- (a) $\bar{x} = 1, s_n = \frac{2}{\sqrt{15}}$ (b) $\bar{x} = 0, s_n = \frac{\sqrt{28}}{5}$
- (a) $\bar{x} = 12.1, s_n = 1.90$
(b) $\bar{x} = 0.263, s_n = 0.137$
- (a) 17, 21 (b) 17, 20
(c) 16.5, 20.5 (d) 16, 20
- (i) $\bar{x} = 26.1, s_n = 20.4$ (ii) $\bar{x} = 253, s_n = 151$
(b) (i) $\bar{x} = 6.38, s_n = 5.23$

ANSWER HINT(4)

the first group starts at 0, not -0.5!

- (ii) $\bar{x} = 102, s_n = 5.78$
- (c) (i) $\bar{x} = 6.58, s_n = 5.11$ (ii) $\bar{x} = 15.3, s_n = 9.85$